ENDING RFQ VANETIPS WITH QUADRUPOLE SYMMETRY

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Abstract

A new technique for ending RFQ vanetips with quadrupole symmetry is presented. A smooth transition is made from full modulation to zero modulation over a distance slightly shorter than a normal cell. The vanes can be extended for a short distance at zero modulation to control the output transverse phase-space ellipses. In addition to eliminating the nonzero on-axis potential at the RFQ exit, this method facilitates matching the output beam into succeeding structures or transport systems.

Introduction

The vanetips in most RFQ linacs are terminated abruptly at the end of a full cell, with one pair of vanes being closer to the axis than the other pair. This configuration produces a time-varying on-axis potential through which the beam must pass, giving rise to an uncertain change in the energy of the output beam. In 1984 I proposed a method for ending the vanetips with what I called an "exit fringe- field region" [1], in which the vanetips followed the equipotentials defined by a special potential function. This technique allowed one to calculate the electric fields in the exit region, thereby removing the uncertainty in the output energy of the beam. Two drawbacks of this technique were: 1) the vanetips in the horizontal and vertical planes differed in length; and 2) although the two potential functions agreed at the vanetips, the fields at the interface between the final acceleration cell and the exit fringe-field did not make a smooth transition. At the 1992 linac conference, Iwashita and Fujisawa[2] proposed ending RFQs near the middle of the final accelerating cell, near the quadrupole symmetry point. Their objective was to produce an output beam having more desirable characteristics in the transverse phase planes. They assumed that the fields in this final cell could still be calculated from the standard two-term potential function used in designing RFQs, in spite of the fact that the vanes were terminated at mid cell, but stated that detailed fringing field calculations were going to be made. In the absence of these detailed calculations, I would doubt the validity of the fields in this region obtained from the two-term potential function.

I now believe that the "proper" way to terminate RFQs is with the vanetips having quadrupole symmetry, but not by simply terminating the vanetips at mid-cell in the final accelerating cell. I propose terminating the vanetips with what I call a "transition cell", one in which a smooth transition is made from full modulation at the beginning to zero modulation at the end. It turns out that the length of such a cell is slightly less than a normal cell. The fields in this transition cell, derived from a three-term potential function, are continuous with the fields at the end of the previous cell, and the on-axis potential at the end of this transition cell would be zero. The RFQ could be terminated at this point, or it could be continued for a short distance with zero modulation. The reason for doing so would be to produce more desirable characteristics in the output transverse phase-space ellipses. An output radial matching section could be added by flaring out the vanetips, thereby producing (nearly) identical properties in x-x' and y-y'. All of the features have attractive properties for several applications:

1) Because the on-axis potential goes to zero in a prescribed way, the uncertainty in the output energy of the beam is removed. This feature alone is, in my opinion, a good enough reason to end all RFQs with a transition cell.

2) The usual way of terminating RFQs produces a highly convergent beam in one transverse plane and a highly divergent beam in the other plane, requiring a quadrupole lens very close to the output of the RFQ. By extending the vanes with zero modulation for a short distance, the designer has more control over the output transverse phase-space ellipses. The beam can be made to exit the RFQ having the characteristics it would normally have in the center of a quadrupole, for example. This would allow an external quadrupole to be spaced farther from the end of the RFQ, and this quadrupole would also be weaker. Another application is that a long RFQ could be divided into two shorter ones. The first RFQ would terminate with a transition cell followed by a short zero-modulation section; the second RFQ would begin with a short zero-modulation section followed by a transition cell followed by a normal acceleration cell. A low-current beam would require nothing but a short drift distance between these two RFQs.

3) If an output radial matching section is added, producing an axisymmetric output beam, then both positive and negative ions accelerated by the RFQ either separately or simultaneously would be matched to the succeeding magnetic transport system. Both the positive and the negative beam exit the RFQ having the same characteristics; that is, both are diverging in the same transverse plane and therefore need to pass through a quadrupole focusing in that plane. With an axisymmetric beam, it doesn't matter which plane is focused first.

