Tools for the Design of High-Current Linacs

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Abstract

With the programs usually used to design linear accelerators, beam dynamics parameters are calculated from data describing the accelerator structure. The desired phase advances (with and without space charge) in the transverse (σ_t and σ_{0t}) and longitudinal (σ_l and σ_{0t}) planes are reached after several iterations on the structure parameters.

Codes which use the opposit procedure have been written. The phase advances are first chosen thanks to a diagram which gives the tune depressions versus the phase advances without space charge :

 $\sigma_t/\sigma_{0t} = f(\sigma_{0t})$ and $\sigma_l/\sigma_{0l} = f(\sigma_{0t}, \sigma_{0l})$.

Once σ_{0t} and σ_{0l} are chosen (then σ_t and σ_l determined), the structure parameters are calculated. This procedure applied to the design of a RFQ is described, some comments on its use for DTL and high-energy structures are also presented.

1. INTRODUCTION

The phase advances per focusing period (with, without space charge) of the transverse (σ_t, σ_{0t}) and longitudinal (σ_t, σ_{0t}) oscillations, and the related tunes $(\upsilon = \sigma/2\pi)$, are crutial parameters for the design of high-current linear accelerators. In fact, the beam dynamics is strongly dependent of these four phase advances and the σ_{0t} and σ_{0t} values determine the structure parameters (transverse and longitudinal focusing forces).

For the beam dynamics, they determine not only the motion stability but also the emittance growths [1] and the halo formation [2 – 5] which can produce intolerable beam losses in high-current accelerators. The beam behaviour is governed by the choice of the zero-current phase advances (σ_{0t} , σ_{0l}) and "tune" spreads ($\sigma_{0t} - \sigma_t$, $\sigma_{0l} - \sigma_l$) which determine both type and number of the resonances which can overlap and induce a chaotic behaviour. Tools centered on these phase advances have then been developed for the design of high-current linacs.

Section 2 presents a method to calculate the phase advances with space charge (σ_t and σ_l) when the phase advances without space charge (σ_{0t} and σ_{0l}) and some basic parameters of the accelerator are fixed. From this first step, the designer can use a diagram which gives the tune depressions versus the phase advances without space charge : $\sigma_t/\sigma_{0t} = f(\sigma_{0t})$ and $\sigma_l/\sigma_{0l} = f(\sigma_{0t}, \sigma_{0l})$.

The next step consists to determine the accelerator characteristics as a function of the σ_{0t} and σ_{0t} values. Tools which help the designer to find acceptable values for the accelerator free parameters (rf field levels, cavity geometry, transverse focusing strength ...) have been developed. They will be presented with some details in section 3 for RFQ,

more simply for DTL and high-energy structures in sections 4 and 5 respectively.

2. TUNE DEPRESSIONS IN THE TRANSVERSE AND LONGITUDINAL PLANES

An uniformly charged ellipsoid with semiaxes $a = \sqrt{a_x.a_y}$ and b in the transverse and longitudinal directions respectively is used as bunch model. The phase advances with space charge (defined over one $n\beta\lambda$ long transverse focusing period) are then given by [6]:

$$\sigma_t^2 = \sigma_{0t}^2 - \frac{Q.(1-ff)}{2a^2.b} \quad (1)$$

$$\sigma_t^2 = \sigma_{0t}^2 - \frac{Q.ff}{a^2b} \quad (2)$$

where the bunch form factor f is a function of b/a [7], and

$$Q = \frac{3.n^2.Z_0.q.I.\lambda^3}{4\pi.m_0c^2.\gamma^3} \quad \text{with} \quad Z_0 = 377\Omega$$

The *Q* parameter is then fixed by the choice of the beam current (*I*), the particle rest mass (m_0c^2) , charge (*q*) and energy $(W \to \beta, \gamma)$, the frequency $(f_{hf} \to \lambda = c/f_{hf})$ and of the *n* value.

