PISCES II : 2.5-D RF CAVITY CODE, AN EXTENSION OF SUPERFISH

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Abstract

The RF cavity code PISCES II can evaluate all the eigenfrequencies and fields for arbitrarily shaped axially symmetric RF cavities. The solutions include symmetric (m=0) and asymmetric modes (m>0) with 2.5-D technique of assuming the sin $m\theta$ and cos $m\theta$ dependencies. Using Finite Element Method with Nedelec elements, the electric or magnetic components ((E_θ,Er,Ez) or (H_θ,Hr,Hz)) are calculated[1]. The resulted eigenvalue system has many zero-eigenvalue solutions, which can be filtered out by using zero-filter technique from the set of solutions. The eigensolutions of the specified number are obtained simultaneously from non-zero lowest frequency.

Introduction

The original PISCES code was written in early 1980's for studying DISK-AND-WASHER (DAW) structures which have dipole modes close to the operating frequency. Because SUPERFISH can calculate only axi-symmetric modes, the results were not enough for analyzing such cavities. ULTRAFISH was once planned to develop at LosAlamos, but has not been available yet. Because URMEL[2] was developed and began to be distributed widely shortly after the DAW study started, PISCES had been abandoned since then. Because URMEL uses rectangular mesh for the calculation, the approximated boundary is different from that of the SUPERFISH, and the comparison is not straightforward. Recently DAW research is restarted, and PISCES is written for studying DAW.

Formulation

The differential equation for electric field $\vec{E}$ or magnetic field $\vec{H}$ to be solved are [3,4],

$$\nabla\times\nabla\times\vec{E}+k^2\vec{E}=0,\ \nabla\cdot\vec{E}=0\ (\text{in}\ \Omega), \tag{1}$$

or,

$$\nabla\times\nabla\times\vec{H}+k^2\vec{H}=0,\ \nabla\cdot\vec{H}=0\ (\text{in}\ \Omega), \tag{2}$$

where $k^2=\omega^2\varepsilon\mu$ and Ω is the entire volume. In vacuum space, $k^2=\omega^2/c^2$, where c is the speed of light. Boundary conditions are

$$\vec{E}\times\vec{n}=0 \quad \text{or} \quad \vec{H}\times\vec{n}=0 \tag{3}$$

on electric boundaries (Γe) for metal surfaces,

$$\vec{E}\cdot\vec{n}=0 \quad \text{or} \quad \vec{H}\cdot\vec{n}=0 \tag{4}$$

on magnetic boundaries (Γm) for symmetry plane, and

$$E_{left}=e^{i\varphi}E_{right} \quad \text{or} \quad H_{left}=e^{i\varphi}H_{right} \tag{5}$$

on periodic boundaries (Γp), where $\vec{n}$ denotes the outward normal on the boundary., and φ is the phase advance in the cell.[5,6] The periodic boundary is not a real boundary but only for a convenience of defining a problem. Because either $\vec{E}$ or $\vec{H}$ can be used as the field variable, only the electric field will be shown hereafter. Integrating Eq.(1) over Ω after multiplying by $\delta\vec{E}$ (virtual electric field), we get

$$\int_\Omega \delta\vec{E}\cdot\nabla\times\nabla\times\vec{E}dv=-k^2\int_\Omega\delta\vec{E}\cdot\vec{E}dv, \tag{6}$$

and applying Green's theorem, the following relations must hold for any δE :

$$\oint_\Gamma\left(\nabla\times\vec{E}\right)\times\delta\vec{E}\,d\vec{s}-\int_\Omega\left(\nabla\times\vec{E}\right)\cdot\left(\nabla\times\delta\vec{E}\right)dv =-k^2\int_\Omega\delta\vec{E}\cdot\vec{E}dv \tag{7}$$

$$\vec{E}\times\vec{n}=0 \text{ and } \delta\vec{E}\times\vec{n}=0 \text{ on } (\Gamma e), \tag{8}$$

$$\vec{E}\cdot\vec{n}=0 \text{ and } \delta\vec{E}\cdot\vec{n}=0 \text{ on } (\Gamma m), \tag{9}$$

The term in the surface integration of Eq. (7) becomes zero on either (Γe) or (Γm) because of the boundary condition of Equ's. (8) or (9).

