SIGN-ALTERNATING PHASING IN ACCELERATING SYSTEMS OF RF RECTANGULAR APERTURE QUADRUPOLES

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Abstract

Sign-alternating phasing in accelerating systems of RF rectangular aperture quadrupoles is considered. Variants of a focusing period configuration with different field symmetry in accelerating gaps are proposed. A method of analysis and optimization of the radial-phase stability region in a drift tube linac with an arbitrary period structure is developed. On the basis of the method focusing period structure providing the maximum transmission efficiency in the linac is chosen. Comparison of focusing properties of channels with different types of RF focusing is conducted.

Introduction

Beam focusing by RF rectangular aperture quadrupoles proposed by G.M. Anisimov and V.A. Teplyakov [1] in 1963 and later by F. Fer and P. Lapostolle [2] has a merit of high accelerating gradient, large design simplicity of drift tubes, possibility for decreasing of drift tubes diameters (due to lenses absence in them) and no need of complex equipment for lenses supply. However a severe restriction is imposed on phase width of separatrix with application of RF rectangular aperture quadrupoles in an autophasing linac. The problem is caused mainly by strong defocusing action of the accelerating field. To decrease the defocusing factor a proposal to combine the RF quadrupole focusing and sign-alternating phasing have been made more than once [2, 3].

Below variants of a focusing period configuration with different field symmetry in accelerating gaps are proposed. A method of analysis and optimization of the radial-phase stability region in a drift tube linac with an arbitrary period structure is developed. On the basis of the method focusing period structure providing the maximum transmission efficiency in the alternating phase quadrupole focusing (APQF) linac is chosen.

Variants of a focusing period structure

We introduce the following notations for elements composing the APQF linac period. Each element contains a drift half-tube with a slit at the output, an accelerating gap and a drift half-tube with a slit at the input (Fig.1). The elements are classified into quadrupole $Q^i_{x,y}$ and dipole $D^i_{x,y}$ ones depending on the direction of slits at the input and the output of gaps. The subscript x or y is corresponding to the slit direction at the element gap input and the

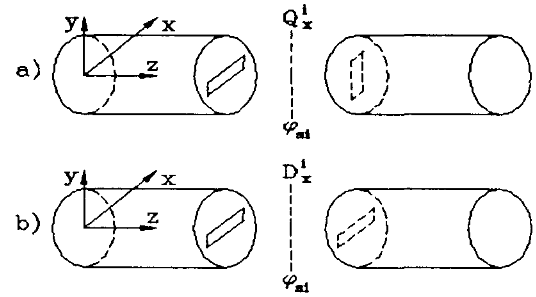

Fig. 1: The quadrupole (a) and dipole (b) elements of APQF linac.

TABLE 1
APQF Period Structures

type of elements	number of elements	structure of period
quadrupole	4	$Q^1_x Q^2_x Q^1_y Q^2_y$
	4	$Q^1_x Q^2_x Q^3_y Q^4_y$
	6	$Q^1_x Q^2_x Q^3_x Q^1_y Q^2_y Q^3_y$
	6	$Q^1_x Q^2_x Q^3_x Q^4_y Q^5_y Q^6_y$
quadrupole and dipole	4	$D^1_x Q^2_x D^1_y Q^2_y$
	4	$D^1_x Q^2_x Q^3_y D^4_y$
	6	$D^1_x Q^2_x Q^3_x D^1_y Q^2_y Q^3_y$
	6	$D^1_x Q^2_x Q^3_x D^4_y Q^5_y Q^6_y$
dipole	4	$D^1_x D^2_y D^1_y D^2_x$
	4	$D^1_x D^2_y D^3_x D^4_y$
	6	$D^1_x D^2_x D^3_x D^1_y D^2_y D^3_y$
	6	$D^1_x D^2_x D^3_x D^4_y D^5_y D^6_y$

superscript i is corresponding to the synchronous phase value φ_{si} in the centre of the element gap. Quadrupole Q^i_x and dipole D^i_x elements are shown in Fig.1. Using the elements configuration of any APQF period may be described. Examples of linac periods containing 4 and 6 gaps are given in Table 1 and in Fig.2.

