PHASE AND AMPLITUDE STABILIZATION OF BEAM-LOADED SUPERCONDUCTING RESONATORS*

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Abstract

A model has been developed to analyze the static and dynamic behavior of superconducting accelerating cavities operated in self-excited loops in the presence of phase and amplitude feedback, ponderomotive effects, and beam loading. This is an extension of an earlier analysis of the stabilization of superconducting cavities [1] which has been the basis of the control system of several superconducting accelerators but did not include beam loading. Conditions have been derived to ensure static and dynamic stability in the presence of ponderomotive effects (coupling between the mechanical and electromagnetic modes of the cavity through the radiation pressure). Expressions for the effect of fluctuations of cavity frequency and beam amplitude and phase on the cavity-field amplitude and phase and beam-energy gain have been obtained.

Introduction

One of the early challenges in the application of rf superconductivity to particle accelerators, especially ion accelerators, was in the control and stabilization of the phase and amplitude of the accelerating fields in the large number of independent cavities. Ambient noise and microphonics can cause frequency variations which are larger than the bandwidth of the resonators. Additionally, even in the absence of microphonics, the radiation pressure exerted by the rf field on the cavity wall leads to a frequency shift which usually is larger than the bandwidth. Such nonlinear behavior can lead to instabilities: a monotonic instability where the field amplitude jumps between two stable values, and an oscillatory instability where energy is transferred between the electromagnetic and mechanical modes of the resonator yielding a sustained sinusoidal variation of the cavity frequency and the cavity wall displacement.

The problem of phase and amplitude stabilization of low-velocity superconducting structures was successfully solved in the 1970s by two different methods. The first one involved the direct control of the cavity frequency by using a voltage-controlled variable reactance which could be electrically connected to or disconnected from the cavity [2]. By adjusting the repetition rate and the duty cycle of the control of the reactance, the phase of the fields in the cavity could be made to track an external phase reference.

The other method did not attempt to control the cavity frequency. Instead, the cavity was purposely overcoupled in order to broaden its bandwidth and it was operated in a self-excited loop. The cavity eigenfrequency was left unperturbed, but the loop oscillation frequency was controlled by introducing an additional amount of phase shift in the loop [1]. This method has the advantage of providing stable operation in the unlocked state since the loop frequency automatically tracks the resonator frequency. This method has been used in a number of low-velocity ion superconducting accelerators [3-6] and also in an electron accelerator [7].

*Work supported by the U.S. Department of Energy under Contract W-31-109-ENG-38 and by the U.S. Army Strategic Defense Command.

In all the existing superconducting ion accelerators, the beam current was small enough that the beam did not affect the dynamics of the fields in the cavities. However, work is in progress on the application of rf superconductivity to high-current ion accelerators where the beam itself is expected to affect the cavity field. Beam loading can either simplify or complicate the stabilization of superconducting cavities. Clearly, if the current is high enough, the loaded bandwidth can be much larger than the frequency variations, and the cavity can simply be driven by an external rf source and the fields will be stable. On the other hand, there can be currents where the loaded bandwidth will be larger than the intrinsic bandwidth while still being comparable to the frequency fluctuations due to microphonics and ponderomotive effects. In this case, the effects of the beam current, microphonics, and ponderomotive effects will need to be included in the analysis of the stabilization system.

In this paper we extend the analysis of the phase and amplitude stabilization of superconducting resonators operated in self-excited loops which was developed in [1] to include arbitrary amounts of beam loading.

Loop Equations

A resonator operating in a self-excited loop is shown schematically in Fig. 1. The resonator acts as a bandpass filter; its output is sent through a phase shifter, a limiter, and an attenuator and then used as its input. The loop phase shifter is used to establish oscillation and sets a constant difference between the resonator eigenfrequency and the loop frequency. If the resonator frequency is ω_c, the loop will oscillate at the frequency ω given by

$$\omega = \omega_c \left[1 + \frac{\tan \theta_l}{2Q} \right] ,$$

where θ_l is the loop phase shift and Q is the loaded quality factorof the resonator . The limiter and the attenuator provide a constant drive amplitude for the resonator independent of the resonator field amplitude. Amplitude feedback is provided by adding a signal in phase which is controlled by the amplitude error. Phase feedback is provided by adding a signal in quadrature which is controlled by the phase difference between the resonator and an external reference of frequency ω_r.

