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3d and r, z Particle Simulations of Heavy Ion Fusion Beams*

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Abstract

The space-charge-dominated beams in a Heavy Ion beam driven inertial Fusion (HIF) accelerator must be focused onto small (few mm) spots at the fusion target, and so preservation of a small emittance is crucial. The nonlinear beam self-fields can lead to emittance growth; thus, a self-consistent field description is necessary. We have developed a multi-dimensional time-dependent discrete-particle simulation code, WARP, and are using it to study the behavior of HIF beams. The code's 3d package combines features of an accelerator code and a particle-in-cell (PIC) plasma simulation. Novel techniques allow it to follow beams through many accelerator elements over long distances and around bends. We have used the code to understand the emittance growth observed in the MBE-4 experiment at Lawrence Berkeley Laboratory (LBL) under conditions of aggressive drift-compression. We are currently applying it to LBL's planned ILSE experiments, and (most recently) to an ESQ injector option being evaluated for ILSE. The code's r, z package is being used to study the axial confinement afforded by the shaped ends of the accelerating pulses, and to study longitudinal instability induced by induction module impedance.

I. Introduction

Heavy-ion particle accelerators are attractive candidates as drivers for inertial fusion energy applications.[1] However, in a fusion driver it is necessary to transport a much larger current than has been achieved in existing ion accelerators, and the physics of high-current beams is considerably more complicated than that of the beams in conventional ion accelerators. This is especially the case for the recirculating induction accelerator being studied at Lawrence Livermore National Laboratory (LLNL) and at LBL as a lower-cost alternative to a linear driver for fusion energy.[2, 3] A variety of numerical tools are employed in the study of HIF beams.[4]

WARP was developed specifically for the study of space-charge-dominated beams. In an HIF driver, such beams must be accelerated and transported over large distances, and undergo a number of manipulations, which may include: transport around bends (needed to enter the target chamber, or for recirculation); transport through imperfectly aligned focusing elements; non-steady acceleration; injection into rings; merging; and splitting.

This work has been described in the *Proceedings of the International Symposium on Heavy Ion Inertial Fusion*, Dec. 3–6, 1990,[5, 6, 7] and elsewhere.[8, 9, 10] In this paper we briefly review the code concept, methods, and applications. These applications include studies of: beam drift-compression in a misaligned lattice of quadrupole focusing magnets; beam equilibria, and the approach to equilibrium; the MBE-4 experiment recently concluded at LBL;[11] and 3d simulations of bent-beam dynamics relevant to planned ILSE experiments.[12]

The code's newest capabilities include a model for an ES-Q injector in 3d, using a beam formed "by injection," and an improved r, z package, incorporating a model of module impedances which can drive longitudinal instability.

II. Code Overview

The WARP code contains a number of distinct parts, including: a 3d PIC package, called WARP3D or WARP6, which uses a "warped Cartesian" mesh in x, y, s to describe bends; an axisymmetric r, z PIC package, WARPRZ; an envelope equation solver (used for loading a "matched" beam); and facilities for initialization, diagnostics, etc. The code uses BASIS,[13] which provides a code development system that facilitates modular construction of programs, and a powerful interactive user interface.

The code's model accelerator "lattice" consists of a fully general set of finite-length (for the most part, sharp-edged) focusing and bending elements. The electric and magnetic fields of these elements (which have properties such as location, strength, etc. specified by the code's user) are computed algebraically at each particle location at each timestep. In combination with the self-fields, these applied fields are used in the Lorentz force law to advance the particle velocity timestep-by-timestep. Each multipole component (azimuthal harmonic) of the applied field is handled separately; for flexibility, different multipoles can overlap axially.

The simulation takes place in the laboratory frame. The computational mesh fills a moving window and is laid down anew at each timestep. The self-field is assumed electrostatic and is usually obtained by an FFT solution of Poisson's equation; boundary conditions are usually those of a square metal pipe. Round pipes (via capacity matrices) and internal conductors are options. The fields from electrostatic quadrupoles were originally an idealiza-

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tion, with a perfect sharp-edged axial dependence and only quadrupole and dodecapole terms; this remains an option.

