NORMAL MODE ANALYSIS OF THE FOUR ROD RFQ AS A SYSTEM OF TEM TRANSMISSION LINES

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Abstract

The normal mode analysis technique for TEM transmission lines is applied to 4-rod RFQ resonators which consist of four rod electrodes and the electrode-supporting posts. Using this approach, any field in the resonator is expressed by the superposition of four TEM modes. This model is applied to one-section of 4-rod RFQ resonator. Formulas for the resonance frequency, flatness of the longitudinal voltage distribution in RFQ-channel, RF power, Q-value and the specific shunt impedance are obtained. The results of calculations are in good agreement with MAFIA and experimental data. Two modifications of 4-rod RFQ resonator with matching section and with frequency variation are proposed.

Introduction

In this paper we develop normal mode analysis for transmission lines (TL) to be applied to any 4-rod RFQ resonator [1-6]. The 4-rod RFQ resonators consist of two main elements: four quadrupole electrodes (rods) and the electrode-supporting posts (stems). There are many configurations of them, which differ by both shapes of these main resonator elements (circular rods, vane-like rods, "spear-shaped beams" and so on for four quadrupole electrods and straight, triangular and cylindrical stems and spiral supports for the electrode-supporting posts) and their mutual layouts.

According to the previous reports [7,8], these resonators can be simulated by resonant circuits, which consist of 4-conductor shielded transmission line (4CSTL) loaded by sets of the impedance corresponding to the stems. It is supposed that the characteristics of 4-rod RFQ resonators are dominated by the 4CSTL. Concerning supporting posts, only the way of their connection to the 4CSTL and their equivalent impedance are essential.

This approach assumes that only TEM waves propagate along all conductors of resonant circuits. The TEM waves are described by the telegraph equations[9] for voltage and currents on the conductors of TL. The fields of TEM waves at given cross-section can be obtained from a two-dimensional (2D) static problems with boundary condition defined by the voltage on conductors. Because in general case the ratios of voltage change along TL, these 2D-problems also will be different for each cross-section. To avoid the longitudinal dependence of these 2D-problems and transverse field distributions, the normal mode analysis technique for TL is applied. This technique introduces the normal TEM waves, the ratios of currents and voltage of which are constant along TL, that is 2D-problems for the normal waves do not depend on the position along TL. The full field of transmission line is the superposition of these normal modes.

In this paper, *propagating* normal TEM modes and the examples of their superposition corresponding to *resonator* modes of 4-rod RFQ are presented. The quantitative results for one-section of the "Alternate stems 4-rod RFQ resonator" are compared with MAFIA calculations and experimental data. Two modifications of 4-rod RFQ resonator with a "simple" matching section and with frequency variation are also proposed.

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The qualitative consideration of 4-rod RFQ resonators

In a transmission line with N conductors there are N normal TEM modes of propagation[9]. For the case of the 4CSTL [7,8] there are the following four normal TEM modes: the coaxial, quadrupole and two dipole modes (fig.1).

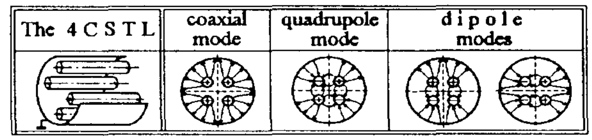

Fig.1. E-lines of the normal TEM modes in the 4CSTL.

In the indefinite 4CSTL, these normal TEM modes can propagate with any amplitudes at any frequencies. But if the conductors of the 4CSTL will be loaded by some impedance, it becomes a resonance circuit with several resonator modes at a discrete set of frequencies, while TEM waves of each *resonator* mode are the superpositions of the *propagating* normal TEM modes of the 4CSTL. The amplitudes of these field components depend on the configurations of resonators. This means that each resonant mode of 4-rod RFQ resonator can be described by the equivalent combination of the normal TEM modes. Therefore, such combination can distinguish a type of a resonator. Some cases of the mode combination in the 4CSTL and corresponding 4-rod RFQ resonators are discussed in the following section.

Figure 2 shows the voltage distributions in the 4CSTL which correspond to normal TEM modes of the indefinite 4CSTL.

