AN APPROACH TO DESIGN OF A ROBUST FOCUSING CHANNEL

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The long quadrupole channels were studied in the works [1,2] as well as the equations describing the beam transverse dimensions increase under random perturbations. In present report these investigations is extended to focusing channels of arbitrary type and structure.

The equation of perturbed motion may be presented in the following way:

$$x'' + B(x)x + f(x,z) = 0 \qquad (1)$$

and $B(z)$ is the periodic function with a unit period.

Assuming that $x = e^{i\mu z}\cdot(a(z) + ib(z))$ is the solution of the non-perturbed equation, and $a(z)$ and $b(z)$ are periodic functions with a unite period, the equation (1) may be brought into the following form:

$$\begin{cases} r' = \rho\cdot\sin(\mu z+\eta+\psi)\cdot f\left(z\rho\cdot\cos(\mu z+\eta+\psi), z\right) \\ r\eta' = \rho\cdot\cos(\mu z+\eta+\psi)\cdot f\left(r\rho\cdot\cos(\mu z+\eta+\psi), z\right) \end{cases} \qquad (2)$$

where $\rho^2 = a^2 + b^2$ is the amplitude and $\psi = arctg(b/a)$ is the phase of the Flauket function, μ – frequency of smooth oscillations.

Let us consider noncoherent oscillations, in which case the equation of particles motion may be presented as follows:

$$r'' - \tilde{G}(z)\cdot r = 0$$

Assuming $\tilde{G}(z) = G(z)\cdot(1 + \alpha(z))$ and using the main equation of the perturbed motion in the form (2), one obtains:

$$\begin{cases} r' = r\cdot\alpha\ G\ \rho^2\sin^2(\mu z + \eta + \psi) \\ \eta' = \alpha\ G\ \rho^2\cos^2(\mu z + \eta + \psi) \end{cases}$$

with the initial condition $r(0) = 0$, $\eta(0) = \varphi_o$.

If $\eta(z)$ is the solution of the second equation, the first one may be resolved as follows:

$$\theta(z) = \frac{r(z)}{r_o} =$$

$$\exp\left(\int_0^z \alpha G\rho^2\sin(\mu t+\eta+\psi)\cdot\cos(\mu t+\eta+\psi)\ dt\right) = \exp F(z)$$

from which it is clear that the distribution of the random variable $\theta(z)$ is determined by the distribution of the random variable $F(z)$.

The $F(N)$ function may be presented as:

$$F(N) = \sum_{k=1}^{N} F_k =$$

$$\sum_{k=1}^{N}\int_{k-1}^{k} \alpha G\rho^2\sin(\mu t+\eta+\psi)\cdot\cos(\mu t+\eta+\psi)\ dt$$

while

$$F_k = \sum_{i=1}^{M}\frac{1}{2}\alpha_i G_i\rho_i^2\varepsilon\cdot\sin 2(\mu t_i + \eta_i + \psi_i),$$

where M is the number of perturbed elements per period, $\alpha_i G_i$ is the field error in the i-th element, $t_i = k-1+\xi_i$, ξ_i – is the middle of the i-th element, $\rho_i = \rho(\xi_i)$, $\eta_i = \eta(t_i)$, $\psi_i(\xi_i)$, ε is the element relative length. (The perturbed element may be represented either by a focusing lens or by an accelerating gap).

One obtains with the accuracy up to α^2:

$$\eta(N) = \varphi_o + \sum_{k=1}^{N}\eta_k = \varphi_o + \sum_{k=1}^{N}\int_{k-1}^{k}\alpha G\rho^2\cos^2(\mu t+\psi+\varphi_o)$$

$$\eta_k = \sum_{i=1}^{M}\alpha_i G_i\rho_i^2\varepsilon\cos^2(\mu t_i+\varphi_o+\psi_i).$$

One may assume the $F(z)$ variable to have normal distribution with the mathematical expectation $M[F] = \sum_{k=1}^{N} M[F_k]$ and dispersion $D[F] = \sum_{k=1}^{N} D[F_k]$.