Two other models for electrostatic quadrupoles are available. A set of rods, complete with self-field image effects, can be modeled; each group of four rods is handled by its own capacity matrix (the image coupling between rods at different axial locations is small). Most recently, electrostatic quadrupoles and ESQ injector structures can be modeled in some generality, with plates attached to the quadrupole elements, holes in the plates through which the beam can pass, etc. This latter model uses an iterative (successive over-relaxation) solution technique.

Usually the entire beam is loaded at the beginning of a run ($t = 0$), with guidance from the envelope solution for initial particle positions and velocities. For injector studies, the beam is formed by injection; particles are continually created along an equipotential surface as time advances.

To model driver-scale beams (which have speeds up to about $c/3$), we plan to use Lorentz transformations (at least in simple straight systems) to obtain the lab-frame self-$\mathbf{E}$ and $\mathbf{B}$ needed to advance the particles. We currently use $\mathbf{E}_{\text{self}}$ directly, a good approximation for the slower beams of near-term experiments.

In leapfrog motion, if a particle were to land within a sharp-edged focusing or bending element on four steps while its neighbor did so on only three, they would receive dramatically different impulses. Thus, the advance incorporates "residence corrections" for element forces which account for the fraction of the velocity advance step actually spent within the element.[8] This allows much bigger steps than otherwise would be possible.

We have developed a family of techniques for modeling bends. These are based upon following a particle's position and velocity in a sequence of rotated inertial (laboratory) frames. An "exact" method, which is symplectic and independent of aspect ratio, has been described previously, for both 3d and 2d (transverse) applications[10], and successfully applied (with modifications for relativistic beams).[14] Here, we summarize the inexact, "simplified" method now in use.[5, 8]

The radius of curvature of the reference orbit (usually the vessel centerline) is $r_* \equiv h^{-1}$. Time is the independent variable for particle orbits. The conventional (for accelerator codes) independent variable s is in WARP a dependent variable for orbits, as are x, y. In straight sections, $s \equiv z$, while in bends, $s \equiv -r_*\theta$. The "radial" coordinate is $x \equiv r - r_*$; the unit vectors $\hat{x}$ and $\hat{s}$ evolve as a particle moves, and are different for each particle. The axial speed is $v_z = -r\dot{\theta}$ (we use subscripts z and s interchangeably). The axial position is advanced in time using:

$$ds/dt = -r_*\dot{\theta} = (r_*/r)v_z \ . \tag{1}$$

A particle's velocity vector rotates because of the rotation of the coordinate axes. Due to this alone, the rate of change of the velocity angle is:

$$\frac{d}{dt}\arctan\left(\frac{v_x}{v_z}\right) = -\dot{\theta} = \frac{v_z}{r_* + x} \ . \tag{4}$$

We thus need only augment the dipole (bending) field at each particle position with a "pseudo-gyrofrequency":

$$B_y \Leftarrow B_y - \frac{m}{q}\frac{v_z}{r_* + x} \tag{5}$$

where m is the particle's mass and q its charge. This folds the necessary back-rotation into existing coding. The algorithm is inexact because v_z and x change during the step, but is accurate enough for our needs; "residence corrections" on entry to and exit from bends are necessary.

Poisson's equation in "warped" coordinates is [15]:

$$\frac{1}{1+hx}\frac{\partial}{\partial x}\left((1+hx)\frac{\partial\phi}{\partial x}\right) + \frac{\partial^2\phi}{\partial y^2} + \frac{1}{1+hx}\frac{\partial}{\partial s}\left(\frac{1}{1+hx}\frac{\partial\phi}{\partial s}\right) = -4\pi\rho \ . \tag{6}$$

Expanding the derivatives, we solve this iteratively. At each iteration the 3d FFT Poisson solver inverts the dominant "Cartesian" second derivative terms. One term, proportional to $(\partial h/\partial s)(\partial\phi/\partial s)$, is included by a simple finite difference, assuming the change in h at bend entry/exit can be spread in s slightly. The iteration converges rapidly, in two or three passes. It is necessary to obtain the true charge density from the "conventional" ρ_c collected from the particles, using $\rho = \rho_c r_*/r$, since (in a bend) the separation in s of zones varies with x. Also, the axial field is $E_z = -(r_*/r)\partial\phi/\partial s$.

