STATIONARY PHASE SPACE DENSITY DISTRIBUTIONS FOR HIGH CURRENT BEAMS WITH LINEAR FOCUSING

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Abstract

Beam dynamics calculations find that different initial phase space distributions - subject to internal space charge forces and an external linear focusing force - evolve to a stationary density. Properties of the stationary density are studied via numerical solutions of the Vlasov equation for beams with cylindrically symmetry.

Introduction

A charged particle beam transport system or linear accelerator can be approximated by a linear radial focusing force when the discrete magnetic elements (e.g. quadrupole lenses) are averaged over in the longitudinal direction. For a cylindrically symmetric beam each particle experiences a radial acceleration given by the sum of the internal space charge force that tend to expand the beam and the external focusing force which contains the expansion

$$a(r) = \frac{K\,Q(r)}{r} - br \qquad (1)$$

where Q(r) is the fraction of charge per unit length (N) contained within r (Gauss'law).

The constant

$$K = \frac{2Ne^2}{4\pi\varepsilon_o\, m\, \gamma^3} \qquad (2)$$

is related to the beam perveance; the positive constant b is the external focusing strength. The evolution of the transverse phase space density function, f(r,v,t), can be followed by numerical solutions of the radial Vlasov equation

$$-\frac{\partial f}{\partial t} = v\,\frac{\partial f}{\partial r} + a\,\frac{\partial f}{\partial v} \qquad (3)$$

where v is the radial velocity. The radial density is

$$g(r) = \int f\, dv\ . \qquad (4)$$

Previous calculations[1,2,3] have shown that the stationary density has a uniform core, a skin thickness determined by the Debye length, and sometimes a halo. The emittance of the beam grows during the evolution of the input beam to the stationary beam.

Energy Moments

Operator moments of the phase space distribution are given by

$$\langle O \rangle = \iint O\, f\, dv\, dr\ . \quad (5)$$

Three transverse energy moments of the beam are of interest:

kinetic

$$KE = (1/2)\langle v^2 \rangle$$

internal potential

$$PE = K \langle Q(r)/r \rangle$$

external potential

$$PEX = (b/2) \langle r^2 \rangle$$

Stable Distributions

For K = 0 (the low current limit) the phase space density distribution evolves as independent particle motion in a harmonic oscillator external potential. The stationary distribution is given by

$$f_o = N_o \exp(-r^2/r_o^2 - v^2/v_o^2) \quad (6)$$

with $b = v_o^2/r_o^2$.

Other distributions rotate about the origin in phase space without changing shape or emittance. The period of rotation is given by $T_o = 2\pi/b^{1/2}$. During the rotation

$$\Delta KE = -\Delta PEX,\ \Delta PE = 0.$$

For K > 0 the repulsive space charge force changes the shape of f as it evolves. The stationary distribution can be approximated as

$$f_K = N_K \frac{\exp(-v^2/v_o^2)}{\left[\ 1 + \exp\left(\frac{r^2-r_o^2}{r_1^2}\right)\right]}\ . \quad (7)$$

with $r_o^2 = K/b$ and $v_o^2 \approx r_1^2/4$. A consistent set of parameters for K = 1, b = 1 is $r_o = 1.$, $r_1^2 = 0.1$, $v_o^2 = 0.025$.

Fig. 1 shows these two stationary distributions and their associated accelerations. In general stationary phase space distributions have a maximum entropy for a given total energy[4,5].

Other K > 0 distribution are found to evolve to the f_K shape but undergo monopole (breathing mode) vibrations. See fig. 2. The energies satisfy

$$\Delta(KE + PE) = -\Delta PEX.$$

There is some indication of halo formation at large r. This may be due to the abrupt change in the acceleration, a(r), from near zero in the core to linear outside the beam. This nonlinearity acting on the monopole vibration may eject particles into the halo region.

Halo formation and emittance growth can be avoided by injecting a stable beam, eq. (7), into the focusing channel.

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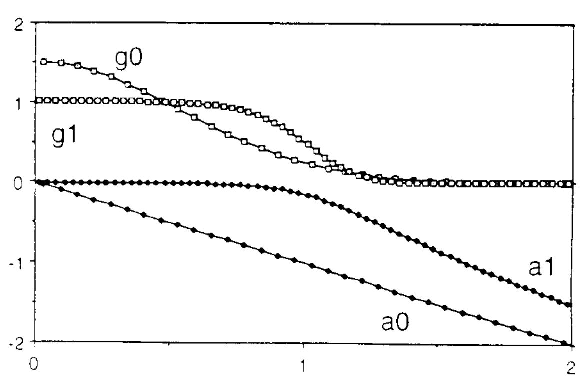

Fig. 1 Stable radial distributions for K = 0 and K = 1 and their net accelerations.

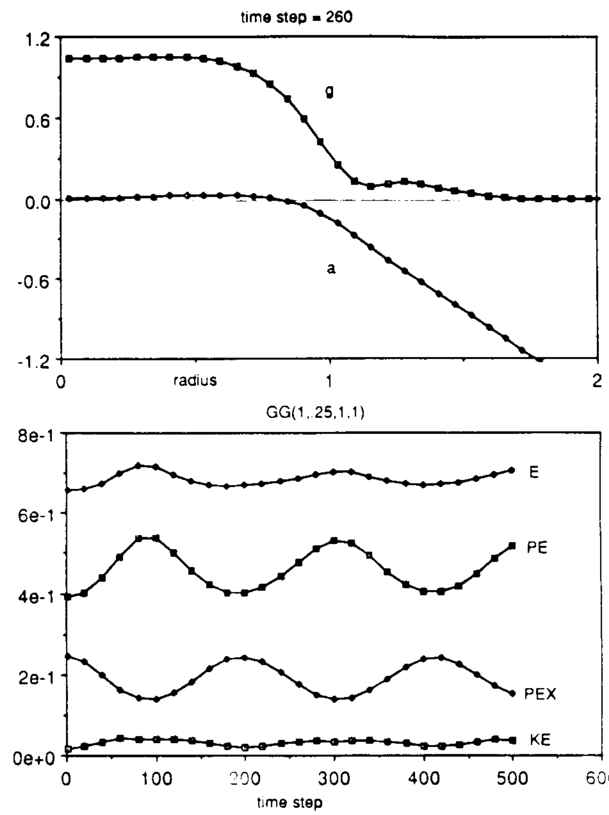

Fig. 2 Evolution of an initially gaussian distribution. The top figure shows the approach to the stationary radial distribution with a uniform density core. The acceleration is near zero in the core, but becomes nonlinear on the edge of the beam creating a halo. The bottom figure shows the three oscillations of the energy moments with (approximate) conservation of the total energy, E.