ANALYSIS OF THE LONGITUDINAL INSTABILITY OF INTENSE BEAMS IN A TRANSPORT CHANNEL WITH COMPLEX WALL IMPEDANCES*

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Abstract

The longitudinal instability of intense beams in a transport channel with complex wall impedances is analyzed based on the framework of the Vlasov theory. The dispersion equation derived characterizes the instability with all the relevant parameters including the space charge, the beam energy spread, the real and imaginary parts of the complex wall impedances. For the beam without energy spread, the growth rates of the slow waves are discussed. For the beam with energy spread, the illustrative examples show the stable regions determined by the beam and wall properties.

Introduction

The longitudinal instability of intense beams is a very important issue in particle accelerators and other applications such as microwave devices [1,2]. In recent years, the problem received new attention in connection with the role of the longitudinal instability in induction linear accelerators which are a promising driver for Heavy Ion Inertial Fusion [3-5]. When the heavy ions are accelerated by the induction gaps, the beam sees the generally complex gap impedances along the accelerators. The interaction between the intense beams and these impedances causes the longitudinal instability.

Following our previous work [6,7], this analysis of the longitudinal instability of intense beams in a channel with complex impedances is based on the Vlasov theory, taking into account the beam energy spread. The dispersion equation for the cold beam is expressed as an impedance balance equation and the growth rates of the slow waves are discussed. The stability due to Landau damping for the hot beams is illustrated by examples.

Dispersion Equation

Considering one-dimensional model of a coasting beam, the perturbed distribution function f_1 should satisfy the linearized Vlasov equation:

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}+\frac{q}{m\gamma^3}E_0\frac{\partial}{\partial v}\right)f_1(z,v,t) = -\frac{q}{m\gamma^3}E_1(z,t)\frac{\partial f_0(v)}{\partial v} \quad , \qquad (1)$$

where q/m denotes the ratio of the charge and mass of the charged particles, γ is the relativistic factor, E_0 is the net unperturbed field which is supposed to vanish, $f_0(v)$ is the unperturbed distribution function, and $E_1(z,t)$ is the induced longitudinal electrical field acting on the beam particles. Under the long wavelength condition, the field E_1 can be calculated as [8]

$$E_1(z,t) = -\frac{g}{4\pi\varepsilon_0\gamma^2}\frac{\partial\Lambda_1(z,t)}{\partial z}+E_w(z,t) \quad , \qquad (2)$$

where $\Lambda_1(z,t)$ is the perturbed line charge density, $E_W(z,t)$ is the induced field on the pipe wall of the transport channel, ε_0 is the permittivity of free space, and g is a geometric factor of order unity. Assuming a wave solution in the form of $e^{i(\omega t-kz)}$ for all the perturbed quantities, the induced field on the wall of the transport channel can be related to the perturbed beam current and the wall complex wave impedance $Z_w^*(k,\omega)$ per unit length as

$$E_w(k,\omega) = -Z_w^*(k,\omega)\Lambda_0\int v\, f_1(k,v,\omega)\,dv \quad , \qquad (3)$$

where Λ_0 is the unperturbed line charge density.

Solving Eqs. (1-3) along with the continuity equation in the complex (k,ω) domain leads to the dispersion equation

$$\frac{q\Lambda_0}{m\gamma^3}\left[\frac{-kg}{4\pi\varepsilon_0\gamma^2}-i\frac{\omega}{k}Z_w^*(k,\omega)\right]\int\frac{df_0(v)}{dv}\frac{dv}{(\omega-kv)} = 1 \quad . \qquad (4)$$

In what follows below, we discuss only the space-charge wave solutions of Eq. (4) under a sinusoidal perturbation of frequency ω_0. In this case, Eq. (4) can be written as

$$K\left[X_s'+i\frac{k_0^2}{k^2}(R'+iX')\right]\frac{v_0^3}{c}\int\frac{df_0(v)}{dv}\frac{dv}{(v-\omega_0/k)} = 1 \quad , \qquad (5)$$

where K is the generalized perveance of the beam, $k_0=\omega_0/v_0$ is a characteristic wave number with v_0 being the average beam velocity, $R'=R^*\lambda_0/Z_0$ and $X'=X^*\lambda_0/Z_0$ are the normalized real and imaginary parts of the wall complex impedance with $\lambda_0=2\pi/k_0$ and Z_0=377 ohms, and X_s' is the normalized space charge impedance defined by

$$X_s' = X_s^*\frac{\lambda_0}{Z_0} = \frac{g}{2\beta\gamma^2} \quad . \qquad (6)$$

Here X_s^* is the space charge impedance per unit length, which is a function of the beam energy and the frequency ω_0.

