BEAM POSITION MONITOR SENSITIVITY FOR LOW-β BEAMS

Robert E. Shafer, MS H808
Los Alamos National Laboratory
Los Alamos, NM 87545 USA

Introduction

At low velocities, the EM field of a particle in a conducting beam tube is no longer a TEM wave, but has a finite longitudinal extent. The net effect of this is to reduce the coupling of the high-frequency Fourier components of the beam current to BPM (beam position monitor) electrodes, which modifies the BPM sensitivity to beam displacement. This effect is especially pronounced for high-frequency, large-aperture pickups used for low-β beams. Noninterceptive beam position monitors used in conjunction with high-frequency RFQ (radio-frequency-quadrupole) and DTL (drift-tube-linac) accelerators fall into this category.

The source of the effect can be understood by referring to Figure 1. For slow particles, the longitudinal extent of the EM field approaches the electrostatic limit. In this case, the field lines from a point charge fan out longitudinally to satisfy Maxwell's equations at the beam-tube boundary. For slow beams, the field (and beam image current) of a moving particle is a simple nonrelativistic velocity transformation from the electrostatic case. The field lines of a moving charge thus have a finite longitudinal extent, which reduces the high frequency Fourier components of the image currents. For very relativistic ($\beta = 1$) beams, the Lorentz contraction compresses the field lines into a thin pancake, simultaneous with the particle itself. For off-center low-β beams, the azimuthal wall-current distribution is therefore dependent on frequency, particle velocity, aperture, and the beam coordinates.

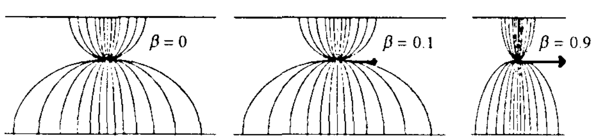

Fig. 1. E-field lines for an off-center point charge in a conducting beam tube for $\beta = 0$, $\beta = 0.1$ ($\gamma^{-1} = 0.995$) and $\beta = 0.9$ ($\gamma^{-1} = 0.44$) particles. Note that the longitudinal extent of the field lines are less, and hence the frequency components are higher, on the beam tube wall nearer the charge.

When testing a BPM with a thin wire excited with either pulses or high-frequency sinusoidal currents, the EM wave represents the principal (TEM) mode in a coaxial transmission line, which is equivalent to a highly relativistic ($\beta = 1$) beam. Thus wire measurements are not suitable for simulating slow particle beams in high-frequency diagnostic devices that couple to the image currents in the beam tube wall. Attempts to load the thin wire either capacitively or inductively to slow the EM wave down have met with limited success.

In general, the analytic expressions in use to represent the response of cylindrical-geometry BPMs to charged-particle beams make several assumptions:

1. The BPM electrodes are flush with and grounded to the surface of the conducting beam tube.
2. The beam is a line source (pencil beam).
3. The longitudinal extent of the EM field of a beam particle at the beam tube wall is zero, corresponding to a highly relativistic beam.

For highly relativistic beams, these simple approximations, based on solving the 2-D Laplace equation, seem to be adequate as long as the beam size is reasonably small compared to the beam tube aperture [1].

The purpose of this paper is to make some quantitative estimates of the corrections to the conventional approximations when a BPM is used to measure the position of low velocity (low-β) beams.

The Low-β Effect

In general, a repetitive time-domain beam current or bunch $I_b(t)$ may be represented by a Fourier series expansion in the frequency domain

$$I_b(t) = \langle I_b \rangle \Big[1 + 2\sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n) \Big] \qquad (1)$$

where $\langle I_b \rangle$ is the average dc current, A_n is a bunch-shape-dependent form factor, ω_0 is the bunching frequency, n is the harmonic number, and ϕ_n is the phase of the n^{th} harmonic. A point charge or very short beam bunch corresponds to $A_n = 1$ and $\phi_n = 0$ for all n. Other bunch shapes (Gaussian, parabolic, triangular, etc.) have values of A_n less than 1 [2].