Finite Element Model

Because only the axisymmetric boundary problems are considered, we can assume sin $m\theta$. and cos $m\theta$ dependencies of E_r, E_z and E_θ components, and then the problem can be reduced to two-dimension problem:

$$E_\theta \sin m\theta,\ E_r \cos m\theta,\ E_z \cos m\theta \tag{10}$$

Then (E_θ, E_r, E_z) are only functions of r and z. The field variables are (rE_θ, E_r, E_z) for $m\geq1$ and (E_θ, E_r, E_z) or (E_θ, H_θ) for m=0. The case for $m\geq1$ will be explained here.

Only the triangular element is used in PISCES II. The shape functions used are the conventional simple linear one for the θ component and two dimensional Nedelec elements[7,8] for the E_r, E_z components. (See Fig. 1) Only the tangential component of E_r, E_z is assigned on the line, and is constant along the line. Then $\vec{E}$ and $\nabla\times\vec{E}$ can be written as

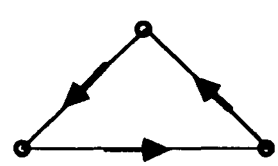

Fig. 1 Finite Element. θ components are assigned at the vertices. The tangential components are assigned on the line.

$$\vec{E}=\begin{bmatrix}E_\theta\\ E_r\\ E_z\end{bmatrix}=\vec{N}\cdot\begin{bmatrix}r\vec{E}_\theta\\ \vec{E}_{rz}\end{bmatrix}=\begin{bmatrix}\frac{1}{r}\vec{N}_\theta & 0\\ 0 & \vec{N}_r\\ 0 & \vec{N}_z\end{bmatrix}\cdot\begin{bmatrix}r\vec{E}_\theta\\ \vec{E}_{rz}\end{bmatrix}, \text{ and} \tag{11}$$

$$\nabla\times\vec{E}=\vec{N}'\cdot\begin{bmatrix}r\vec{E}_\theta\\ \vec{E}_{rz}\end{bmatrix}=\begin{bmatrix}0 & \partial_z\vec{N}_r-\partial_r\vec{N}_z\\ \frac{-1}{r}\partial_z\vec{N}_\theta & \frac{1}{r}\partial_\theta\vec{N}_z\\ \frac{1}{r}\partial_r\vec{N}_\theta & \frac{-1}{r}\partial_\theta\vec{N}_r\end{bmatrix}\cdot\begin{bmatrix}r\vec{E}_\theta\\ \vec{E}_{rz}\end{bmatrix}$$
$$=\begin{bmatrix}0 & \partial_z\vec{N}_r-\partial_r\vec{N}_z\\ \frac{-1}{r}\partial_z\vec{N}_\theta & \frac{-m}{r}\vec{N}_z\\ \frac{1}{r}\partial_r\vec{N}_\theta & \frac{m}{r}\vec{N}_r\end{bmatrix}\cdot\begin{bmatrix}r\vec{E}_\theta\\ \vec{E}_{rz}\end{bmatrix}, \tag{12}$$

where $\vec{N}$, $\vec{N}'$, $\vec{N}_\theta$, $\vec{N}_z$, and $\vec{N}_r$ are the shape functions, and $r\vec{E}_\theta$ and $\vec{E}_{rz}$ are the field variable.
The element matrix equation is

$$\int_e \vec{N}'^T\cdot\vec{N}'\, rdrdz=-k^2\int_e \vec{N}^T\cdot\vec{N}\, rdrdz, \tag{13}$$

where symbol e is the element volume and T denotes matrix transpose. The integrations are performed numerically up to 11th order precision. The singularity in the integrand on the axis, is not serious because the real divergent terms are eliminated by the boundary condition on the axis. By assembling all element matrices and applying the boundary condition, finally we get the general eigenvalue equation.

$$\vec{M}\cdot\vec{x}=k^2\vec{K}\cdot\vec{x}, \tag{14}$$

where $\vec{M}$ and $\vec{K}$ are large sparse symmetric matrices, and $\vec{x}$ is an eigenvector for the field variables. Usually several eigensolutions starting from the smallest one but zero are of interest. Unfortunately, this eigenvalue problem has many zero eigenvalue solutions, and then special care should be taken.