In [1] it was found advantageous to operate the loop slightly off resonance on the low frequency side ($\theta_l < 0$); this introduced a small amount of coupling between phase and amplitude feedback which could be used to damp the microphonics. In the present model we add a feedback phase shifter (θ_f) which can be used to provide the same amount of coupling while still operating the unlocked self-excited loop on resonance ($\theta_l = 0$).

In the case of high-current accelerators, the fields in the resonator will also be affected by the beam which is assumed to have the phase φ_b with respect to the external reference. The differential equation for the resonator field is then

$$\ddot{v} + 2\,\frac{1+\beta}{\tau_0}\,\dot{v} + \omega_c^2 v = \frac{2}{\tau_0}\,\dot{v}_g - \frac{2}{\tau_0}\,\dot{v}_b\ ,$$

where $v_b = V_b \exp\left[i(\omega_r t + \varphi_b)\right]$,

$$v_g = V_{p0}\, 2\,\beta^{1/2}\left[1 + \exp(i\,\theta_f)\,(\Delta v_g + i\Delta t)\right]\exp[i\,(\theta_l + \alpha)]\ ,$$

$$V_b = \frac{i_b R_{sh}}{2} \quad \text{and} \quad V_{p0} = \sqrt{R_{sh} P_{inc}}\ ,$$

and i_b is the beam current, R_{sh} is the resonator shunt impedance, P_{inc} is the power driving the resonator, Δv_g is the additional "in phase" signal providing amplitude feedback, Δt is the additional "in quadrature" signal providing phase feedback, β is the coupling coefficient, and τ_0 is the intrinsic decay time of the cavity.

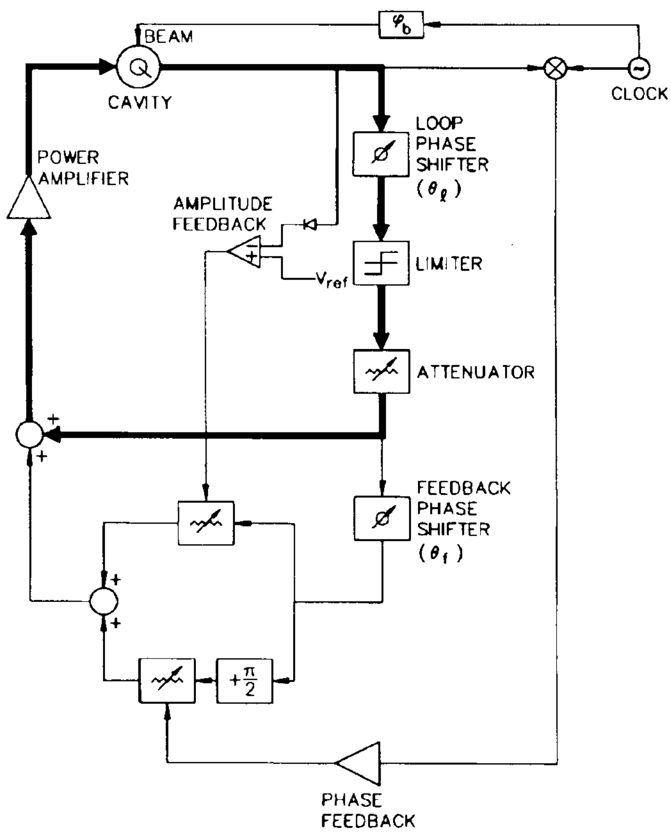

Fig. 1 Block diagram of a resonator operating in a self-excited loop in the presence of beam loading with phase and amplitude feedback.