When the APQF linac is designed the necessity for equalization of tunes on transverse coordinates arises in each focusing period. In the case when the focusing period length S is twice as large than the phasing period length S_φ (Fig.2a) the equality condition of transverse tunes is fulfilled if the accelerating field strength is increased proportionally to the synchronous particle velocity and widthes

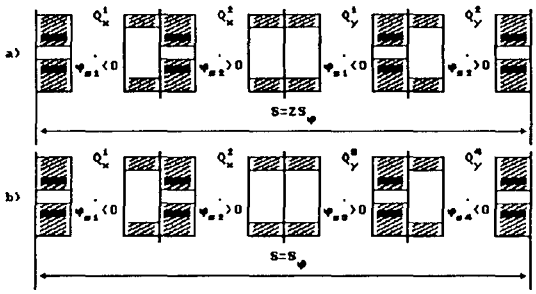

Fig. 2: The examples of APQF period structures with $S = 2S_\varphi$ (a) and $S = S_\varphi$ (b).

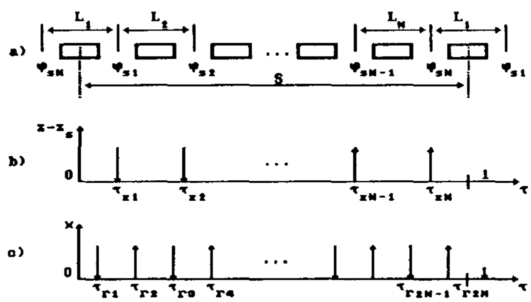

Fig. 3: The drift tube linac period (a) and the structure of phasing (b) and focusing (c) forces.

of phasing (dephasing) gaps are the same. Under $S = S_\varphi$ (Fig.2b) validity of this condition may be provided only approximately and the gap width, the field strength and the synchronous phase are adjusted for each accelerating gap.

Analysis and optimization of a radial-phase stability region in a drift tube linac with arbitrary period structure

We consider a drift tube linac period containing N accelerating gaps with a different symmetry of electric field (Fig.3a). To analyse and optimize phase and radial stability regions we use of thin lens approximation.

In paper [4] a general formula for the potential function of averaged phase particle oscillations in an accelerating system consisting of an arbitrary periodic sequence of N thin lenses located at τ_{zi} (Fig.3b) have been derived

$$V(\psi) = \sum_{i=1}^{N} \eta_{0i}\left[\sin(\psi + \varphi_{si}) - \psi\cos\varphi_{si}\right] - $$
$$-\frac{1}{48}\sum_{i=1}^{N}\sum_{j=1}^{N} \eta_{0i}\eta_{0j}\left(6\tau_{zij}^2 - 6\left|\tau_{zij}\right| + 1\right) \times$$
$$\times\left[4\cos\varphi_{sj}\cos(\psi + \varphi_{si}) - \cos(2\psi + \varphi_{si} + \varphi_{sj})\right] \quad (1)$$

where $\tau = z/S$ is the dimensionless longitudinal coordinate, $\tau_{zij} = \tau_{zi} - \tau_{zj}$, ψ is the phase difference of nonsynchronous and synchronous particles. In the hard edge approximation to the axis field distribution parameter η_{0i} is defined by expressions

$$\eta_{0i} = 4p_{0i}\sin\pi\alpha_i; \quad p_{0i} = \frac{eZE_{gi}\lambda}{2A\mathcal{E}_{0p}\beta_i\gamma_i^3}k_F \qquad (2)$$

where e and $\mathcal{E}_{0p}$ are the charge and the rest energy of a proton, Z and A are the charge and the mass numbers of the accelerated ion, λ is the wavelength of the rf field, E_{gi} is the rf field amplitude in the ith gap, α_i is the ith gap coefficient, β_i and γ_i are the mean reduced velocity and the mean Lorentz factor in the ith cell,

$$k_F = \sum_{i=1}^{N} k_i,$$

k_i is the accelerating mode number of the ith cell.