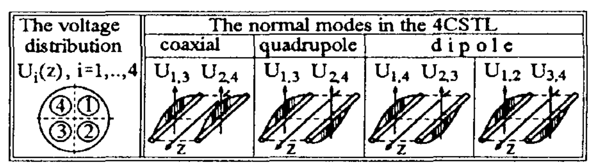

Fig.2. The voltage distribution in the 4CSTL.

The points on conductors with zero voltage can be shorted to the shield providing a resonant circuits. Then non-uniformity of voltage distribution along structure may be reduced by "bending the low voltage parts"[10]. The resulting resonant circuits are the model for "In-line stems RFQ" resonator[8] (fig.3). Thus, this resonator has four dominant resonant modes, while each of them corresponds to one of four normal TEM modes in the 4CSTL.

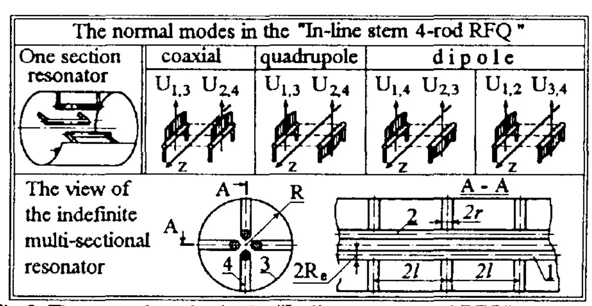

Fig.3. The normal modes in the "In-line stems 4-rod RFQ" resonator.

The practically important case of mode combination is presented in fig.4. This is the superposition of the standing waves of coaxial and quadrulpole TEM modes shifted along longitudinal direction as large as $\lambda/4$ (λ - the wavelength), while the voltage amplitude of coaxial mode is larger by factor $2(Z_1/Z_2)^{1/2}$ than the quadrupole mode, where Z_1 and Z_2 are the wave impedances of the coaxial and quadrupole modes, respectively. The following relation exists $Z_1/Z_2=C_2/C_1$, where C_1 and C_2 are linear capacitances of the coaxial and quadrupole modes, respectively.

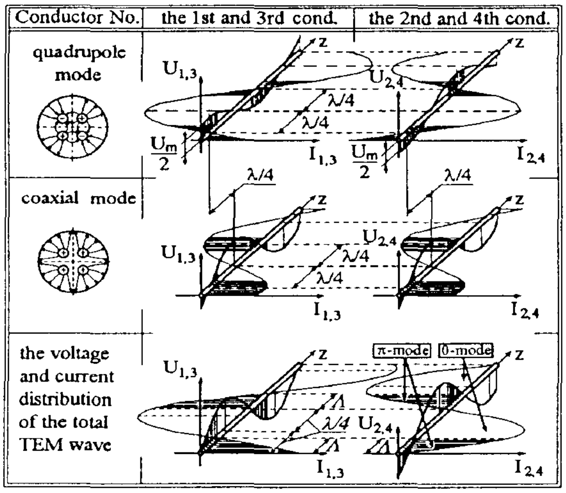

Fig.4. The coaxial and quadrupole mode superposition in the 4CSTL which corresponds to the "Alternate stems 4-rod RFQ" resonator.

As a result from figure 4, there is a set of transverse planes perpendicular to the z-axis of the 4CSTL in which either voltage or current on each conductor of the 4CSTL is equal to zero (fig.5). Then in these planes, the conductors with zero voltage are connected to the shield and the conductors with zero current are cut to open-circuits. The segments between neighboring planes are resonant circuits modeling the one section of the "Alternate stems 4-rod RFQ " resonator. The short segments with length $\Lambda(\Lambda<\lambda/4)$ correspond to π-mode relative to neighboring rods (RFQ operating mode) and the long ones with length $\lambda/2-\Lambda$ correspond to 0-mode (in-phase neighboring rods). The sum of the lengths of π and 0 mode segments is equal to exactly $\lambda/2$. It should be noted, that the principal difference of these segment lengths from $\lambda/2$ is caused by the difference of the wave impedance between coaxial and quadrupole modes.