If we define $v = V\exp(i\alpha)$ where V is the real amplitude of the resonator field and α its absolute phase, then, after separating real and imaginary parts and neglecting changes of amplitude and frequency during one rf period, the equations for the resonator field amplitude and phase are

$$\tau_0 \dot{V} + (1+\beta)V = 2\beta^{1/2} V_{p0}\left[\cos\theta_l + \Delta v_g \cos(\theta_l + \theta_f) - \Delta t \sin(\theta_l + \theta_f)\right] - V_b \cos\varphi_s$$

$$V\tau_0(\omega - \omega_c) = 2\beta^{1/2} V_{p0}\left[\sin\theta_l + \Delta v_g \sin(\theta_l + \theta_f) - \Delta t \cos(\theta_l + \theta_f)\right] - V_b \sin\varphi_s\ ,$$

where φ_s is the instantaneous phase between field and beam and $\dot{\alpha} = \omega$, the instantaneous loop frequency.

Steady State

When the loop is unlocked and the beam is off, the steady state amplitude is found from the above equations

$$V_0 = \frac{2\,\beta^{1/2}}{1+\beta}\, V_{p0} \cos\theta_l\ .$$

When the beam is on and the loop is locked to the frequency ω_r, the amount of steady state amplitude and phase feedbacks which are required to still maintain the amplitude V_0 in the resonator are

$$\Delta v_{g0} = \cos\theta_l \cos(\theta_l + \theta_f)\left[\frac{b}{1+\beta}(1 + y_0 y) + (y_r - y_l)\,y\right]\ ,$$

$$\Delta t_0 = \cos\theta_l \cos(\theta_l + \theta_f)\left[\frac{b}{1+\beta}(y_0 - y) + (y_r - y_l)\right]\ ,$$

where $y_0 = \tan\varphi_0$, $y_l = \tan\theta_l$, $y = \tan(\theta_l + \theta_f)$ and the detuning y_r and the beam loading coefficient b are

$y_r = \tan\theta_r = \frac{\tau_0}{1+\beta}(\omega_r - \omega_c)$ and $b = \frac{V_b \cos\varphi_0}{V_0}$, where φ_0 is the nominal phase between field and beam as indicated in Fig. 2.

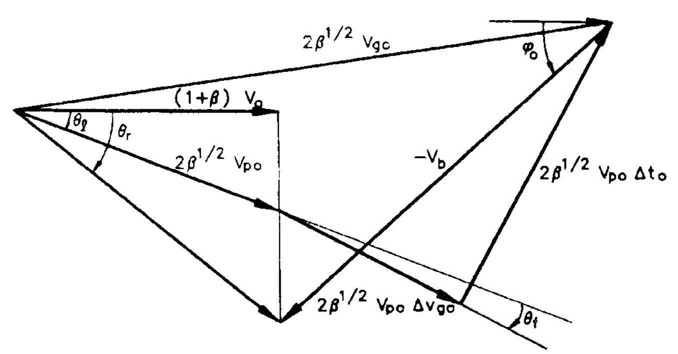

Fig. 2 Phase relationships between the signals driving the resonator.

The detuning y_r and the cavity coupling coefficient β can be optimized to minimize the amount of forward power required to provide an energy gain $V_0 \cos\varphi_0$ to a beam of current i_b with phase φ_0. The optimal values are $\beta = b + 1$ and $y_r = -\frac{b}{b+2}\tan\varphi_o$.

Transient Analysis

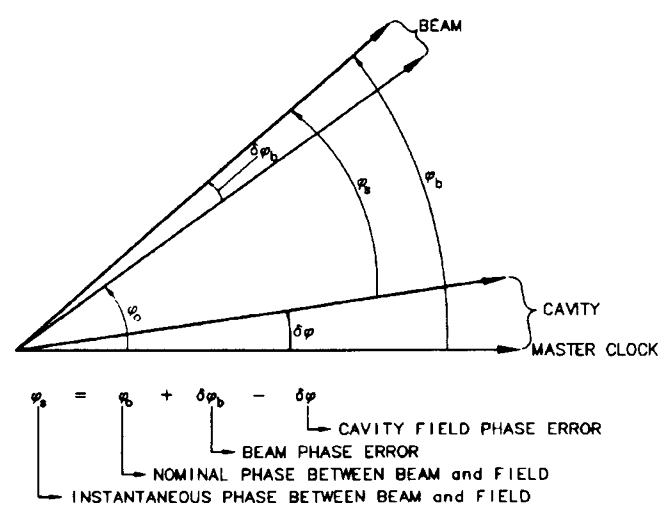

Fig. 3 Phase relationship between the beam and the cavity field.