To compute the tune depressions solely as a function of the phase advances without space charge, a and b must also be expressed as a function of known parameters. The bunch mean radius is determined from the total transverse normalized emittance :

$$\varepsilon_{tn} = \frac{a^2 \sigma_t \gamma}{n \lambda}$$

The bunch half length can be related to the synchronous phase, for example such that :

$$b=\frac{\gamma \cdot \beta \lambda \cdot |\phi_s|}{2\pi}$$

So, for each σ_{0t} value, an optimization routine computes σ_t which verifies equation (1). The function $\sigma_t = f(\sigma_{0t})$ being known, the optimization routine can be used again to solve equation (2) which can be written in the form $\sigma_t = f(\sigma_{0t}, \sigma_{0t})$. Figure 1 shows two diagrams drawn using this method, they allow the designer to know what the σ_{0t} and σ_{0t} values are which must be selected to reach the desired tune depressions.

The next step consists to determine the accelerator parameters which give the chosen phase advances. Obviously, this choice must lead to a physically realizable design and must be studied for each kind of structure.

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Fig.1 : tune depressions versus σ_{0t} and σ_{0l} for : $f_{rf} = 352 \text{ MHz}, \phi_s = -30^\circ, I = 100 \text{ mA}, \varepsilon_{bn} = 1 \text{ mm.mrd and} :$ -(a) - n = 1, W = 300 keV (RFQ)-(b) - n = 2, W = 20 MeV (DTL)

3. HIGH-CURRENT RFQ DESIGN

Ref. [8] gives a detailed description of most of the formulae which will be used in this section. The notations will be the same than those used in this fundamental paper. The radius parameter of the pole tip will be noted "a" and the maximum transverse radius of the bunch "a max beam".

3-1- Parameters at the end of the gentle buncher.

At the end of the gentle buncher, the bunch is formed but the energy is still close to the injection one. This is therefore the critical point of the structure, the point where the space-charge forces are the most predominant.

The first constraint to deal with is the machine aperture (a). It must be greater than the beam size (a $_{max beam}$) and large enough to minimize the multipolar fields and the image charge effects. The designer can fix the (a) value to :

a = k_a . a max beam (k_a > 1)
with [9]: a max beam =
$$\sqrt{\frac{\lambda \varepsilon_{tn}}{\sigma_t} \cdot \frac{\pi \sqrt{2} + \sqrt{\sigma_{0t}^2 + .5\sigma_{0l}^2}}{\pi \sqrt{2} - \sqrt{\sigma_{0t}^2 + .5\sigma_{0l}^2}}}$$
 (3)

Once "a" known, the parameters of the radial and longitudinal dynamics : AV and XV respectively, can be calculated. The RFQ main parameters are then given by :

$$V = XV + AV I_0(ka)$$

$$X = XV/V, r_0 = a / \sqrt{X} \text{ and } A = AV/V$$

The next step consists to draw a diagram which gives the peak field ($E_{\mu} = KV/r_0$ in Kilpatrick unit) and the

modulation factor (m) as a function of the phase advances without space charge in order to enable a choice which is physically realizable. The modulation factor is computed through an optimization routine which solves the equation :

$$A = \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)}$$

Figure 2 shows a diagram obtained with this method. It must be used in connection with the charts $\sigma_t = f(\sigma_{0t})$ and $\sigma_t = f(\sigma_{0t}, \sigma_{0t})$ (section 2) in order to choose the best compromize between beam dynamics and feasibility.



Fig.2 : m and $E_{f}E_{Kilpatrick}$ as a function of σ_{0t} and σ_{0t} with : $f_{hf}=352$ MHz, W = 300 keV, $\phi_s = -30^\circ$, $\varepsilon_{tn} = 1$ mm mrd, I = 100 mA, K = 1.36, and $k_s = 1.3$.

3-2- Acceleration section parameters.

