THE INTERACTION OF THE CHARGED PARTICLES BUNCHES[1] AND WAVEGUIDE - CAVITY STRUCTURES

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Abstract

The charged particles dense short bunches radiation when they are crossing the cavity or passing along the waveguide with the dielectric medium is considered. Two different problems are discussed: the cylindrical finite Q-factor cavity geometry optimization problem and the electromagnetic wake-field formation in the waveguides.

Introduction

Field formation in cavities and waveguides by relativistic electron bunches traversing the cavity or passing along the axis of dielectric-loaded waveguide is due to the well-known effects of Cherenkov or transition radiation (TR). In that case the dispersion and resonance properties of these systems turn out to be determinative in the nature of exciting fields as well as in attaining high effective field strengths in them.

When a cavity is traversed by a pretty long periodic train of bunches, then the cavity resonance excitation at their repetition rate takes place. If the cavity geometry is chosen such that only one of the eigen-modes would exist in it at that frequency, then one can achieve effective energy take-off from the bunch in the form of cavity excitation on this mode. On this ground one can execute optimization of the cavity parameters depending on the parameters of the bunches.

The charged bunch motion in the dielectric at a velocity exceeding the wave phase velocity in the medium is accompanied by Cherenkov radiation which in free space represents a cylindrical linearly polarized wave. In a waveguide this physical phenomenon acquires new properties. If we abstract ourselves from the medium dispersion, we must claim that the whole spectrum of Cherenkov waves will propagate in the waveguide at the same phase and group velocities and induce on its axis considerable electric field strengths as a result of summation of all propagating modes. The picture of the ideal model is broken when dealing with the real medium possessing dispersion as well as in case of a channel cut through in the dielectric along the waveguide axis. Account of the medium dispersion leads, on the other hand, to limitation of the number of Cherenkov modes from above.

The present talk is devoted to the investigation of the mechanism of these physical phenomena.

1. The research described in this publication was made possible in part by Grant No RYO 000 from the International Science Foundation

TR in a Cavity Traversed by a Train of Charged Bunches

Let a cavity be traversed (generally speaking, nonsimultaneously) by two different trains of bunches defined by currents $\vec{j}_1$ and $\vec{j}_2$. Owing to the TR effect these currents will excite fields $\vec{E}_1$ and $\vec{E}_2$ in the cavity. Making use of the Pointing theorem, for the energy change inside the cavity $\partial W/\partial t$ one can write down the following equation (energy balance equation)

$$\frac{\partial W}{\partial t} = -\int_V \vec{j}_1(\vec{E}_1+\vec{E}_2)dV - \int_V \vec{j}_2(\vec{E}_1+\vec{E}_2)dV \qquad (1)$$

In (1) integration is carried out over the cavity volume V.

Following the terminology generally adopted in the wake-field theory, we call the train corresponding to current $\vec{j}_1$ a driving beam and a single bunch corresponding to current $\vec{j}_2$ - a driven beam.

The beam with $\vec{j}_1$ loses energy and gives it to the cavity: $\int_V \vec{j}_1\vec{E}_1 dV < 0$. If then the driven bunch enters the cavity so that the condition $\int_V \vec{j}_2\vec{E}_1 dV > 0$ would be satisfied, it will accelerate in field $\vec{E}_1$, taking off the energy accumulated in the cavity. Simultaneously current $\vec{j}_2$ will excite in the cavity TR fields and $\int_V \vec{j}_2\vec{E}_2 dV < 0$. Suppose that the driven bunch enters the cavity after the latter was traversed by the last bunch of the driving train. Then $\int_V \vec{j}_1\vec{E}_2 dV = 0$. Thus we imply that every bunch from the driving train increases the energy accumulated in the cavity at the expense of its TR whereas the driven bunch falling into the cavity in antiphase with the driving bunches takes off part of that energy and accelerates leaving its portion of TR in the cavity.

Rewrite, therefore, Eq. (1) in the form:

$$\frac{\partial W}{\partial t} = \left|\int_V (\vec{j}_1-\vec{j}_2)\vec{E}_1 dV\right| + \left|\int_V \vec{j}_2\vec{E}_2 dV\right| \qquad (2)$$

The last term in (2) represents energy of the TR generated by the driven bunch in the cavity. If further we assume that in (2)

$$\int_V \vec{j}_1\vec{E}_1 dV = \int_V \vec{j}_2\vec{E}_1 dV \qquad (2a)$$

then the driven single bunch takes off all the energy accumulated by the driving train of bunches in the cavity. From (2) and (2a) we have the following:

- to current $\vec{j}_2$ corresponds the maximum charge which can be accelerated in the field produced by the bunch train;

- after the passage of the driven bunch with maximum charge defined from (2a) the cavity will turn out filled with energy

$$W = \int_t \left| \int_V \vec{j}_2 \vec{E}_2 dV \right| dt \qquad (2b)$$

at the expense of the TR of the driven bunch. In (2b) integration is done over the time of passage of the driven bunch.

