# ANALYTICAL SCALING LAWS AND MAIN GUIDE-LINES FOR THE DESIGN OF RF GUN CAVITIES

L.Serafini and C.Pagani
University of Milano and INFN
Via Celoria 16, 20133 Milano, Italy.


## Abstract

In this paper the results of an extensive analytical study of the beam dynamics in RF guns are reported: removing some approximations of Kim's model[1], such as the absence of higher spatial harmonics, the relevant beam properties at the gun exit (emittances, compressibility, etc) are derived as functions of the RF cavity characteristics. By means of the reported formulas one can find, once chosen the desired beam performances, the optimum value for the first half cell length, the high harmonic content and the laser pulse shape. A possible method for the neutralization of the space charge induced emittance is also proposed.


## 1. - Introduction

Laser-driven RF guns for generating high brightness electron beam have been built and tested in the past years at a number of laboratories[2], indicating that such injectors are the most promising sources both for future linac-based Free Electron Lasers in the X-UV domain and electron-positron colliders in the TeV energy region. For both applications intense electron beams are required with very low emittance and low energy spread.

It is well known[1,6] that the beam brightness achievable by RF guns is limited mainly by two effects:

- the emittance growth due to space charge forces, which produce a distortion of the transverse phase space;
- the time (or phase) dependence of RF transverse forces, which produces a transverse momentum strongly correlated to the longitudinal position in the bunch.

Once chosen the frequency and field of the RF gun, for a fixed bunch charge some optimum values for R and L can be found which minimize the emittance growth[6]. An increase of the bunch length L causes actually a decrease of the space charge contribution to the emittance but increases at the same time the emittance growth due to RF field. The same holds for the bunch radius R.

In a standard RF gun it is not possible to decrease the beam emittance simply by decreasing the bunch charge density, i.e. using longer and larger bunches, since the RF induced emittance blow-up becomes the dominant effect. As shown in the following, the possibility to cancel this RF contribution allows automatically to damp down also the space charge emittance simply by using larger sizes for the bunch (for a given bunch charge).

In particular, the space-charge emittance growth becomes vanishing for a uniform distributed cigar-like bunch, since the space-charge field becomes linear in the limit of very low astect ratio A, A≡R/L. The use of uniform cigar-like bunches, together with some technique to damp down the RF emittance growth, will allow, as shown below, to reach ultra-low emittance, high charged, bunches.

Again, separating the two contributions to the space charge induced emittance growth (longitudinal space-charge field variation inside the bunch and non linear transverse components of the space-charge force) it will be shown that a proper correction of the non linear term will produce a strong damping of the total emittance growth in the domain of ultra-short bunches ($A \gg 1$)

## 2. - Correction of the RF Induced Emittance

The basic mechanism of the RF induced emittance blow-up consists in the correlation between the exit transverse momentum and the injection phase, as given by the formula:

$$p_r = \alpha k r \cdot \left(\sin\langle\phi\rangle + \Delta\phi \cdot \cos\langle\phi\rangle - \frac{\Delta\phi^2}{2}\sin\langle\phi\rangle\right) \quad (1)$$

which gives the well known fan-like shape of the transverse phase space distribution at the gun exit. In this expression: $\alpha \equiv eE_o/2m_oc^2k$ ($E_o$ is the RF cathode peak field and $k=\omega_{RF}/c$), r is the radial position of a generic electron of the bunch and $\phi$ its exit phase (defined as $\phi \equiv \omega T_f - kL + \phi_o$, L being the gun length, $\phi_o$ the injection phase at the cathode and $T_f$ the exit time). The exit phase $\phi$ comes out to be given by: $\phi = \phi_o + 1/2\alpha\sin\phi_o$. It is supposed to be slightly distributed around an average exit phase of the bunch $\langle\phi\rangle$, such that $\phi = \langle\phi\rangle + \Delta\phi$ .