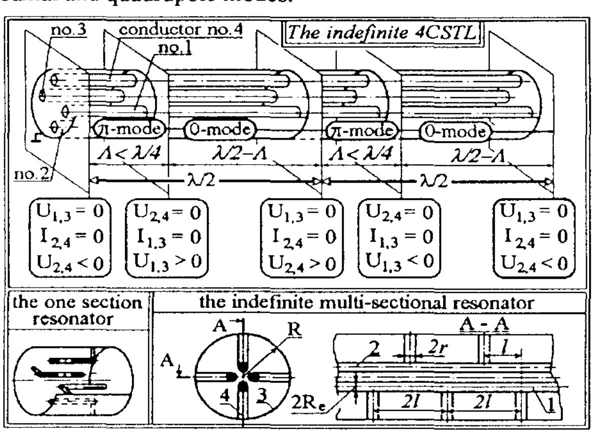

Fig.5. The transformation from the 4CSTL's resonant circuits to the "Alternate stems 4-rod RFQ" resonator.

In contrast to above, the analogous combination of two dipole modes, which have exactly the same wave impedance, results in the segments with equal length $\Lambda=\lambda/4$. These segments are the resonant circuits modeling the two dipole resonator modes with the same frequencies, fields of which is different only by the space rotation. Thus, one section of the "Alternate stems 4-rod RFQ" resonator has four dominant resonator modes. Two combinations of coaxial and quadrupole normal TEM modes correspond to π and 0 resonator modes and others two combinations correspond to two dipole modes. For one section with length l it can be obtained that $f_\pi+f_0=2f_d$, where f_π, f_0 and f_d are frequencies of π-mode, 0-mode and dipole ones, while $f_d \approx (c/4l)$, where c is velocity of light.

One section of the "Alternate stems 4 rod RFQ" resonator

The geometry and the equivalent circuit of the one-module of the "Alternate stems 4-rod RFQ" resonator is shown in fig.6. The resonator consists of four non-modulated round rods in a cylindrical tank. It is assumed that dipole resonator modes are absent[8], i.e. currents and voltages of opposite rods are the same. Then, the two opposite electrodes can be treated as a single conductor. Hence, we have the section of the two-conductor shielded transmission line (2CSTL), which is terminated by the opened or shorted circuits.

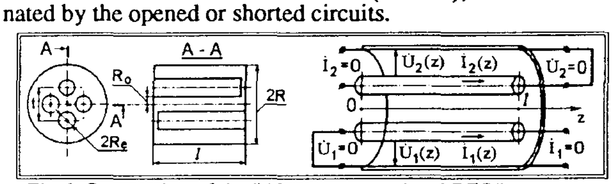

Fig.6. One section of the "Alternate stem 4-rod RFQ" resonator.

The solution of telegraph equations for voltage and current on the conductors of the 2CSTL with corresponding boundary conditions (see fig.6) gives the longitudinal distribution of voltage and currents (as example, see hatched areas in fig.4), resonance conditions for l/λ and voltage flatness δ [11,12]:

$$l/\lambda = (1/\pi)\cdot arctg\sqrt{C_1/4C_2}\,;\quad \delta=(1-\chi)/(1+\chi),\ \chi=\cos(kl/2) \quad (1)$$

Power loss P is calculated from the distribution of the surface current density j_s on the resonator surface. For our model, the longitudinal distribution of j_s is determined by the current distribution derived from the telegraph equations. The transverse distribution of js coincides with the surface charge density distribution σ obtained from the solution of the corresponding electrostatic 2D-problems for normal TEM modes (fig.7). The distribution of σ presented in fig.7 shows that the surface current densities on the conductors are not uniform in azimuth. This non-uniformity of current distribution on the conductors of the 4CSTL determined by so-called the proximity effect due to close spacing of the conductors [13]. The total power loss P is the sum of the loss P_1 on the 4CSTL and the loss P_2 on end plates of resonator [12]:

$$P_1/l = A_0\cdot[F_1+F_2]\,;\qquad P_2/l = 8\cdot A_0\cdot C_1\cdot C_2\cdot \ln(R/3R_e)\,;$$

$$F_1=[C_1^2\cdot g_+/\sin^2(kl/2)]\cdot[0.25\cdot K_{1,R}+(R/R_e)\cdot K_{1,S}]\,; \quad (2)$$