The main results of some investigations were reported at the EPAC'94[1]. We would like to mention only a few of them.

Owing to the cavity finite quality factor Q the number of effectively radiated bunches is $N=3Q/\pi k$ ($k= 1,.2,3,...$ is the harmonics number of the driving bunches repetition rate). Then we claim that the train may be assumed large if $N>3Q/\pi k$. Optimizing the geometry of the cavity for each harmonic one may attempt to reach very high strength of the electric field in it. If immediately after the driving train an accelerated point-like bunch flies into the cavity so as it falls into the accelerating field, then on the cavity output one can observe electrons with energy gain. The use of the set of the cavities will result in acquired energy by a factor of n higher. For the different optimizing dimensions of radius R and height a of the copper cavity and for the Yerevan LTF beam parameters [2] (the driving bunches length is 1 cm, radius - 0.5 cm, the number of particles per bunch $\cong 10^9$.) some values are listed in Table 1.

TABLE 1
The Cavity Optimized Parameters

k	R (cm)	a (cm)	Q	$<E_z>$ (MV/m)	dW/dz (MeV/m)
1	3.83	2.75	13 000	205	230
2	1.91	1.275	7 000	228	300
3	1.275	0.7	4 000	200	300

Using (2a) we can estimate the maximum charge which can be accelerated in such a cavity as 10^{12} - 10^{13} particles per bunch.

Vavilov-Cherenkov Effect in a Waveguide

If the Cherenkov radiation in free space represents a cylindrical wave going from the charge trajectory, then in waveguide this wave multiply reflects from its walls, which results in occurrence of wave packets consisting of the waveguide eigenmodes which excite on discrete frequencies [3]

$$\omega_n = \frac{\lambda_n v}{\sqrt{\beta^2 \varepsilon - 1}}, \quad \beta^2 \varepsilon - 1 > 0 \qquad (3)$$

where λ_n - eigenvalues of the waveguide cross-section, v - velocity of a bunch, $\beta = v/c$. The condition of occurrence of Cherenkov radiation in the waveguide is the fulfillment of the equality

$$\omega_n = \gamma_n v, \qquad (4)$$

where $\gamma_n = (\varepsilon\omega_n^2/c^2 - \lambda_n^2)^{1/2}$ is proper constant of wave propagation in the waveguide, and the condition (4) itself is an analogue of the well-known formula from the Cherenkov radiation theory:

$$\cos\theta = 1/\beta\varepsilon^{1/2} \qquad (4a)$$

It follows from (4) and (4a) that the phase and group velocities of the Cherenkov waves respectively are:

$$v_{ph} = v, \quad v_{gr} = v/\beta^2\varepsilon \text{ and } v_{ph}\cdot v_{gr} = c^2/\varepsilon \qquad (5)$$

Thus, if the medium is assumed dispersionless, then all the Cherenkov modes turn out to propagate in the waveguide at the same phase and group velocities and radiate at same angles θ to its axis (4a). As a result of that, the mentioned wave packets will not scatter as the wave propagates, but will form spikes that follow the moving radiating charge at a velocity $v_{gr}<v$. The distance between the charge and the first spike can be found by calculating the time for which the wave propagating at a velocity $c/\varepsilon^{1/2}$, reflecting from the walls falls at the axis:

$$\Delta t = \frac{2L}{v_{gr}\sqrt{\beta^2\varepsilon - 1}},$$

where L - distance between the trajectory of the charge motion and the waveguide walls.

The distance between the radiating charge and the first spike we can obtain multiplying that time by the difference between the phase and group velocities (since $v_{ph}=v$):

$$\Delta z = \Delta t(v - v_{gr}) = 2L\sqrt{\beta^2\varepsilon - 1}.$$

The subsequent spikes turn out really equidistant, which can be illustrated by the following qualitative reasons:

$$\gamma_n\ \Delta z = 2L\lambda_n$$

In case of, for example, a round cylindrical waveguide with circular cross-section of radius R , $\lambda_n = \mu_{0n} R$, where μ_{0n} is the *m-th* root of the zero-order Bessel's function, which is well approximated by formula $\mu_{0n} \cong \pi(n - 1/4)$. Then

$$m\cdot(\gamma\Delta z) \cong 2\pi m(n - 1/4) \qquad (6)$$

The phases of all the modes forming the first spike differ from those of the second spike by $\pi/2$, and from (6) follows that wave phase relations will recur for each fifth spike (m=4). Note that such a picture is peculiar to namely circular waveguide wherein the reflected rays organize a focal line along its axis. In case of, e.g. a planar waveguide the second spike is a reflection of the first, and the third recurs the first one.