The loop equations can be linearized around the steady state using (cf. Fig. 3)

$$V = V_0(1 + \delta v) \quad , \quad \Delta v_g = \Delta v_{g0} + \delta v_g \quad ,$$
$$\Delta t = \Delta t_0 + \delta t \quad , \quad V_b = V_{b0}(1 + \delta v_b) \quad ,$$
$$\varphi_s = \varphi_0 + \delta\varphi_b - \delta\varphi \quad ,$$

to provide the differential equations for the deviations from steady state which are

$$\tau_0 \delta\dot{v} + (1+\beta)\delta v = \frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}(1+\beta)(\delta v_g - y\,\delta t) - b\,\delta v_b + b\,y_0(\delta\varphi_b - \delta\varphi) \quad , \tag{1}$$

$$\tau_0(\delta\omega - \delta\omega_\mu - \delta\omega_{ex}) - (1+\beta)y_r\delta v = \frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}(1+\beta)(y\,\delta v_g + \delta t) - by_0\delta v_b - b(\delta\varphi_b - \delta\varphi) \quad . \tag{2}$$

Applying the Laplace transform to Eqs. (1) and (2), the system can be represented in the block diagram form shown in Fig. 4 where the transfer functions are

$$G_{aa} = \frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}\frac{1}{1+\tau s} \quad , \qquad G_{ta} = -\frac{\sin(\theta_l+\theta_f)}{\cos\theta_l}\frac{1}{1+\tau s} \quad ,$$

$$G_{a\omega} = \frac{1}{\tau}\frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}\left[y - \frac{y_r}{1+\tau s}\right] \quad , \qquad G_{t\omega} = \frac{1}{\tau}\frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}\left[1 + \frac{y\,y_r}{1+\tau s}\right] \quad ,$$

$$G_\mu = -\frac{2\,\Omega_\mu^2 k_\mu V_0^2}{s^2 + \frac{2}{\tau_\mu}s + \Omega_\mu^2} \quad , \qquad G_{ba} = -\frac{m}{1+\tau s} \quad , \qquad G_{\varphi a} = -\frac{m\,y_0}{1+\tau s} \quad ,$$

$$G_{b\omega} = \frac{m}{\tau}\left[-y_0 + \frac{y_r}{1+\tau s}\right] \quad , \qquad G_{\varphi\omega} = -\frac{m}{\tau}\left[1 + \frac{y_0 y_r}{1+\tau s}\right] \quad ,$$

F_a : Amplitude Feedback, $\qquad$ F_φ : Phase Feedback ,

$\tau = \dfrac{\tau_0}{1+\beta}$: Loaded amplitude decay time,

$m = \dfrac{b}{1+\beta}$: Beam matching coefficient.

G_μ represents the coupling between the field amplitude and cavity frequency which is responsible for the ponderomotive instabilities [1]. Ω_μ is the frequency of the mechanical mode of the cavity, and τ_μ is its decay time.