General Eigenvalue Solver for Large Sparse Symmetric Matrices [9]

Because the matrices are sparse, only non-zero elements are stored by list vector technique. The method for the eigenvalue solver is based on the subspace method[10] and zero filter technique. The conjugate gradient method (CGM) is used to solve the simultaneous linear equations. Unfortunately, the condition number of $\vec{M}$ is getting large as the number of mesh increases, and then the convergence of the CGM goes very slow. Preconditioned CGM (PCGM) which utilizes the preconditioning technique is being considered.

Components of PISCES II

There are three stages in PISCES II. The first one is the preprocessor that prepares the mesh data for the field solver PISCES II. For the time being, the same mesh data is used as POISSON / SUPERFISH Rel. 4 (PS4) [11]. MESHNET reads TAPE35 from LATTICE and writes out an input file for PISCES II. NETREF can modify the input file to subdivide the mesh at any place. Unlike the PS4, PISCES II can handle topologically non-uniform triangular mesh, because of the Finite Element Method. The mesh generator NET is planned to generate the mesh data directly from the input data for AUTOMESH. The third one is the post processor DISPLAY which displays the graphical information of field vectors and so on interactively.

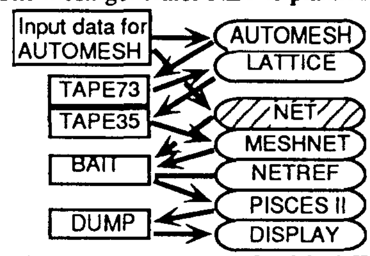

Fig. 2 Components of PISCES II

Examples

The relative error of $k^2=\omega^2/c^2$ of the second lowest mode in the spherical cavity and that of TM_{110} mode in disk-loaded wave guide (the radius of cylinder = 10.779 mm, length of a period = 8.7474 mm, disk thickness = 2.0 mm, radius of beam hole = 4.5 mm) are shown in Fig. 3. The former corresponds to the first mode of the dipole modes (m=1) in PISCES II calculation, and the right frequency can be obtained analytically. The latter was calculated with the periodic boundary at the phase advance of 0° for both electric field and magnetic field as the variables. The frequency of the sphere converges within 1.4×10^{-5} at the mesh size of 0.05 cm, which corresponds to 72568 elements or 144359 unknown variables.

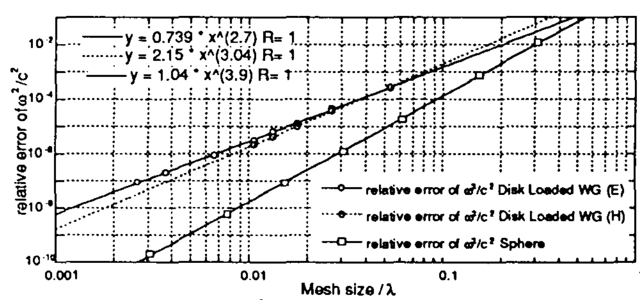

Fig. 3 Relative errors of k^2 of the second lowest mode in a spherical cavity and the lowest dipole mode in a disk-loaded wave guide as a function of the mesh size/λ.

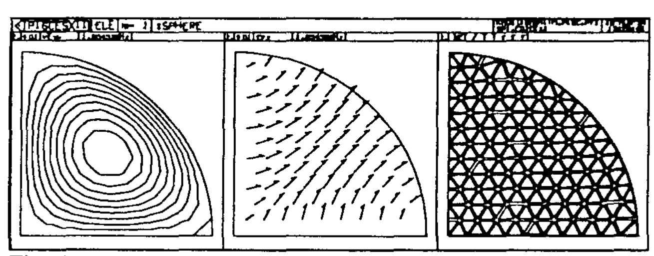

Fig. 4 Right to left: contour plot of rE_θ, arrow plot of Er,Ez vector, and mesh plot for the sphere at mesh size=1cm.

