ANALYSIS OF DETUNED STRUCTURE BY OPEN MODE EXPANSION

M. Yamamoto[1], T. Higo, and K. Takata
[1]The Graduate University for Advanced Studies
KEK, National Laboratory for High Energy Physics
1-1, Oho, Tsukuba-shi, Ibaraki-ken, 305, Japan

Abstract

A method called "open mode expansion" was extensively developed to analyze a periodic disk-loaded structure and a detuned structure by including as many higher modes as needed. In this method, the electromagnetic field is developed in terms of the open modes in each cell of the structure. These open modes are treated by taking the coupling between any two modes into account. The coefficients of the expansion can be calculated from an eigenvalue equation. The calculated dispersion relations and kick factors for a uniformly periodic structure excellently agree with those by the field matching technique. The 1200 resonant modes and the resultant wake field in a detuned structure of 150 cells were calculated and the results are presented.

Introduction

In order to achieve a high luminosity, a multi bunch operation of 90 bunches with bunch spacing 1.4 nsec is designed for the Japan Linear Collider (JLC)[1]. In this operation, the emittance growth by the long-range transverse wake field is one of the serious problems. In order to suppress this growth, a detuned structure[2] was proposed where the wake fields of the cells are canceled within a structure with 200 cells by spreading the frequencies of the relevant modes in the cells.

To design a realistic detuned structure for linear collider, the wake field in the structure must be calculated precisely because the cancellation should be of the order of 100.

The wake field in the structure can be estimated in terms of the resonant modes in it. In such a structure as a detuned disk-loaded structure for the linear collider, the long-range transverse wake field is mainly composed of the modes in the lowest two pass bands. In this case, the calculation of only these modes is needed. Based on this understanding, an equivalent circuit model was developed to analyze the detuned structure[3]. However, the importance of the higher modes was also discussed[4]. As an approach which takes them into account, two field matching codes were developed[5,6], though there still need a revision for the realistic analysis of the detuned structure with larger number of cells. In order to incorporate those higher order modes in more realistic manner, an open mode expansion of the electromagnetic field in a cell was developed. We call the method "open mode expansion". This method was originally described by Bevensee and applied to simple cases[7].

A brief description of this method and the result of this method for a periodic structure and a detuned structure are described in the present paper.

Open mode expansion

Infinitely periodic structure

In the case of infinitely periodic structure, all the electromagnetic field can be calculated by only one-cell field with a boundary condition of arbitrary phase advance per period ϕ according to Floquet's theorem. The electric field $\boldsymbol{E}(\boldsymbol{r})$ within a cell can be developed in terms of the open modes,

$$\boldsymbol{E}(\boldsymbol{r}) = \sum_{j=1}^{n} a_j \acute{\boldsymbol{e}}_j \qquad (1)$$

where the a_j is the coefficient of the open mode $\acute{\boldsymbol{e}}_j$ and n is the number of open modes to be included in the analysis. The boundary condition of this open mode $\acute{\boldsymbol{e}}_j$ is defined as magnetic short at both disk holes shown in Fig.1. Following Maxwell's equation and taking some approximations, the eigen value equation can be expressed as the following[8].

$$\omega^2 \boldsymbol{a} = \boldsymbol{X}\boldsymbol{a} \qquad (2)$$

$$\boldsymbol{a} = (a_1, a_2, a_3, \cdots, a_n)^T \qquad (3)$$

$$\boldsymbol{X} = \boldsymbol{X}_0 + \boldsymbol{X}_C \cos(\phi) + i\boldsymbol{X}_S \sin(\phi) \qquad (4)$$

Here ω is a frequency and $\boldsymbol{X}_0$, $\boldsymbol{X}_C$ and $\boldsymbol{X}_S$ are the $n \times n$ matrices. By taking some approximations, matrix $\boldsymbol{X}$ becomes Hermitian even in the case of finite number of modes. In the following, all the calculated results are compared to those by field matching technique, which is precise enough for evaluating the frequencies and kick factors for uniformly periodic structures.

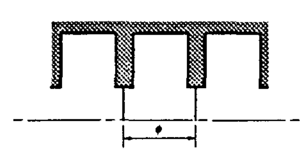


Figure 1: Cell geometry of infinitely periodic structure.

The dispersion relations calculated by this open mode expansion are shown in Fig.2, showing quite good agreement with those of the field matching technique. The disagreement of the highest dispersion curve is due to the process necessary for keeping the matrix $\boldsymbol{X}$ Hermitian.