The rms amplitude of a single frequency harmonic n of the beam current is then

$$I_b(n\omega_0) = \sqrt{2}\,\langle I_b \rangle A_n \qquad (2)$$

Beam position is often determined by the BPM response to a single frequency harmonic. If the BPM electronics responds to a range of frequencies, then the BPM response is calculated by summing the Fourier beam current components over the appropriate frequency harmonics, as per Eq. (1).

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Using a single Fourier frequency component of the beam current, the azimuthal distribution of the beam tube wall image currents may be derived by solving the Laplace equation for an off-center pencil beam with velocity βc. For low-β beams, the 3-D Laplace equation must be used when the current modulation wavelength is comparable to the aperture. For a line current $I_b(\omega)$ at (r,θ), the rms image current density i_w at frequency ω and azimuthal position α on the inner wall of a conducting cylindrical beam tube of radius b is then [3]

$$i_w(\omega,r,\theta,\alpha)=\frac{A_n(I_b)}{\sqrt{2}\pi b}\left[\frac{I_0(gr)}{I_0(gb)}+2\sum_{m=1}^{\infty}\frac{I_m(gr)}{I_m(gb)}\cos[m(\alpha-\theta)]\right] \quad (3)$$

where $I_m(arg)$ represents the modified Bessel function of order m, and

$$g=\frac{2\pi}{\gamma\lambda}=\frac{n\omega_0}{\beta\gamma c}=\frac{\omega}{\beta\gamma c}. \quad (4)$$

Here, λ is the beam current modulation wavelength corresponding to a frequency $\omega = n\omega_0$, and γ is the Lorentz contraction factor. It is important to note that the index n is the frequency harmonic number corresponding to λ, and m is the separation constant resulting from solving the 2-D differential equation in r and θ. To obtain the time-domain image current distribution for a wide-bandwidth BPM, Eq. (3) must be inserted into Eq. (1) and summed over both indices. Note that g contains the index n.

Integrating Eq. (3) over two opposing BPM electrodes of angular width ϕ placed at 0° and 180° as shown in Fig. 2, we get the rms wall image currents I_{WR} and I_{WL} on the R (right) and L (left) BPM electrodes at a single frequency $\omega = n\omega_0$:

$$I_{WR}(\omega,r,\theta,\phi)=\frac{A_n(I_b)\phi}{\sqrt{2}\pi}\left[\frac{I_0(gr)}{I_0(gb)}+\frac{4}{\phi}\sum_{m=1}^{\infty}\frac{1}{m}\frac{I_m(gr)}{I_m(gb)}\sin[m(\phi/2-\theta)]\right]$$

(5), and (6):

$$I_{WL}(\omega,r,\theta,\phi)=\frac{A_n(I_b)\phi}{\sqrt{2}\pi}\left[\frac{I_0(gr)}{I_0(gb)}+\frac{4}{\phi}\sum_{m=1}^{\infty}\frac{1}{m}\frac{I_m(gr)}{I_m(gb)}\sin[m(\pi+\phi/2-\theta)]\right]$$

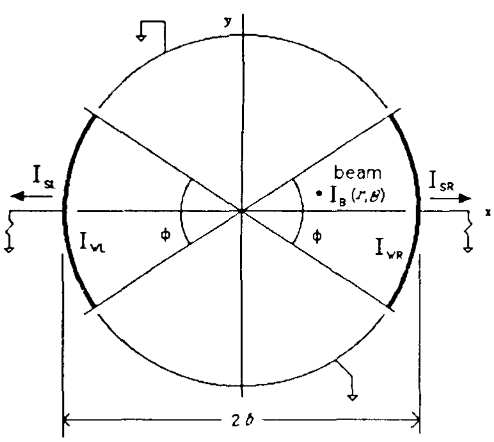

Fig. 2. Cross section of the beam position monitor. The beam current I_b is located at r,θ, the half aperture is b, and the electrode width is ϕ. The induced wall currents on the right and left electrodes are I_{WR} and I_{WL} respectively. These wall currents give rise to the signal currents I_{SR} and I_{SL}.