Figure 4 shows the graphical output from DISPLAY for the spherical cavity with a 10 cm radius. Figure 5 shows that for the disk-loaded wave guide with phase advance of 90°. Both the real part and the imaginary part of the first four modes are plotted. There are doubly degenerated modes in the periodic boundary problems except for the phase advance of 0° and 180°. As seen in Fig. 5, the mesh density at each convex corner is increased by NETREF for the higher accuracy. The "converged frequency" f_0 (=15934 MHz) of the disk-loaded wave guide was estimated by extrapolating the calculated frequencies $f(\Delta x)$ to the mesh size of zero assuming the following relation;

$$f(\Delta x) = f_0 + a \cdot \Delta x^b, \tag{15}$$

where a and b are the fitting parameters.

Summary

Finite Element Method with Nedelec element is useful for 2.5D electro-magnetic field analysis of cavity resonators. In the current version, the computing speed is not enough, because the convergence of the CGM depends on the condition number of the matrix , which often becomes large for the higher mesh density problem. Using PCGM is being considered.

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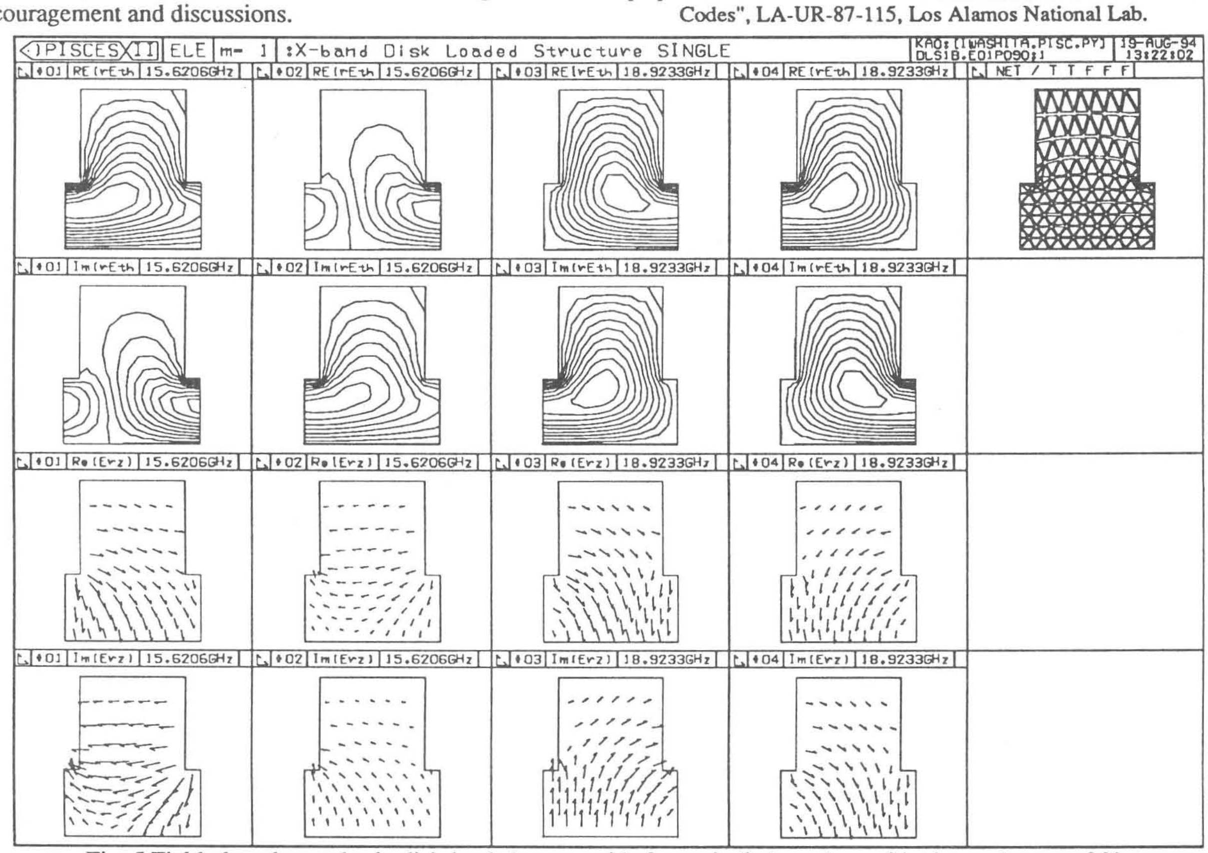

Fig. 5 Field plots the modes in disk-loaded wave guide for periodic boundary with phase advance of 0°.