$$F_2=[(4\cdot C_2^2\cdot g_-/\cos^2(kl/2)]\cdot[K_{2,R}+0.5\cdot K_{2,S}];$$

$$A_0=(c^2\cdot R_s\cdot U_m^2)/(16\pi\cdot R_e);\ \ g_\pm=0.5\cdot[1\pm\sin(kl)/kl],$$

where R_s is the surface skin resistance; the coefficients $K_{m,n}$ ($m=1$ for coaxial mode or $m=2$ for quadrupole one; $n=R$ for Rods or $n=S$ for Shield) characterize the proximity effect on the conductors of the 4CSTL for each normal mode. The additional RF power loss from this effect is up to 60%. The calculations showed that $P_2<<P_1$ and $F_2<F_1$. Because F_1 and F_2 define power loss from coaxial and quadrupole mode, respectively, it can be concluded that main power loss caused by co-

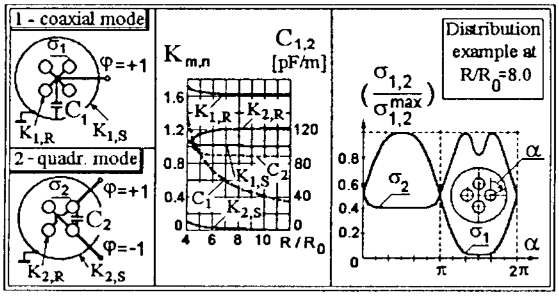

Fig.7. Parameters of coaxial and quadrupole normal modes.

axial mode. It should be noted that coaxial mode doesn't take part in creating RFQ fields, i.e. in this view point it is parasitic, but its presence is essentially necessary providing resonator with small transverse dimensions and available flatness.

The efficiency of RFQ structures is described by the specific shunt-impedance, ρ [1], defined by $\rho=U^2/(2P/l)$, where $U=U_m/\cos(kl/2)$ is the voltage between neighboring rods at $z=l/2$. Q-value is expressed by the next formula [12]:

$$Q_0 = (\pi/8)\cdot f_\pi \cdot \rho \cdot (16C_2^2 - C_1^2)/\sqrt{C_1 \cdot C_2} \qquad (3)$$

For resonator geometry with l=0.28m, R_e=R_0=0.01m, R=0.08m (see fig.6), evaluations according above formulas are compared with calculations provided by MAFIA code [14]. The evaluated values for f_π, Q_0, ρ and δ are 121 MHz, 6700, 83kΩ·m and ±3.1%, respectively, while corresponding values obtained by MAFIA are 119MHz, 6390, 90kΩ·m and ±2.5%. The relation between frequencies of resonator modes is satisfied within 1% (f_d=261MHz, f_0=407MHz, i.e. 526≈522). The experimental data presented in paper of Ref.6 (fig.5) confirms also this relation.

Applications of normal mode technique

As application of the described technique, two modifications of 4-rod RFQ resonators are presented. First one is the 4-rod RFQ with a "simple" matching section. The main idea is illustrated by figures 8 and 9. The cophased superposition of coaxial and qudrupole TEM modes with the same voltage amplitudes results in λ/4-resonator(fig.8). One pair of rods has the zero (as shield) potential and another one has the voltage distribution which is sinusoidally increasing along z-direction.

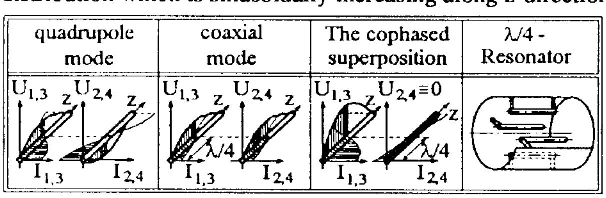

Fig.8. λ/4- resonator with the cophased superposition of the quadrupole and coaxial modes.

Then combination of this λ/4-resonator with the "Alternate stems 4-rod RFQ " resonator produces the modified resonator, which has initial part with increasing RFQ voltage(fig.9). This initial part can be used as matching section. Note, that this matching section is originally free from the unwanted longitudinal fields and has no complicated electrode configuration as the usual "Alternate stems 4-rod RFQ" resonators[15,16]. Unfortunately, the application of the present modified resonator is limited to such a region where the wavelength is around a few meters. It is caused by the fact that the distance between neighboring stems is equal to λ/4 and becomes untolerably long at larger wavelength.