III. Summary of Applications

Drift Compression: (current enhancement resulting from a head-to-tail velocity gradient or "tilt"): Relatively small misalignments of the focusing quadrupoles can lead to significant off-axis displacements. Image forces and fringing fields can then induce emittance growth. We seek to learn how fast and how much the beam may be compressed without unacceptable emittance degradation. The details of the errors, in a system of ILSE scale, are significant; different random-number seeds for the offsets lead to widely varying displacements.[9]

Equilibration: We are examining, in 3d and r, z, the transfer of thermal energy between transverse and longitudinal motions. For certain ranges of physical parameters, a beam initialized colder in z (axially) than in x, y (transversely) is observed to heat rapidly in z until T_z is a large fraction of $T_{x,y}$. This appears to be a collective process.[9]

Axial Confinement, Nature of Equilibria: To follow a finite-length beam for a long time, it is necessary to apply an axial confining force. This is done using shaped ends of the accelerating pulses, or "ears." We have modeled (in 3d) near-equilibrium beams that remain "quiescent" over runs as long as 175 lattice periods without significant emittance degradation in the simulation.[6]

Simulations of the MBE-4 Experiment: In this LBL experiment, emittance growth has been observed to accompany aggressive drift compression. Using WARP, we have confirmed that this results from nonlinearities in the focusing fields which are sampled by the particles to a greater extent when the beam grows "fat" as a result of the compression.

Longitudinal Stability with Finite Gap Impedance: A known instability is associated with the impedance of the accelerating modules. While careful design and techniques such as feed-forward stabilization should afford suppression of the instability, it is important to be able to model (in a causal, self-consistent way) multidimensional effects such as wave reflection at the bunch ends and radial variations of the interaction between particles and modules. Such effects may be especially significant in the driver-relevant low growth rate regime.[7, 16]

Other applications of the (r, z) model include studies of equilibration processes, of the equilibrium axial dependence of the emittance at the beam ends, and of axial confinement using "ears."

Bent-beam dynamics: We have examined beam behavior in a variety of lattices which incorporate bends; these include models of the 180° bend planned as an ILSE experiment, and both round and racetrack-shaped recirculator configurations.[17]

A lattice we have considered is similar to one proposed for an ILSE experiment.[18] For this system, the phase advances per lattice period are $\sigma_0 = 72°$, $\sigma = 20°$, and dipoles (20 cm) and quadrupoles (20 cm) alternate in a FOBODOBO lattice with full period 1.2 m. The first dipole begins at $z = 2.6$ m, the last ends at 16.6 m (after 180° of bending), and we ended the runs at 18 m (900 steps). We considered axially-cold and -hot ($T_z \sim T_\perp$) beams. The emittance of the axially-hot beam grows; that of the axially-cold beam does not. An axially-hot straight beam in a similar lattice without dipoles does not appear to suffer emittance growth, nor does an emittance-dominated beam in a bend. The beam centroid locations (away from mid-pulse) move radially during their transit of the bend because of the head-to-tail velocity "tilt;" nonetheless, they are re-injected nearly along the centerline of the straight section which follows, due to the "first-order achromat" design.

ESQ injector: We have recently begun modeling this class of systems in 3d.[19] One item of interest is an "energy effect" associated with the fact that the electrode potential differences are comparable to the beam energy, at least at the low-energy end. This leads to some emittance growth, which may be reduced through careful design.

References

[1] T. J. Fessenden and A. Friedman, *Nucl. Fusion* **31**, 1567 (1991).

[2] J. J. Barnard, A. L. Brooks, F. Coffield, F. Deadrick, L. V. Griffith, H. C. Kirbie, V. K. Neil, M. A. Newton, A. C. Paul, L. L. Reginato, W. M. Sharp, J. Wilson, S. S. Yu, and D. L. Judd, Lawrence Livermore National Laboratory Report UCRL-LR-108095, May 1992.