The phase width of the averaged separatrix is

$$\Delta\Psi_{sep} = \psi_c - \psi_k \qquad (3)$$

where ψ_c is the saddle coordinate of the potential function (1) and ψ_k obeys equation

$$V(\psi_k) = V(\psi_c). \qquad (4)$$

The phase advance of linear longitudinal oscillations per a focusing period is given by [5]

$$\cos\mu_z = 1 + \frac{1}{2}\sum_{i=1}^{N}\xi_{zi} + \frac{1}{2}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\xi_{zi}\xi_{zj}\tau_{zji}\left(1 - \tau_{zji}\right), \qquad (5)$$

$\xi_{zi} = \eta_{0i}\sin\varphi_{si}$ are the refraction coefficients of a longitudinal particle trajectory.

To determine a radial stability region we use a formula for transverse phase advance μ_t (t=x,y) per a focusing period consisting of an arbitrary sequence of thin lenses [5]. When the period contains $2N$ lenses located at τ_{rn} (Fig.3c) the formula takes the form

$$\cos\mu_t = 1 + \frac{1}{2}\sum_{n=1}^{2N}\xi_{tn} + \frac{1}{2}\sum_{n=1}^{2N-1}\sum_{m=n+1}^{2N}\xi_{tn}\xi_{tm}\tau_{rmn}\left(1 - \tau_{rmn}\right) \qquad (6)$$

where $\tau_{rmn} = \tau_{rm} - \tau_{rn}$. In the hard edge approximation to the axis field distribution the refraction coefficients $\xi_{x,yn}$ of a transverse particle trajectory are

$$\xi_{x,yn} = \begin{cases} -h_{x,yn}p_{0i}\cos(\varphi_{si} + \psi - \pi\alpha_i) & \text{if } n = 2i - 1 \\ +h_{x,yn}p_{0i}\cos(\varphi_{si} + \psi + \pi\alpha_i) & \text{if } n = 2i. \end{cases} \qquad (7)$$

The parameters $h_{xn} = 1$ for gaps with axial field symmetry; $h_{xn} = 0, 2$ for gaps with quadrupole and dipole field symmetry; $h_{yn} = 2 - h_{xn}$.

The limits $\psi_m^{\mp}$ of the radial stability region are calculated from conditions $\left|\cos\mu_{x,y}\left(\psi_j^- < 0, \psi_j^+ > 0\right)\right| = 1$;

$$\psi_m^- = \max_j \psi_j^-; \quad \psi_m^+ = \min_j \psi_j^+. \qquad (8)$$

We consider a problem of finding of a maximum size of a radial-phase stability region.

As it follows from (1)–(8) functions $\Delta\Psi_{sep}$ and $\psi_m^{\mp}$ depend on the arguments N, h_{xn}, k_i, φ_{si}, α_i, p_{0i}. We present the quantities φ_{si}, α_i, p_{0i} in the following form $\varphi_{si} = \varphi_0 + \varphi_{Ai}$, $\alpha_i = \alpha_0(1+\varepsilon_{\alpha i})$, $p_{0i} = p_0(1+\varepsilon_{pi})$ where φ_0, α_0, p_0 are the mean values of φ_{si}, α_i, p_{0i} respectively and φ_{Ai}, $\varepsilon_{\alpha i}$, ε_{pi} may be both positive and negative.

Under the given structure of the focusing period (i.e. at the known N, h_{xn}, k_i and α_0) and fixed values of the longitudinal μ_z phase advance and the transverse μ_{xs} phase advance for the synchronous particle the widthes of stability regions depend only on the parameters

$$\varphi_{A1}, \dots, \varphi_{AN-1};\ \varepsilon_{\alpha 1}, \dots, \varepsilon_{\alpha N-1};\ \varepsilon_{p1}, \dots, \varepsilon_{pN-1}. \qquad (9)$$

Now the optimized problem may be formulated. Varying the parameters of linac period (9) to find the maximum of phase width of separatrix

$$\Delta\widehat{\Psi}_{sep} = \max \Delta\Psi_{sep} = \max(\psi_c - \psi_k) \qquad (10)$$

in which limits radial particle oscillations are stable

$$\psi_m^- \le \psi_k, \quad \psi_c \le \psi_m^+ \qquad (11)$$

and values of phase advances for the synchronous particle in planes $x\tau$ and $y\tau$ are the same

$$\mu_{xs} = \mu_{ys}. \qquad (12)$$

So search of optimal parameters of the focusing period is reduced to the finding of an extremum for function (10) with additional constraints (11), (12) i.e. to the nonlinear programming problem.