Taking the standard definition of rms normalized emittance[4], we calculate the RF induced emittance blow up, assuming that the phase distribution is symmetric with respect to $\langle\phi\rangle$ (i.e. $\langle\Delta\phi\rangle = \langle(\Delta\phi)^3\rangle = ... = 0$):

$$\varepsilon_x^{RF} = \varepsilon_{min} + \alpha k \langle x^2\rangle \sqrt{\langle(\Delta\phi)^2\rangle}\, \left|\cos\langle\phi\rangle\right| \quad (2)$$

where

$$\varepsilon_{min} = \frac{\alpha k \langle x^2\rangle}{2}\sqrt{\langle(\Delta\phi)^4\rangle - \langle(\Delta\phi)^2\rangle^2}$$

The longitudinal emittance $\varepsilon_z$ comes out to be, for a uniform distributed bunch of length $\Delta\phi = kL$:

$$\varepsilon_z = \varepsilon_{z\,min} + \frac{\alpha\,\Delta\phi^3}{4\sqrt{2 \cdot 5!}\,k}\left|\pi(N+1/2)\sin\langle\phi\rangle + \cos\langle\phi\rangle\right|$$

where:

$$\varepsilon_{z\,min} = \frac{\alpha\,\Delta\phi^4}{2 \cdot 5!\sqrt{21}\,k}\sqrt{1+\pi^2(N+1/2)^2}$$

The second order term in $\Delta\phi^2$ of the expression for $p_z$[6], is the main source of longitudinal emittance blow up. This term vanishes only at an average exit phase defined by

$$\cot\langle\phi\rangle = -\pi\,(N+1/2) \quad (3)$$

Unfortunately the solution of eq. (3) approaches $\pi$ as the number of cell N becomes larger: it comes out that the average exit phase required to minimize the longitudinal rms emittance blow up is far from the one required ($<\phi> = \pi/2$ ) to minimize the transverse emittance at the gun exit.

Since the longitudinal emittance blow up is substantially due to the curvature of the longitudinal phase space, this has a relevant effect on the possibility to increase the peak current of the bunch via a magnetic compression applied downstream the gun exit.

Defining the compressibility C as the ratio between the bunch length and its minimum length achieviable by applying an idel magnetic compression at the gun exit (i.e. the maximum peak current increase), we get:

$$C = \frac{\sqrt{\frac{20}{3}\left(\pi(N+1/2)\cos<\phi>-\sin<\phi>\right)^2 + \Delta\phi^2\left(\pi(N+1/2)\sin<\phi>+\cos<\phi>\right)^2}}{\Delta\phi\left|\left(\pi(N+1/2)\sin<\phi>+\cos<\phi>\right)\right|} \qquad (4)$$

It turns out that, for a fixed natural bunch length $\Delta\phi$, C increases significantly as $<\phi>$ grows above the prescribed $<\phi> = \pi/2$. That implies a higher transverse emittance, due to the first order term in eq. (2)

To solve this problem two different solutions have been proposed so far: a lengthening of the first half cell[7] and a correction of the extra-emittance contribution, due to the first order term, by means of an unsymmetrical cell[5] added downstream the RF gun cavity.

### 2.1 - Lengthening of the first half cell

The divergence at the exit of the first half cell can be found by calculating the ratio $D = p_r / p_z$ as a function of the parameter $\delta$, giving the relative change of the first half cell with respect to the standard $\lambda/4$ length.

It can be shown[3] that a shorter first half cell increases the exit divergence, while the viceversa holds for longer cells: at $\delta$=0.3 (i.e. a 30% longer cell) the exit divergence is decreased by 6% .

The compressibility can be computed by the same procedure used above for the ideal $\lambda/4$ cell: since the condition to have a minimum rms transverse emittance is still $<\phi> = \pi/2$, it is interesting to compute the gain in compressibility at such exit phase, i.e. the ratio $g = C' / C$ , where C' is the compressibility of the longer (or shorter) half-cell and C is the compressibility given in (4) for the standard half-cell. We found:

$$g = \frac{1 + \delta(4/3+\pi^2/8) + \frac{5\delta^2\pi^2}{32}}{1 + \frac{5\delta}{4} - \delta^2(5/16+\pi^2/24)} \qquad (5)$$

g is less than 1 for $\delta<0$ while grows above 1 for $\delta>0$: that implies a gain in compressibility for longer half-cells. In particular, a half-cell whose length is increased by a factor 1.3 is able to give a compressed current nearly 50 % higher than a standard $\lambda/4$ cell.

It must be stressed, however, that the injection phase is still locked at a fixed value as long as a minimum rms transverse emittance is requested at the gun exit. Moreover, the varied length of the half-cell implies that non linear transverse components will appear in the RF field, as shown in ref. [3].

### 2.2 - Unsymmetrical Cell downstream the gun cavity.

To have together compressibility (i.e. linear longitudinal phase space) and a transverse emittane close to the minimum value (i.e. $<\phi> \approx \pi/2$), the technique of using a unsymmetrical cell downstream the gun cavity has been suggested[5]. Since this technique has been estensively reported and the scaling laws have been given, we just recall that the effect is mainly produced by the spatial harmonic content, introduced by the unsymmetrical cell, in a way similar to that reported in the next paragraph, in which a multi-mode cavity is described.