[3] S. S. Yu, J. J. Barnard, G. J. Caporaso, A. Friedman, D. W. Hewett, H. Kirbie, M. A. Newton, V. K. Neil, A. C. Paul, L. L. Reginato, W. M. Sharp, T.F. Godlove, R.O. Bangerter, C.G. Fong, and D.L. Judd, *Particle Accelerators* **37-8**, 489 (1992).

[4] A. Friedman, J. J. Barnard, D. A. Callahan, G. J. Caporaso, Y.-J. Chen, J. F. DeFord, W. M. Fawley, D. P. Grote, I. Haber, K. D. Hahn, E. Henestroza, D. W. Hewett, D. D.-M. Ho, A. B. Langdon, E. P. Lee, A. N. Payne, C. C. Shang, W. M. Sharp, H. D. Shay, L. Smith, and S. S. Yu, "Simulation of Heavy Ion Fusion Beams," to appear in *Proc. IAEA Tech. Comm. Mtg. on Advances in Simulation and Modeling of Thermonuclear Plasmas*, Montreal, June 15–17, 1992; LLNL Report UCRL-JC-110193, 1992.

[5] A. Friedman, D. P. Grote, D. A. Callahan, A. Bruce Langdon, and I. Haber, *Particle Accelerators* **37-8**, 131 (1992).

[6] D. P. Grote, A. Friedman, and I Haber, *ibid.*, p. 141.

[7] D. A. Callahan, A. B. Langdon, A. Friedman, D. P. Grote, and I. Haber, *ibid.*, p. 97.

[8] Alex Friedman, David P. Grote, and Irving Haber, *Phys. Fluids B* 4, 2203 (1992).

[9] A. Friedman, R. O. Bangerter, D. A. Callahan, D. P. Grote, A. B. Langdon, and I. Haber, *Proc. 2nd European Particle Accelerator Conference*, Nice, June 12–16, 1990.

[10] A. Friedman, *Proc. 13th Conf. on Numerical Simulation of Plasmas*, Santa Fe NM, 1989; LANL, R. J. Mason, Ed.

[11] T. J. Fessenden et. al., *Proc. of the 1987 Particle Accelerator Conf.*, Pp. 898-902, IEEE Cat. No. 87CH2387-9 (1987).

[12] T. Fessenden, R. Bangerter, D. Berners, J. Chew, S. Eylon, A. Faltens, W. Fawley, C. Fong, M. Fong, K. Hahn, E. Henestroza, D. Judd, E. Lee, C. Lionberger, S. Mukherjee, C. Peters, C. Pike, G. Raymond, L. Reginato, H. Rutkowski, P. Siedl, L. Smith, D. Vanecek, S. Yu, F. Deadrick, A. Friedman, L. Griffith, D. Hewett, M. Newton, and H. Shay, "ILSE, The Next Step toward a Heavy Ion Induction Accelerator for Inertial Fusion Energy," *Proc. 14th Int. Conf. on Plasma Physics and Controlled Nuclear Fusion Research*, IAEA, Wurzburg, Germany, Sept. 30-Oct. 7, 1992.

[13] Paul F. Dubois et. al., "The Basis System," LLNL Document M-225 (1988).

[14] S. Slinker, J. Krall, M. Lampe, and G. Joyce, in *Conference Record of the 1991 IEEE Particle Accelerator Conference*, San Francisco, edited by L. Lizama and J. Chew (IEEE, NJ, 1991), p. 242.

[15] K. L. Brown and R. V. Servranckx, in *Physics of High Energy Particle Accelerators*, AIP Conf. Proc. **127**, 75 (1983).

[16] D. A. Callahan, A. B. Langdon, A. Friedman, and I. Haber, submitted to *Proc. APS Div. of Plasma Phys. Mtg.*, Seattle, Nov. 16–20, 1992.

[17] J. J. Barnard *et. al.*, this conference.

[18] E. P. Lee, *Nucl. Inst. Meth. in Plasma Research* **A278**, 178 (1989).

[19] D. P. Grote and A. Friedman, submitted to *Proc. APS Div. of Plasma Phys. Mtg.*, Seattle, Nov. 16–20, 1992.