Choice of focusing period structure

We consider the APQF periods (Table 1) with the following parameters: $N = 4$, $k_i = 0.5$, $0.1 < \alpha_0 < 0.3$, $\mu_{xs} \approx \mu_z = 60°$, $-90° < \varphi_{Ai} < 90°$, $-0.5 < \varepsilon_{\alpha i} < 0.5$, $-0.5 < \varepsilon_{pi} < 0.5$. The optimization problem (10)-(12) was solved on a mesh with a variable step for each parameter. As studies show a larger value for the permissible amplitude of phase oscillations may be achieved in systems with $S = S_\varphi$ than in systems with $S = 2S_\varphi$. The maximum phase width of separatrix $\Delta\widehat{\Psi}_{sep}$ in that limits radial particle oscillations are stable as a function of mean gap coefficient α_0 is shown in Fig.4 for different configurations of APQF periods with $S = S_\varphi$. Period $Q_x^1 Q_x^2 Q_y^3 Q_y^4$ containing only gaps with a quadrupole field symmetry has the best focusing properties (curve 1, Fig.4) and period $D_x^1 D_y^2 D_x^3 D_y^4$ with a dipole field symmetry has the worst ones (curve 3, Fig.4). Period $D_x^1 Q_x^2 Q_y^3 D_y^4$ involving both quadrupole and dipole gaps occupies an intermediate position by focusing properties. To compare different types of RF focusing function $\Delta\widehat{\Psi}_{sep}(\alpha_0)$ is plotted in Fig.4 also for conventional alternating phase (APF) period (curve 4)

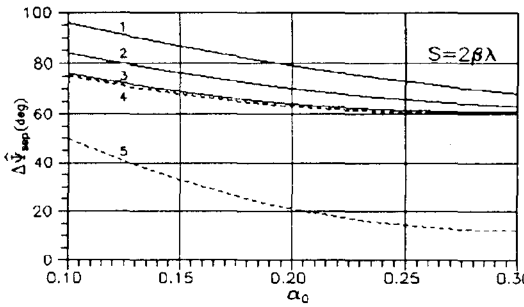

Fig. 4: The phase width of the radial stable separatrix vs the mean gap coefficient for APQF(1–quadrupole, 2–quadrupole and dipole, 3–dipole field symmetry), APF(4) and $Q_x^1 Q_x^1 Q_y^1 Q_y^1$(5) periods.

and for autophasing period $Q_x^1 Q_x^1 Q_y^1 Q_y^1$ (curve 5). The values of $\Delta\widehat{\Psi}_{sep}$ for period $Q_x^1 Q_x^2 Q_y^3 Q_y^4$ are significantly more than for $Q_x^1 Q_x^1 Q_y^1 Q_y^1$ (cf. curves 1,5 in Fig. 4) and exceed the corresponding values for the APF period 13-28% in the studied range of α_0 (cf. curves 1,4 in Fig. 4).

Function $\Delta\widehat{\Psi}_{sep}$ decreases with α_0 for all considered period configurations (Fig. 4). In the APF linac decrease of $\Delta\widehat{\Psi}_{sep}$ may be accounted for by reduction of the radial stability region due to increasing degree of compensation of defocusing and focusing forces at the input and output of drift tubes. In the APQF linac this effect is caused by presence of drift tubes with the same direction of input and output slits. Since there are only two such tubes in period $Q_x^1 Q_x^2 Q_y^3 Q_y^4$ in this case compensation effect has the least influence and curve $\Delta\widehat{\Psi}_{sep}(\alpha_0)$ goes the highest. Hence advantages of the APQF linac (with the period structure $Q_x^1 \dots Q_x^{N/2} Q_y^{N/2+1} \dots Q_y^N$) over the APF linac in the $\Delta\widehat{\Psi}_{sep}$ value must increase with the number of gaps due to decreasing of the relative number of tubes with slits in the same direction.

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