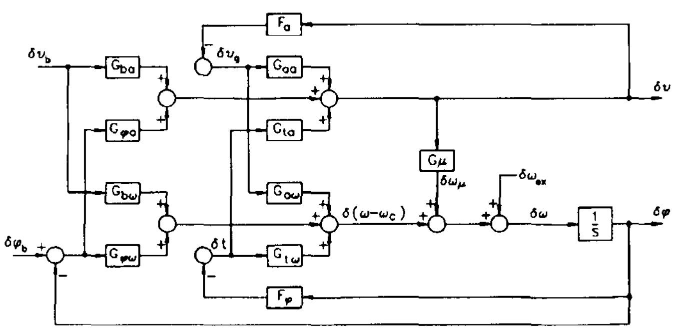

Fig. 4 Transfer function representation of the system shown in Fig.1

The residual amplitude and phase errors due to fluctuations of the cavity eigenfrequency ($\delta\omega_{ex}$), beam current (δi_b) and beam phase ($\delta\varphi_b$) are

$$\delta v = D^{-1}\{-\delta\omega_{ex}(G_{\varphi a} + F_\varphi G_{ta}) + \delta v_b[G_{ba}(s + G_{\varphi\omega} + F_\varphi G_{t\omega}) - G_{b\omega}(G_{\varphi a} + F_\varphi G_{ta})] + \delta\varphi_b[G_{\varphi a}(s + G_{\varphi\omega} + F_\varphi G_{t\omega}) - G_{\varphi\omega}(G_{\varphi a} + F_\varphi G_{ta})]\} \quad ,$$

$$\delta\varphi = D^{-1}\{\delta\omega_{ex}(1 + F_a G_{aa}) + \delta v_b[G_{b\omega}(1 + F_a G_{aa}) - G_{ba}(F_a G_{a\omega} - G_\mu)] + \delta\varphi_b[G_{\varphi\omega}(1 + F_a G_{aa}) - G_{\varphi a}(F_a G_{a\omega} - G_\mu)]\} \quad ,$$

with $D = (1 + F_a G_{aa})(s + G_{\varphi\omega} + F_\varphi G_{t\omega}) - (G_{\varphi a} + F_\varphi G_{ta})(F_a G_{a\omega} - G_\mu)$.

The stability of the system in the presence of ponderomotive coupling can be determined from D. In particular, if one assumes simple proportional amplitude and phase feedback ($F_a = k_a$ and $F_\varphi = k_\varphi$), then the conditions for monotonic and oscillatory stability are respectively

$$-\left[y - \frac{my_o}{K_\varphi}\right]k_\mu V_0^2 < \frac{1}{\tau}B_{mo}(K_a, K_\varphi) \quad ,$$
$$\left[y - \frac{my_o}{K_\varphi}\right]k_\mu V_0^2 < \frac{1}{\tau_\mu}B_{os}(K_a, K_\varphi) \quad , \tag{3}$$

with

$$B_{mo}(K_a, K_\varphi) = \frac{1}{2}\left[K_a(1+y^2) + 1 + yy_r - \frac{m}{K_\varphi}[K_a(1+yy_0) + 1 + y_0 y_r]\right] \quad ,$$

and

$$B_{os}(K_a, K_\varphi) = \frac{K_a + K_\varphi + 1 - m}{\tau^2\Omega_\mu^2 K_\varphi} \times \left\{\tau^2\Omega_\mu^2 + \left[\frac{(K_a+1)(K_\varphi - m) + (K_a y - my_0)(y_r + K_\varphi y) - \tau^2\Omega_\mu^2}{1 + \frac{2\tau}{\tau_\mu} + K_a + K_\varphi - m}\right] \times \left[\frac{(K_a+1)(K_\varphi - m) + (K_a y - my_0)(y_r + K_\varphi y) - \tau^2\Omega_\mu^2}{1 + \frac{2\tau}{\tau_\mu} + K_a + K_\varphi - m} + \frac{2\tau}{\tau_\mu}\right]\right\} ,$$

where we used $K_a = k_a\dfrac{\cos(\theta_l+\theta_f)}{\cos\theta_l}$ and $K_\varphi = k_\varphi\dfrac{\cos(\theta_l+\theta_f)}{\cos\theta_l}$.

Equations (3) clearly show how beam loading ($m \neq 0$) modifies the region of stability and how phase feedback (K_φ) can be used to reduce the effect of beam loading on stability.

Further work will investigate parameter optimization to reduce the effect of $\delta\omega_{ex}$, δv_b and $\delta\varphi_b$ on δv and $\delta\varphi$.

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