Growth Rates of the Instability for a Cold Beam

A beam without energy spread at a meta-equilibrium state is characterized by the Dirac delta distribution function. Eq. (5) then can be approximated as

$$i\frac{(k-k_0)^2}{k_0^2\beta K} \cong -\left[R'+i(X'-X_s')\right] \qquad (7)$$

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The quantity on the left hand side of Eq. (7) is usually referred to as the normalized electronic impedance of the beam, which is simply equal to the normalized space charge impedance minus the wall impedance. The general solution of Eq. (7) is

$$k = k_r + ik_i = k_0\left(1 \pm \sqrt{\beta K} \cdot \sqrt{X'_s - X' + iR'}\right) , \quad (8)$$

where k_r is the perturbed wave number and k_i is the spatial growth or decay rate depending on the sign of its numerical value. More explicit expressions for Eq. (8) can be obtained in some parameter ranges. For instance, if $X_s' >> R'+X'$, the wave number and growth rate of the slow waves are

$$\begin{cases} k_r = k_0\left\{1 - \left[K\beta\left(X'_s - X'\right)\right]^{1/2}\right\} \\ k_i = k_0\dfrac{R'}{2}\left[\dfrac{K\beta}{\left(X'_s - X'\right)}\right]^{1/2} \end{cases} \quad (9)$$

As an example to apply this result, we consider an induction linear accelerator consisting of many induction gaps, which can be represented by a succession of essentially uncoupled resonators. Each induction gap may be equivalent to a discrete R, C, L parallel resonant circuit occupying an equivalent interaction length z_g along the beam line, as shown in Fig. 1. The resistance R is mainly an external load for beam loading purposes. It also accounts for the energy loss of the beam in the induction gap. The complex wall impedance of such an equivalent circuit is

$$Z_w(k, \omega) = \frac{R}{1 + iR\left(\omega C - \dfrac{1}{\omega L}\right)} . \quad (10)$$

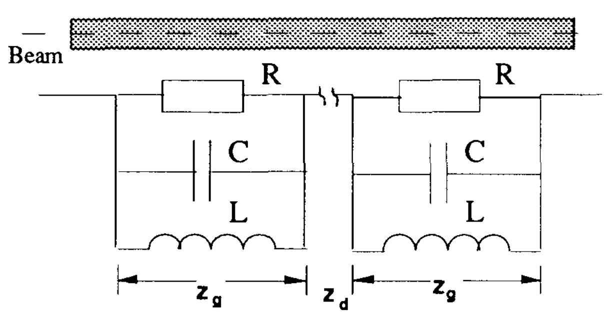

Fig. 1. Circuit model for induction gaps.

For a sinusoidal perturbation of frequency ω_0, the relative real and imaginary parts of this impedance, i.e. $R(\omega_0)/R$ and $X(\omega_0)/R$ as a function of ω_0/ω_r, are plotted in Fig. 2, where $\omega_r=(LC)^{-1/2}$ is the resonance frequency of the resonator and it is also supposed that the components of the resonator satisfies a relation of $R(L/C)^{-1/2}=1$. From Eq. (9), the approximate growth rate per induction gap is given by

$$k_i z_g \approx \pi \frac{R}{Z_0}\sqrt{\frac{K\beta}{X'_s}}\left[1 + R^2\left(\omega_0 C - \frac{1}{\omega_0 L}\right)^2\right]^{-1} , \quad (11)$$

which has a maximum value at the resonance.