The second modification of 4-rod RFQ resonator is a frequency variable 4-rod RFQ (fig.10). It is the variable capacity

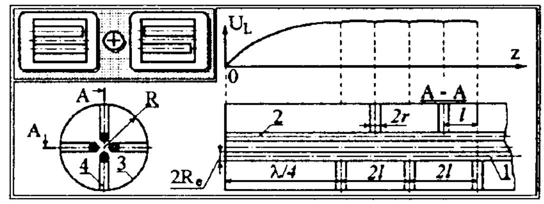

Fig.9. 4-rod RFQ resonator with a "simple" matching section.

method of frequency variation in one section of the "Alternate stems 4-rod RFQ" resonator by four movable vanes. The main idea is the following. According to formula (1) the wavelength is defined by the ratio of linear capacities C_1 and C_2 (or the ratio of the wave impedance of coaxial and quadrupole modes). The effective way of the ratio variation is to increase of one value reducing or at least keeping to be constant another value. In present method, the displacement of four conductive vanes at planes corresponding to the symmetry planes of normal modes disturbs only coaxial mode changing its wave impedance (or capacity C_1).

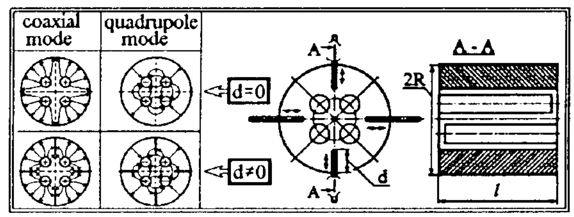

Fig.10. The frequency variable 4-rod RFQ resonator.

Conclusion

The presented equivalent circuit model of 4-rod RFQ resonators has given a clear interpretation of laws and mechanisms governing the physical phenomena in these resonators. This model allows classification of the resonant modes of 4-rod RFQ resonators on the basis of the ratio of the components of the TEM field. The qualitative and quantitative consideration is in good agreement with MAFIA code calculations and experimental data. This approach is to help designing and optimization of 4-rod RFQ resonators.

References

[1] H.Klein, IEEE Trans., vol.NS-30, No.4, pp. 3313-3322, 1983.
[2] A.Schempp, Proc.1992 L.A.C., Ottawa, pp.545-549, 1992.
[3] S.Arai, Inst. for Nucl. Study, techn. report INS-T-464, 1986.
[4] A.Moretti et al., Proc. 1981 L.A.C., Santa Fe, pp.197-199, 1981.
[5] R.M.Hutcheon, Proc. 1984 L.A.C., Darmstadt, pp. 94-96, 1984.
[6] Y.Katayama and H.Takekoshi, Bull.Inst.Chem.Res., Kyoto Univ., vol.61, No.1, pp.1-11, 1983
[7] V.Kapin, Report's Thesises of the 12 USSR Linac Seminar, Kharckov,1989, p.61(*in Russian*).
[8] V.Kapin, presented in EPAC-94.
[9] A.D.Grigorjev, Electrodimamika i tehnika S.V.Ch. (Electrodynamics and RF Technique), M., Vish.shk., 1990 (*in Russian*).
[10] G.Dôme, In.: "Linear Accelerators", Amsterdam, 1970, p.710.
[11] V.Kapin, Report's Annotations of the 11 USSR Linac Seminar, Kharkov,1989, p.39 (*in Russian*).
[12] V.Kapin, Radiation acceleration complexes: collected scientific articles, Moscow: MEPhI, 1991, p.49-58.(*in Russian*).
[13] G.S.Smith and J.D.Nordgard, IEEE Trans., vol.MTT-28, No.8, pp. 887-893, 1980.
[14] R.Klatt et al., Proc.1986 L.A.C., Stanford, pp.276-278, 1986.
[15] S.Arai, GSI report, GSI-83-11, October 1983.
[16] V.A.Andreev et al., Proc.1993 P.A.C., pp.3121-3123,1993.