Normal Cells

In a "normal" acceleration cell one pair of vanes begin with a displacement ***a*** from the axis and end with a displacement ***ma***, while the other pair of vanes starts at ***ma*** and end at ***a***, where ***m*** is called the vane modulation parameter. For design purposes, the fields in the cells are derived from a time-varying two-term potential function that satisfies Laplace's equation (without the time dependence) for boundary conditions that are periodic in z:

$$U(r,\theta,z)=\frac{V}{2}\left[\left(\frac{r}{r_0}\right)^2\cos 2\theta + AI_0(kr)\cos kz\right] \qquad (1)$$

where V is the maximum potential difference between vanes, $k=\pi/L$, with L being the length of a cell, and A and r_0 are constants that depend on the boundary conditions. The vanetip profiles in the horizontal and vertical planes are then given by:

$$\left(\frac{x}{r_0}\right)^2 + AI_0(kx)\cos kz = 1\,; \qquad (2)$$

$$-\left(\frac{y}{r_0}\right)^2 + AI_0(ky)\cos kz = -1. \qquad (3)$$

The constants A and r_0 are determined by specifying $x = a$ at $z = 0$ in Eqn. 2, and $y = ma$ at $z = 0$ in Eqn. 3. Typical vanetip profiles satisfying Eqns. 2 and 3 are shown in Fig. 1. It is obvious that the period of these profiles is $2L$, and that the slopes of the profiles (x' and y') are zero at the end of each cell.

Transition Cell

Suppose a different potential function is used for the final cell, one that agrees with the full modulation at the end of the previous cell and one that gives equally-spaced parallel vanes (zero modulation) at the end of this "transition cell". Instead of using the potential function given by Eqn. 1, use one having the following form in the transition cell:

$$U(r,\theta,z)=\frac{V}{2}\left(\frac{r}{r_0}\right)^2\cos 2\theta$$

$$\pm\frac{V}{2}\left[A_{10}I_0(Kr)\cos Kz \pm A_{30}I_0(3Kr)\cos 3Kz\right] \qquad (4)$$

where $K=\pi/2L'$, with L' being the length of this transition cell. The parameters A_{10}, A_{30}, and K (or L') are determined so that a smooth transition is made from full modulation at the beginning of the cell to no modulation at the end of the cell. The potential function given by Eqn. 4 satisfies Laplace's equation for periodic boundary conditions, where the length of the period is $4L'$. The horizontal and vertical vanetip profiles are given by

$$\left(\frac{x}{r_0}\right)^2 - A_{10}I_0(Kx)\cos Kz - A_{30}I_0(3Kx)\cos 3Kz = 1\,; \qquad (5)$$

and

$$-\left(\frac{y}{r_0}\right)^2 - A_{10}I_0(Ky)\cos Kz - A_{30}I_0(3Ky)\cos 3Kz = -1\,. \qquad (6)$$

The minus sign was used for the second and third terms so that the x vane would start at ma and the y vane would start at a when $z = 0$. Because the second and third terms are zero at $z = L'$, both vanetip profiles are r_0 at this point. However, we also want the slopes of the profiles to be zero at $z = L'$. Differentiating Eqns. 5 and 6 with respect to z and forcing x' and y' to be zero at $z=L'$ gives the following relationship between A_{30} and A_{10}:

$$A_{30}=\frac{I_0(Kr_0)}{3I_0(3Kr_0)}A_{10}. \qquad (7)$$

Typical vanetip profiles satisfying Eqns. 5 and 6, using the constraint given by Eqn. 7, are shown in Fig. 2, which shows periodic profiles with the period length of $4L'$. If a smooth transition is made between the vanetip geometries shown in Figs. 1 and 2, I hypothesize that the potential functions given

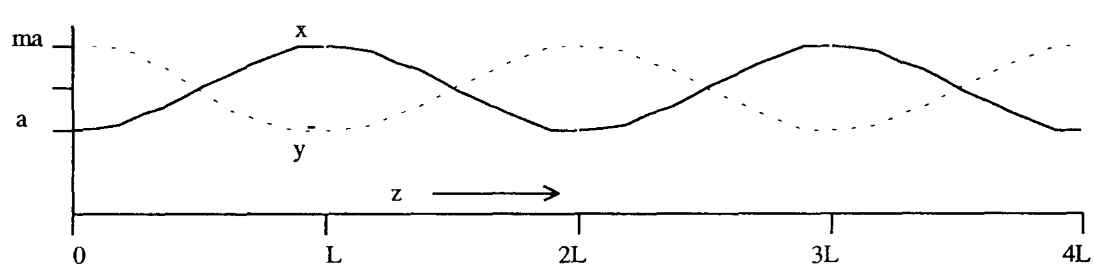

Fig. 1. Periodic vanetip profiles satisfying Eqns. 2 and 3.