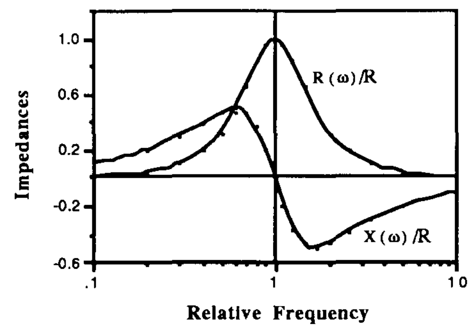

Fig. 2. Impedance of the induction gap circuit model.

Stable Regions for a Hot Beam

The longitudinal instability for hot beams have been studied for circular accelerators [9,10]. Due to Landau damping the stable regions can be found in the half R'-X' plane for certain beam and wall parameters. To illustrate this, we first consider the Lorentz distribution

$$f_0(v) = \frac{1}{\pi\alpha v_0}\left[\left(\frac{v - v_0}{\alpha v_0}\right)^2 + 1\right]^{-1} \quad (12)$$

The parameter α is a real quantity to specify the amount of the velocity spread, and $2\alpha v_0$ is equal to FWHM of the distribution. Carrying out the velocity integration for k_i=0 in Eq. (5), the dispersion equation becomes

$$i\frac{\left[(1 - i\alpha)k_r - k_0\right]^2}{k_r k_0 \beta K} \cong -\left[R' + i\left(X' - \frac{k_r^2}{k_0^2}X'_s\right)\right] , \quad (13)$$

which describes the boundary between the stable and unstable regions of the instability in the R'-X' plane. Note that in Eq. (13) the impedances are normalized with respect to $\lambda=2\pi/k_r$ instead of $\lambda_0=2\pi/k_0$. An explicit expression for the boundary then can be written as

$$X' = \left(1 - \frac{K\beta}{2\alpha}R'\right)\left[X'_s\left(1 - \frac{K\beta}{2\alpha}R'\right) + \frac{\alpha^2}{K\beta}\right] - \frac{K\beta(R')^2}{2\alpha(2\alpha - K\beta R')} . \quad (14)$$

Fig. 3a shows the result from Eq. (14) where the boundary curves are plotted for three different velocity spreads α at a fixed beam perveance K. The stable regions are bounded by the curves and the X' axis. The upper half plane is characterized by the inductive component of the complex impedance, while the lower half plane is capacitive. The stable regions are mainly obtained by the capacitive components. When the beam energy spread is getting smaller, the stable region is narrower and, eventually collapses to the X' axis. Fig. 3b plots the stable regions for a fixed velocity spread but with three different beam perveances K. It is evident that the space charge increases the instability dramatically.

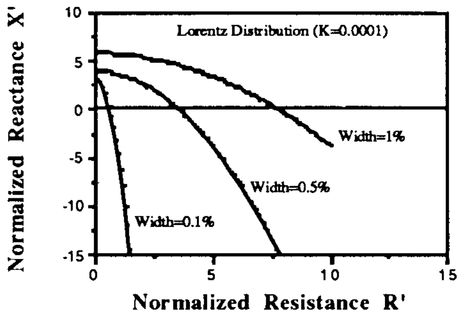

Fig. 3a. Stable regions for the Lorentz distribution in the half R'-X' plane for three different velocity spreads at a fixed beam perveance K, where β=0.3 and g=2 are used in the calculation.

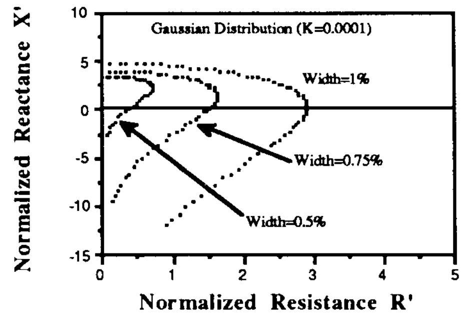

Fig. 4a. Stable regions for the Gaussian distribution in the half R'-X' plane for three different velocity spreads at a fixed beam perveance K, where β=0.3 and g=2 are used in the calculation.