The calculations performed by strict formulae for fields reproduce the above statement not only for a homogeneous dielectric but also for a thin-laminated medium the waveguide is filled with. In this case the waveguide dielectric filling is similar to a uniaxial crystal with permittivities: $\varepsilon_{//}$ and $\varepsilon_{\perp}$. Own frequencies corresponding to eigenmodes of the waveguide are described in that case by expressions [4]

$$\omega_n = (\varepsilon_\perp/\varepsilon_{//})^{1/2}(\beta^2\varepsilon - 1)^{-1/2}\lambda_n v; \ \beta^2\varepsilon_\perp - 1 \geq 0 \qquad (7)$$

and distances between the spikes are:

$$\Delta z_m = 2mL(\varepsilon_\perp/\varepsilon_{//})^{1/2}\cdot(\beta^2\cdot\varepsilon_\perp - 1)^{1/2} \qquad (8)$$

As is known, the Cherenkov radiation also generates when the charged particle flies parallel to the boundary with dielectric at a velocity exceeding the velocity of light in dielectric.

It was shown in [5] that when the charge flies along the axis of narrow channel (with a radius smaller than the radiated wave length), then the Cherenkov radiation field on sufficiently long waves will be such as though the charge moved in dielectric. As the charge moves along the channel axis in the dielectric or uniaxial crystal the waveguide is filled with, the Cherenkov wave will emit again in the form of spikes produced by the wave packets. However, the presence of the channels results in such a violation of the waveguide dispersion properties that the above-described picture gets broken, it smears as the field removes from the source.

This smearing can be observed depending on quantity r/R, where r is the channel radius. The larger this ratio is, the quicker that smearing comes. On the other hand, at amply high values of the channel radius the high frequencies, i.e. modes propagating on these frequencies begin to attenuate still faster. The number of modes effectively participating in Cherenkov radiation formation reduces. This circumstance, in particular, allows to pose the problem of optimization on the choice of the channel radius (at the cost of some decrease in Cherenkov wave amplitude) to restrict the frequency spectrum so that to approach the situation with dispersionless medium as close as possible. In this case the field strengths in homogeneous dielectric E_{zd} and in thin-laminated medium $E_{z\,cr}$ made of the same dielectric plates at $\varepsilon > 1$ turn out bound to each other by the relations :

$$E_{z\,cr} = E_{zd}\,(\varepsilon / \varepsilon_{//}),\ \varepsilon / \varepsilon_{//} > 1,\ \varepsilon > 1 \qquad (9)$$

Further, with increasing ε there also grows (but slower) the value of $\varepsilon_{\perp}$ and $\varepsilon_{//}$, which leads to reduction in factor $\beta^2 - 1/\varepsilon_{\perp}$, to which the electric field transverse component E_r is proportional. Thus in a uniaxial crystal, along with the increase of the longitudinal electric field component there must take place a decrease of its transverse component. However, if we consider the influence of the channel on the formation of radiation field, we can notice that the frequency intervals between the modes for a continuous dielectric $\Delta\omega_d$ and uniaxial crystal $\Delta\omega_{cr}$ relate as

$$\Delta\omega_{cr} / \Delta\omega = [\varepsilon_{\perp}\cdot(\beta^2\cdot\varepsilon - 1)]^{1/2} [\varepsilon_{//}(\beta^2\cdot\varepsilon_{\perp} - 1)]^{1/2} \qquad (10)$$

which leads to a decrease in the number of effectively radiated modes in the case of thin-laminated medium. In virtue of that, the gain in strength E_z can decrease equally as the extent of decrease of radial focusing and defocusing forces.

Conclusion

The beam-beam interaction in the cavity can be considered in non-self-consistent linear approximation. At sufficiently large length of the bunch train in the cavity can be excited only on of its own modes with a frequency corresponding to the bunch train frequency. A train of such cavities is a compact system for heavy current acceleration with high acceleration rate.

The presence of a channel in the waveguide at analysis of the Cherenkov radiation and the account of the medium disperse properties smear the field pattern.

The merit of a uniaxial crystal filling the waveguide with a channel is that the number of effectively radiating modes is simpler to control and to attain high accelerating fields.

We are thankful to Prof. A.M. Sessler (LBL) as well as to Dr. G. Sohngen (DESY) for their permanent attention and great support.

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