The dB ratio of the two electrode wall (and hence signal) currents is then given by

$$\left(\frac{R}{L}\right)_{dB}=\left(\frac{I_{WR}}{I_{WL}}\right)_{dB}=20\,\mathrm{Log}_{10}\left(\frac{I_{WR}}{I_{WL}}\right) \quad (7)$$

which for small displacements may be approximated by (where $x = r\cos\theta$)

$$\left(\frac{R}{L}\right)_{dB}=\frac{160}{\mathrm{Ln}(10)}(1+G)\frac{\sin(\phi/2)}{\phi}\frac{x}{b}+O(x^2) \quad (8)$$

where G is approximately

$$G=0.139\left(\frac{\omega b}{\beta\gamma c}\right)^2-0.0145\left(\frac{\omega b}{\beta\gamma c}\right)^3 \quad (9)$$

For the case where $gr < gb << 1$ the modified Bessel functions become

$$\lim_{g\to 0}\frac{I_m(gr)}{I_m(gb)}=\left(\frac{r}{b}\right)^m \quad (10)$$

In this limit, Eqs. (5), (6), and (8) reduce to the $\beta = 1$ approximation (based on solving the 2-D Laplace equation) [4]

$$I_{WR}(\omega,r,\theta,\phi)=\frac{A_n(I_b)\phi}{\sqrt{2}\pi}\left[1+\frac{4}{\phi}\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{r}{b}\right)^m\sin[m(\phi/2-\theta)]\right] \quad (11)$$

$$I_{WL}(\omega,r,\theta,\phi)=\frac{A_n(I_b)\phi}{\sqrt{2}\pi}\left[1+\frac{4}{\phi}\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{r}{b}\right)^m\sin[m(\pi+\phi/2-\theta)]\right] \quad (12)$$

$$\left(\frac{R}{L}\right)_{dB}=\frac{160}{\mathrm{Ln}(10)}\frac{\sin(\phi/2)}{\phi}\frac{x}{b}+O(x^2) \quad (13)$$

This corresponds to $\gamma = \infty$ in the Bessel factor argument. For large displacements, the actual dB ratio of Eqs. (5) and (6) or (11) and (12) should be calculated directly, rather than using Eq (8) or (13). Eq. (11), (12), and (13) represent the induced electrode currents for a TEM signal on a coaxial wire.

Example

As an example of the low-β effect, consider the response of a b = 10-mm effective radius, $\phi = 45°$ wide electrode BPM operating at 100, 200, 400, and 800 MHz to a proton beam ranging in energy from 1 to 1000 MeV. As shown in Figs. 3 and 4, the BPM position sensitivity (dB per mm) *increases* at low beam energies due to the low-β effect. This is in spite of the fact that the *power* coupled into the BPM electrodes *decreases*. Based on Eq. (9), the $\beta = 1$ ($\gamma = \infty$) approximation is satisfactory (< 3 % error) for values of the Bessel function argument less than about

$$gb=\frac{2\pi b}{\gamma\lambda}=\frac{\omega b}{\beta\gamma c}<0.5 \quad (14)$$

An easy mnemonic for low-β beam position monitor design is then to limit the BPM half-aperture b such that