### 2.3 - Multi-mode RF guns.

In the two previous cases the main result is that the spatial harmonics are capable of correcting a correlated transverse emittance as long as the beam can keep a significant divergence (comparable to the natural divergence in the gun) through the cell. Moreover, if the field amplitude in the cell is comparable to that one in the gun only the first order emittance term possibly present in the beam can be corrected. It looks that the spatial harmonics play the role just to shift the spatial phase of the first harmonic, but they do not produce any net contribution to the momentum change.

Hence we must search for a real harmonic of the fundamental $TM_{010-\pi}$ mode, i.e. a higher mode of the RF gun cavity whose frequency is an integer multiple of the fundamental one. That is equivalent to the assumption that the RF field can be written as:

$$E_z(z,t) = E_o\cos(kz)\sin(\omega t+\phi_o) + E_n\cos(nkz)\sin(n\omega t+n\phi_o) \qquad (6)$$

In this case it can be shown that, for odd harmonics (of the type n=3,7,11,...), the exit longitudinal momentum is linearly correlated up to the fourth order term to the exit phase $\phi$, given that:

$$\alpha_n = (-1)^{(n-3)/2}\frac{\alpha}{n^3} \qquad \text{where:} \quad \alpha_n \equiv \frac{eE_n}{2nkm_oc^2} \qquad (7)$$

Again we compute the rms normalized transverse emittance blow-up using the definition[4]:

$$\varepsilon_x = \sqrt{<x^2><p_x^2> - <xp_x>^2},$$

where < > means an average over the phase space distribution. To compute the transverse momentum we simply take into account the presence of the odd n-th harmonic, under the straight-topping condition. Summing a contribution similar to the one of the fundamental mode:

$$p_r = \alpha kr\cdot\sin\phi + \frac{\alpha kr}{n^2}\sin(n\phi)$$

we get:

$$\varepsilon_x^{RF} = O(<(\Delta\phi)^4>) + \alpha k<x^2>\sqrt{<(\Delta\phi)^2>}\,\left(\frac{n^2-1}{6}\right)\left|<\phi> - \pi/2\right|^3 \qquad (8)$$

The longitudinal rms emittance becomes, for n=3:

$$\varepsilon_z = \frac{\pi\,\alpha\,\Delta\phi^5\,(N+1/2)}{288\sqrt{3}\,k} \qquad (9)$$

which gives a very favourable scaling, implying that the longitudinal phase space distribution is free from non linear distortions: the compressibility is in this case much higher than in the standard case (without harmonic). As shown elsewhere[6], this condition is called "straight topping" since the energy-phase relationship exhibits a slanted but straight top, to be compared with the flat-top typical of different type of RF cavity operation with harmonics .

Lookong at the transverse emittance, we can say that the RF induced emittance blow up is neutralized completely, not only at the optimum phase, but even for average exit phases slightly shifted around the optimum one.

The two relevant effects of the superposition of a third harmonic under the straight topping condition (7) can be summarized as:

- the minimum of the longitudinal and transverse emittance blow up due to RF effects occur at the same injection phase for both the emittances
- the transverse emittance blow up is neutralized up to fourth order terms in $\Delta\phi$.
- the longitudinal emittance blow up is neutralized up to fifth order in $\Delta\phi$

### 3. - Correction of the Space Charge induced Emittance

Here we analyze the domain of ultra-short bunches (with large aspect ratio A), in order to find a technique capable to neutralize the space-charge emittance blow-up.

Using the same approximation as in ref.1, we compute the total transverse momentum $p_r$ given by the space-charge field during the acceleration by the formula:

$$p_r = \frac{\pi}{2\,E_o \sin\phi_o}\, E_r^{sc}$$

stating that $p_r$ is proportional to the radial component of the electrostatic field $E_r^{sc}$ produced by the bunch charge at rest in the laboratory frame, divided by the actual RF field at the cathode surface $E_o \sin\phi_o$ when the bunch is emitted from the cathode.

In our case $p_r$ will be represented by the sum of two contributions, one scaling as the radius and the other one scaling as the cube of the radius, i.e $p_r = p_I\, r + p_{III}\, r^3$ , with $p_I$ and $p_{III}$ functions of the z coordinate inside the bunch.