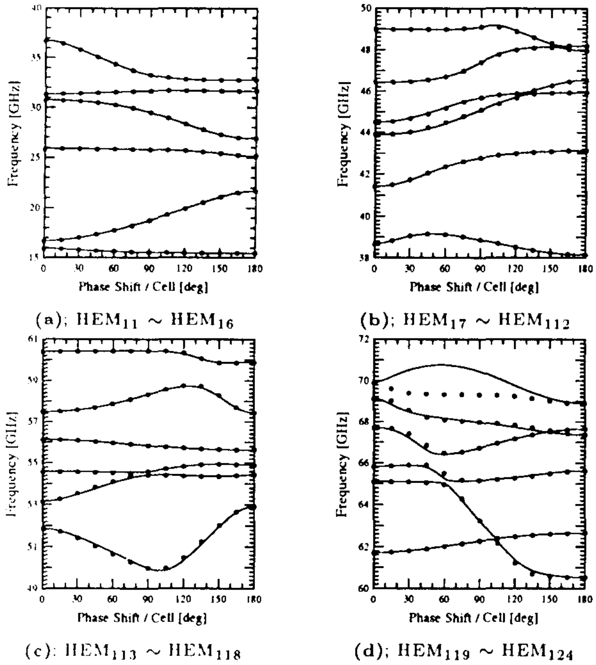


Figure 2: Dispersion relations. Solid lines show the results of the open mode expansion and closed marks show those of the field matching technique.

The kick factor of a dipole mode $\tilde{K}_n$, which shows the strength of the transverse wake field of the mode n, can be calculated as

$$\tilde{K}_n = \frac{c|\int E_z(r=a)e^{-i\frac{\omega_n}{c}z}dz|^2}{4a^2\omega_n U_n}, \tag{5}$$

where $E_z(r=a)$ is the z component of the electric field at disk radius a, ω_n and U_n are the angular frequency and the stored energy of the mode, respectively, and c the speed of light. The kick factors calculated for the modes in the two lowest passbands by the open mode expansion are compared with those of the field matching technique in Fig.3. As shown in the figure, a larger number of open modes should be included to obtain the kick factors which agrees well with those of the field matching technique.

From the above results, the open mode expansion was confirmed to be applicable to rigorous calculation of the frequencies and the kick factors of the infinitely periodic structure.

Detuned structure

In an aperiodic structure such as a detuned structure, the electric field $\boldsymbol{E}(\boldsymbol{r})$ of the resonant modes can also be developed in terms of the open modes in all of the cells constructing the structure as the following[8].

$$\boldsymbol{E}(\boldsymbol{r}) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\acute{\boldsymbol{e}}_{ij}, \tag{6}$$

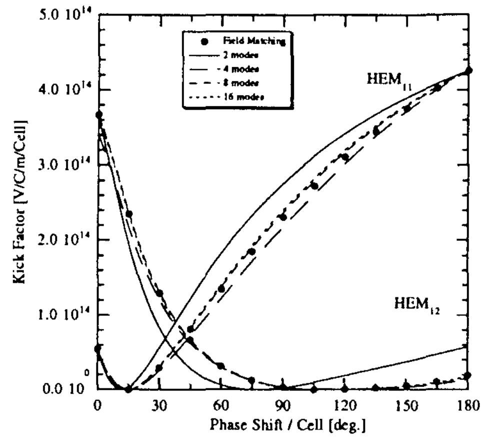


Figure 3: Kick factors of the two lowest modes. Lines show the results of open mode expansions with various numbers of modes. The closed circles show those of the field matching.

where a_{ij} is the coefficient of the j'th open mode $\acute{\boldsymbol{e}}_{ij}$ in the i'th cell, n the number of open modes to be included in the analysis and m the number of cells constructing the structure. The assumed cell geometry is shown in Fig.4. In this case, the coefficients of the open modes a_{ij} and the frequencies of the resonant modes become eigenvectors and eigenvalues of the eigenvalue equation of the same form as Eq.2[8]. The $\boldsymbol{X}$ in this eigenvalue equation becomes the $mn \times mn$ symmetric matrix.

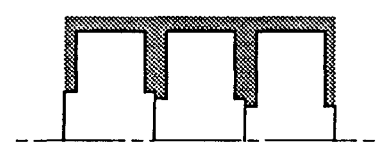


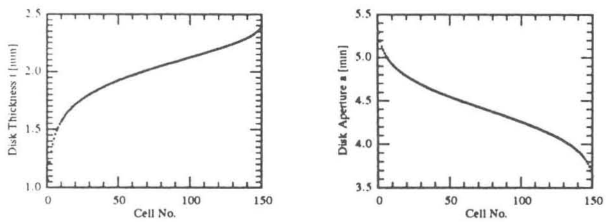
Figure 4: Cell geometry in aperiodic structure.