$$b \leq \frac{\gamma \lambda}{4\pi} \tag{15}$$

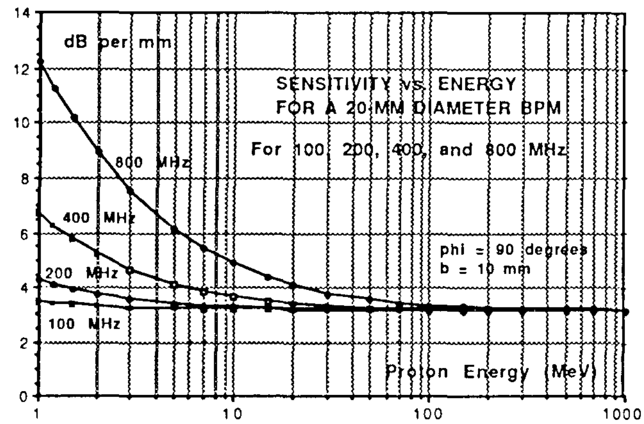

Fig. 3. The displacement sensitivity for a 10-mm radius, 45-degree wide beam position monitor vs. energy for protons bunched at 100, 200, 400, and 800 MHz. The nominal BPM response for highly relativistic (β = 1) particles is about 3.39 dB per mm. For 2.5-MeV protons at 800 MHz, the sensitivity rises to about 9 dB per mm.

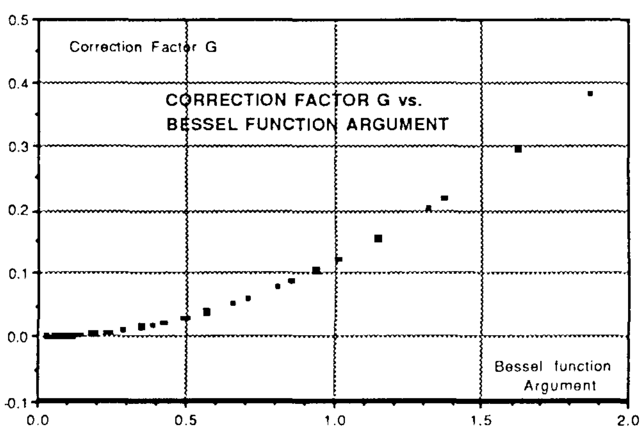

Fig. 4. The correction factor G (Eq. (9)) for the points in Fig. 3, plotted as a function of the Bessel function argument $gb = \omega b / \beta\gamma c$.

Because G in Eq. (8) is a function of frequency, the BPM position sensitivity is frequency dependent. If more than one frequency component is used in the signal processing (e.g., time domain processing), the BPM sensitivity is thus pulse-shape dependent.

Conclusions

1 Low β, high frequency BPMs are required for present-day ion-beam RFQs and DTLs.

2 The position sensitivity (gain) of high-frequency beam-position monitors can be significantly affected (relative to β = 1) when used with low-β ion beams.

3 The position sensitivity is dependent on both the signal processing frequency and the beam velocity, as well as the BPM aperture. If wide-band (time-domain) processing is used, the position sensitivity is pulse-shape dependent.

4 Wire calibrations of the BPM sensitivity cannot measure this effect because the wire signal represents the principal (TEM) mode in a coaxial transmission line that emulates a β = 1 beam.

5 A multiplicative factor that depends on the beam energy, the processing frequency, and the BPM aperture can estimate the magnitude of the effect.

References

[1] R. E. Shafer, "Beam Position Monitoring," in Proceedings of the 1989 Accelerator Instrumentation Workshop (Brookhaven National Laboratory), AIP Conference Proceedings 212 (1990). See Section 5. See also Proceedings of the 1985 Particle Accelerator Conference (Vancouver), p. 1933 (1985).

[2] R. E. Shafer, "Beam Position Monitoring," in Proceedings of the 1989 Accelerator Instrumentation Workshop (Brookhaven National Laboratory), AIP Conference Proceedings 212 (1990). See Table 4.1.

[3] J. H. Cuperas, "Monitoring of Beams at High Frequencies," Nucl. Inst. and Meth. 145, p. 219 (1977). See Eq. (14).

[4] R. E. Shafer, "Characteristics of Directional Coupler Beam Position Monitors," 1985 Particle Accelerator Conference Proceedings (Vancouver), p. 1933 (1985). See Eqs. (4.2), (4.3), and (4.5).