Since the rms normalized emittance growth due to space-charge forces is defined as usual, with the previous general expression for the space-charge imparted momentum $p_r$ , $\varepsilon_x^{sc}$ will be given by:

$$\varepsilon_x^{sc} = \sqrt{\varepsilon_I^2 + \varepsilon_{III}^2 + \varepsilon_c^2}$$

where:

$$\varepsilon_I^2 = \langle x^2\rangle^2 \left(\langle p_I^2\rangle - \langle p_I\rangle^2\right) \qquad \varepsilon_{III}^2 = \langle x^2\rangle\langle x^6\rangle\langle p_{III}^2\rangle - \langle x^4\rangle^2\langle p_{III}\rangle^2$$

$$\varepsilon_c^2 = 2\langle x^2\rangle\langle x^4\rangle\left(\langle p_I p_{III}\rangle - \langle p_I\rangle\langle p_{III}\rangle\right)$$

In order to better understand the role plaied by different components we list separately: $\varepsilon_I$ given by the momentum phase correlation, $\varepsilon_{III}$ given by the spherical aberration effect and the total emittance $\varepsilon_x^{sc}$. These are given in the following as functions of the aspect ratio A, assuming A»1 (i.e. neglecting terms of the type $O(1/A^3)$):

$$\varepsilon_I = \frac{Q}{128\sqrt{5}\,\varepsilon_o E_o \sin\phi_o}\,\frac{1}{A^2 R}$$

$$\varepsilon_{III} = \frac{3Q}{1024\,\varepsilon_o E_o \sin\phi_o}\,\frac{1}{R}$$

$$\varepsilon_x^{sc} = \varepsilon_{III}$$

Q being the bunch charge.

It comes out that the total emittance blow up is dominated for ultra-short uniform bunches by the third order term, which represents a spherical aberration[8] in the transverse phase space, and scales unchanged versus A.

A possible cure to the saturation of the emittance blow up, which exhibits a minimum value at large A, is the exploitation of ultra-short bunches with a parabolic distribution in the transverse direction. The optimum distribution that cancel out the third order component at the centre of the bunch is found[3] to be given by:

$$\rho = \rho_o\left(1 - \frac{1}{3}\frac{r^2}{R^2}\right).$$

Such a charge density distribution can be achieved using a laser pulse which has a constant intensity profile along its longitudinal direction, and a gaussian clipped profile in the transverse direction. Clipping the laser beam at a radius $R = \sqrt{2/3}\,\sigma_r$, a radial distribution is obtained with the requested coefficient for the second order term in $r^2$ and a coefficient of the fourth order term really negligible. .

Repeating the emittance calculation for the optimum parabolic distribution[3], we get:

$$\varepsilon_I = \frac{7Q}{900\sqrt{5}\,\varepsilon_o E_o \sin\phi_o}\,\frac{1}{A R}$$

$$\varepsilon_{III} = \sqrt{\frac{211}{500}}\,\frac{Q}{256\,\varepsilon_o E_o \sin\phi_o}\,\frac{1}{A^2 R}$$

$$\varepsilon_x^{sc} = \varepsilon_I$$

In this case the scaling law for the emittance blow up becomes really favourable, being dominated by the linear term which scales like $A^{-1}$. The optimum parabolic distribution cancels out indeed the third order effect, so that the emittance scales like the inverse of the current!

### References


[1] - K.J.Kim, NIM, A275 (1989), p.201.

[2] - J.S.Fraser and R.L.Sheffield, IEEE, QE-23, 9, 1987, p.1489. C.Travier, "RF Guns: a review", LAL/RT 90-13, June 1990.

[3] - L.Serafini, "Improving the beam quality of RF Guns by correction of RF and space-charge effects", Proc. Advanced Acc. Conepts, June 1992, Port Jefferson, NY.

[4] - P.M.Lapostolle, IEEE Trans. Nucl. Sc. NS-18, no.3 (1971), p.1101.

[5] - L.Serafini et al., "RF Gun emittance correction using unsymmetrical RF cavities", NIM A318 (1992) p.275.

[6] - L.Serafini, M.Ferrario, C.Pagani and R.Rivolta, INFN rep. INFN/TC-91/07 (1991).

[7] - I.S.Lehrman et al., "Design of a high-brightness, high-duty factor photocathode electron gun", Proc. FEL Int. Conf. 1991, Santa Fe, Aug. 1991.

[8] - J.C.Gallardo and R.B.Palmer, BNL-43862, CAP 54-ATF-90J.