The calculation of the kick factors in a 150-cell detuned structure, whose disk dimensions are shown in Fig.5, was carried out. In the present paper, the electric field in a cell was expanded by the eight open modes. The 1200 resonant modes were calculated with about 15 minutes CPU time on HITAC M880. The obtained kick factors are shown in Fig.6. As shown in the figure, it was found that there were eight relatively large peaks corresponding to the eight regions estimated from the dispersion curves in infinitely periodic structure. The coefficients of the open mode expansion in two typical resonant modes are shown in Fig.7. It is evident from this figure that a large number of open modes is needed for the calculation of the higher modes.

From the obtained kick factors, the wake function was calculated using the equation,

$$W(t) = \sum_{j=1}^{mn} 2\tilde{K}_j \sin(\omega_j t). \qquad (7)$$

The wake function of the detuned structure thus obtained is shown in Fig.8. The bump around 20 [nsec] is due to the large kick factors of the modes near 36 [GHz] shown in Fig.6. The expansion coefficient of the mode shown in Fig.7 exhibits a large mixing of the 4'th and the 7'th open modes into the 6'th mode.



(t): Disk thickness distribution (a): Disk aperture distribution

Figure 5: The dimensions of disks of the detuned structure.

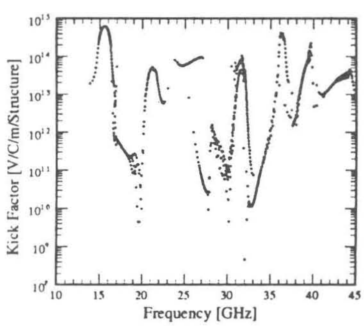


Figure 6: Kick factors of the detuned structure.

Summary

- The open mode expansion was developed to estimate the dispersion relations and the kick factors of the infinitely periodic structure incorporating the modes as high as needed.
- Based on the above method for periodic structure, the open mode expansion was further developed to estimate the wake field of the detuned structure.

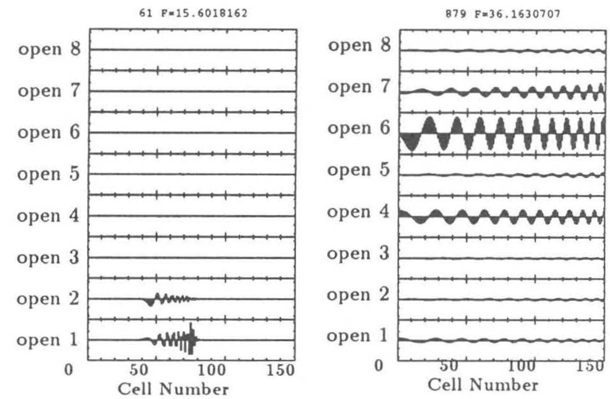


Figure 7: The coefficients of the open mode expansion of the 61'th (15.6GHz, left) and 879'th (36.2GHz, right) modes of the detuned structure.

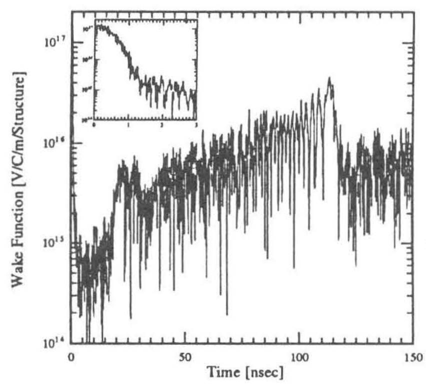


Figure 8: Wake function of the detuned structure.

References

[1] JLC Group, "JLC-I", KEK Report 92-16, 1992.
[2] H. Deruyter *et al.*, "Damped and Detuned Accelerating Structure", Linear Acc. Conf., Albuquerque, USA 1990 and SLAC-PUB-5322.
[3] K. L. F. Bane and R. L. Gluckstern, "The Transverse Wake field of a Detuned X-band Accelerator Structure", Part. Accel. p123 42 1993.
[4] K. Bane, Int. Workshop on Emittance Preservation in Linear Collider, Tsukuba, Japan, KEK Proceedings 93-13, 1993.
[5] S. A. Kheifets, *et al.*, "Transverse Electromagnetic Fields in a Detuned X-band Accelerating Structure", presented in Int. Workshop on Next Generation of Linear Colliders, LC93, SLAC, USA, 1993.
[6] U. van Rienen, "Higher Order Mode Analysis of Tapered Disk-Loaded Waveguides using the Mode Matching Technique", Part. Accel. 173 42 1993.
[7] R. M. Bevensee, "Electromagnetic Slow Wave Systems", John Wiley & Sons, Inc., 1964.
[8] M. Yamamoto, Doctor thesis, to be published.