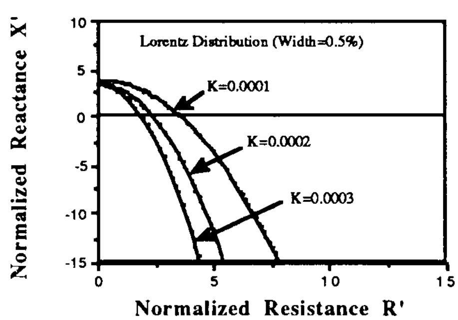

Fig. 3b. Stable regions for the Lorentz distribution in the half R'-X' plane for three different beam perveances at a fixed velocity spread.

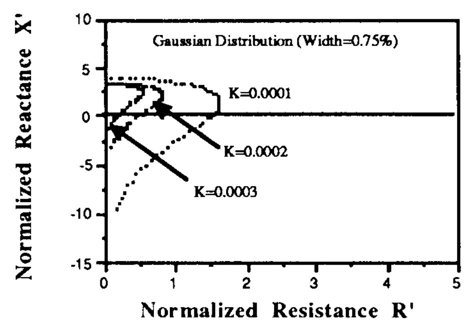

Fig. 4b. Stable regions for the Gaussian distribution in the half R'-X' plane for three different beam perveances at a fixed velocity spread.

For a Gaussian distribution

$$f_0(v) = \frac{1}{\sqrt{\pi}\,\alpha v_0} \exp\left[-\left(\frac{v - v_0}{\alpha v_0}\right)^2\right], \qquad (15)$$

the velocity integral for k_i=0 in Eq. (5) yields

$$\frac{2K\beta}{\alpha^2}\left[X'_s + i\frac{k_0^2}{k^2}(R' + iX')\right][1 + \xi Z(\xi)] = 1 \quad . \qquad (16)$$

Here Z(ξ) is the plasma dispersion function defined by

$$Z(\xi) = \sqrt{\pi}\int_{-\infty}^{+\infty} dx \frac{e^{-x^2}}{x - \xi}, \qquad (17)$$

with $\xi=(k_0/k_r-1)/\alpha$. The boundaries for the stable regions are obtained by numerical solutions of Equation (16) and plotted in Fig. 4a,b. In comparison with the Lorentz distribution, the stable regions are much smaller for the Gaussian distribution under the same beam parameters.

Summary

The one-dimensional Vlasov equation has been used to analyze the longitudinal instability in a linear beam transport channel with general wall impedances. The growth rates of the slow waves for a cold beam are discussed and calculated for the induction gaps. The beam energy spread is a stabilizing factor for beam transport due to Landau damping. It is possible to achieve stable operations in the design of induction accelerators for heavy ion inertial fusion if the right beam parameters and wall properties are chosen.

References

[1] V. K. Neil and A. M. Sessler, Rev. Sci. Instr. **36**(4), p. 429, 1965.
[2] J. R. Pierce, Proc. I. R. E., **35**, pp. 111-123 (1947).
[3] E. P. Lee, in this proceedings.
[4] E. P. Lee and L. Smith, Proc. of the 1991 IEEE Particle Accelerator Conference, San Francisco, CA, May 6-9, 1991, pp. 1737-1739.
[5] L. Smith, in ERDA Summer Study of Heavy Ions for Inertial Fusion, edited by R. O. Bangerter, W. B. Herrmannsfeldt, D. L. Judd, and L. Smith (LBL-5543, 1976), p. 77.
[6] J. G. Wang, M. Reiser, W. M. Guo, and D. X. Wang, Particle Accelerators, Vol. 37-38 (1992), pp. 181-189.
[7] J. G. Wang and M. Reiser, Proc. of the 1991 IEEE Particle Accelerator Conference, San Francisco, May 6-9, 1991, pp. 1821-1823.
[8] A. Hofmann, CERN 77-13, p. 139 (CERN, Geneva, 1977).
[9] A. G. Ruggiero and V. G. Vaccaro, CERN Report ISR-TH/68-33 (1968).
[10] E. Keil and W. Schnell, CERN Report ISR-TH-RF/69-48 (1969).