The code calculates the beam parameters (the four phase advances, $a_{max beam} \dots$) up to the output energy when the structure parameters are kept constant from the gentle buncher output. In order to decrease the length of this section, the same program can also calculate the beam and structure parameters to keep $k_a = a / a_{max beam}$ constant.

3-3- Gentle buncher parameters.

The calculation of this section follows the K-T approach described in ref. [8]. σ_{0l} and " z_b " are kept constant all along the gentle buncher. Its input energy which is chosen by the designer. The beam parameters with space charge are also computed all along this section.

3-4- Shaper parameters.

Once the RFQ input energy and the shaper length chosen, the code calculates the synchronous phase law in such a way that both the shaper output energy and the gentle buncher input one become the same. This is realized with : -i) a linear law for a and m,

-ii) $\phi_s = -90^\circ$ up to a point determined by the code. Beyond this point, ϕ_s is linearly increased to reach the GB input synchronous phase.

3-5- Example of RFQ design.

Figure 3 shows the main parameters of a 100mA RFQ obtained using these procedures. This design is done with : input and output energies : 100 keV - 2.7 MeV, $f_{rf} = 352$ MHz, $\varepsilon_{tn} = 1$. mm.mrd. The structure is 2.4 m long, V = 145 kV and $r_0 = 5.1$ mm.





4. HIGH-CURRENT DTL DESIGN

In opposition to the RFQ case, the longitudinal and radial external focusing forces can be tuned independently in a DTL. In the longitudinal plane, each RF gap is characterized by the parameter :

$$\Delta = \frac{\pi n^2 q \lambda \sin \phi_s}{m_0 c^2 \beta \gamma^3} E_0 \qquad (<0)$$

where E_0 is the effective accelerating gradient. As σ_{0l} is only dependant of Δ , its choice fixes the value of E_0 . For example, in the case of a FODO law of transverse focusing, Δ then E_0 can be computed by solving :

$$\sigma_{0l} = \cos^{-1}(1 + 4\Delta + 2\Delta^2)$$

In the transverse plane, the apperture can be determined using the same method than the one presented in the previous section (formula (3)). The transverse phase advance can be easily computed from the Twiss matrix of the focusing period. For a FODO focusing law, a simple matrix calculus can be done using a thin lense approximation for both quadrupoles and RF gap effects, it gives :

$$\sigma_{0t} = \cos^{-1} \left[1 - \left(\frac{1}{2} - \frac{\Lambda}{3} \right) \Lambda^2 \theta_0^4 - 2\Delta \right]$$
(4)

where $\Lambda = L_q/L_{cell}$ is the quadrupole filling factor and $\theta_0^2 = B'L_{cell}^2/B\rho$ is the strengh of the quadrupoles when their gradients are B' and the beam momentum $B\rho$.

A fixed, θ_0 then B' can be computed as a function of σ_{0t} by solving (4). The machine apperture being known, it is then possible to check the feasibility of the quadrupoles.

5. HIGH-CURRENT CCL DESIGN

Possible choices for the design are even more numerous than for a DTL. The longitudinal phase advance

without space charge is a function of n, E_0 and N : the number of cells per tank. The effective accelerating gradient being imposed by the cost optimisation, the designer can test some values of N in order to determine the total length of the period (α n), then the room available for quadrupoles and diagnostics. This fixes an upper limit for the quadrupole filling factor Λ . In the transverse plane, a calculus similar to the one described for a DTL can be used.

This procedure can be applied from place to place along the CCL to readjust the parameters as the beam energy increases.

6. CONCLUSION

A method which allows to determine the parameters of a high-intensity linear accelerator <u>starting from the phase</u> <u>advances with and without space charge</u> has been presented. As shown, it can be applied to RFQ, DTL and high-energy structures. Up to now, this method has been mainly used to design RFQ linear accelerators. Cross checking with the Los Alamos codes CURLI and RFQUICK [10] has been done in order to verify that both programs lead to consistent results.

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7. REFERENCES

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