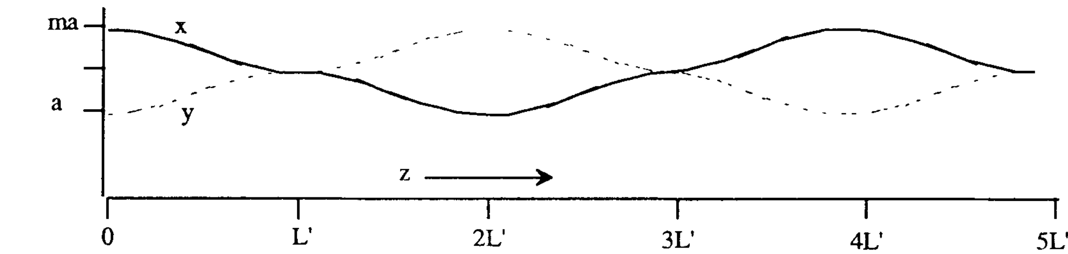

Fig. 2. Periodic vanetip profiles satisfying Eqns. 5 and 6.

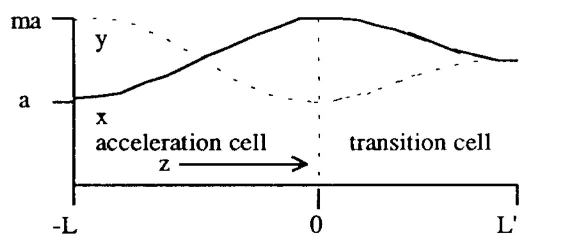

Fig. 3. Vanetip profiles in final acceleration cell followed by a transition cell.

Eqns. 1 and 4 are reasonable approximations for the actual configuration shown in Fig. 3.

The other two free parameters in Eqn. 4, A_{10} and K (r_0 is not a free parameter), are determined numerically from Eqns. 5 and 6 using the boundary conditions

$$U(ma, 0, 0) = V/2 \; ,$$

and

$$U(a, \pi/2, 0) = -V/2 \; .$$

Because $K = \frac{\pi}{2L'}$,

$$U(r_0, 0, L') = \frac{V}{2},$$

and

$$U(r_0, \pi/2, L') = -\frac{V}{2}$$

are satisfied automatically by Eqn. 5. It can be shown, to first order, that

$$K^2 \approx k^2/3,$$

which means that

$$L' \approx \sqrt{3/4}\, L \; .$$

It can also be shown that x'' and y'' obtained from Eqns. 5 and 6 agree with the values obtained from Eqns. 2 and 3 *at the interface between the normal cell and the transition cell*, and that x'' and y'' are zero at the end of the transition cell (at $z=L'$). The smooth blending of the vanetip characteristics with those of its neighboring cells ensure that the fields will indeed make a smooth transition in this so-called *transition* cell.

Summary

A method has been presented for ending RFQs with a transition cell defined by a 3-term potential function. The fields at the beginning of this transition cell blend smoothly with those at the end of the previous accelerating cell; the fields at the end of this cell are the same as for unmodulated vanes. The vanetips end being parallel and with quadrupole symmetry, so the on-axis potential is zero. This feature removes the uncertainty in the output energy of the beam. By continuing the vanetips with zero modulation for a short distance, the RFQ designer can control the point at which the beam exits from the periodic focusing system of the RFQ, and therefore have some control over the output transverse phase-space ellipses. An axisymmetric beam can be produced by adding an output radial matching section, allowing both positive and negative beams accelerated by the RFQ to be matched to a succeeding magnetic transport system.

References

[1] K. R. Crandall, "RFQ Radial Matching Sections and Fringe Fields", Proceedings of the 1984 Linac Conference, GSI-84-11.

[2] Y. Iwashita and H. Fujisawa, "Half End-Cell Geometry of RFQ", 1992 Linac Conference Proceedings, AECL-10728.