Taking $F(N)$ with the accuracy up to α^3 and making allowance to the independent character of the initial perturbations, one obtains:

$$M[F(N)] = M\left\{\sum_{k=1}^{N}\sum_{i=1}^{M}\left(\frac{1}{2}\cdot\alpha_i G_i\rho_i^2\cdot\sin 2(\mu t_i+\varphi_o+\psi_i) + \frac{1}{2}\cdot\alpha_i^2 G_i^2\rho_i^4\cdot\cos 2(\mu t_i+\varphi_o+\psi_i)\cdot\cos^2(\mu t_i+\varphi_o+\psi_i)\right)\right\}$$

which being average over the frequencies, multiple of μ, gives

$$D[F(N)] = M[F(N)] = \frac{1}{8}\ \overline{\alpha^2} G_i^2\rho_i^4 = N\cdot\Delta^2$$

where $\overline{\alpha^2}$ is the dispersion of the initial error.

So, the amplitude growth of the transverse oscillations of a certain particle is given by $\theta = \frac{r(N)}{r_o} = \exp(F(N))$, where random variable $F(N)$ is normally distributed, and

$$M[F(N)] = D[F(N)] = N\cdot\Delta^2$$

The quantity $(G_i^2\rho_i^2)^2$ must be summed up over all perturbed elements of a unit period in order to find Δ^2. With the aim to estimate Δ^2 quantity, one may replace each item of it by the value $\overline{G^2}\ \overline{\rho^4}$ averaged over a period. One may assume that

$$\overline{\rho^4} = \frac{1}{2}\cdot(\rho^4_{max} + \rho^4_{min}) = \frac{1}{2\nu^2}\cdot\left(1 + \frac{1}{\varkappa^4}\right)$$

and finally one obtains that

$$\Delta^2 = \frac{M\overline{G^2}}{2\nu^2}\cdot\left(1 + \frac{1}{\varkappa^4}\right)\cdot\overline{\alpha^2}$$

The quantity G represents refractive force gradient of the element F_r. Finally,

$$M[F(N)] = D[F(N)] = \frac{N_o}{2}\cdot\frac{\overline{F_r^2}}{\nu^2}\cdot\left(1+\frac{1}{\varkappa^4}\right)\cdot\overline{\alpha^2} = N_o\Delta_o^2 \qquad (4)$$

where N_o is the overall number of perturbed elements in the channel.

Repeating the mathematical treatment of Ref.[1] and taking the initial equilibrium crossection in the form of ellipsoid matched with a unit period, one obtains that $\theta_r(N) = \max_{o\le\varphi_o\le 2\pi} \theta(N)$ - is a random quantity having the following distribution:

$$P(\theta_r) = 1 - \exp\left(-(ln\theta_r)^2/(2N_o\Delta_o^2)\right) \qquad (5)$$

It follows, that the beam dimensions growth coefficient will not exceed the α limit with the probability P, if tolerances are defined by the equation

$$N_o\Delta_o^2 = -\frac{(ln\alpha)^2}{2ln(1-p)} \qquad (6)$$

and by the equation (4) for $N_o\Delta_o^2$.

Now we pass on to coherent oscillation equations. Random errors causing them result in deviation of the channel axis from an ideal line. We assume the channel axis $y(z)$ to be a polygonal line consisting of segments connecting centers of neighboring drift tube ends.

Assuming that drift tube ends are displaced independently with dispersion $\overline{\Delta^2}$ and averaging over oscillations with μ frequency, one obtains in a similar manner the dispersion of the beam displacement from the channel axis at the end of a section under consideration

$$D[x(N)] = \frac{N_o}{8}\cdot\frac{\overline{F_r^2}}{\nu^2}\cdot\left(1 + \frac{1}{\varkappa^4}\right)\cdot\overline{\Delta^2} \qquad (7)$$

The amplitude of the center oscillations $\alpha^2 = x^2 + x'^2\rho_o^2$ is distributed as

$$F(\alpha) = 1 - \exp\left(-\alpha^2/\overline{\alpha^2}\right) \qquad (8)$$

where

$$\overline{\alpha^2} = \frac{N_o}{4}\cdot\frac{\overline{F_r^2}}{\nu^2}\cdot\left(1 + \frac{1}{\varkappa^4}\right)\cdot\Delta^2 = N_o\overline{\Delta^2_{oc}} \qquad (9)$$

The transverse oscillations amplitude will not exceed the preset value x with the probability P, if the transverse displacement of the drift tube ends tolerances are found from the equation

$$N_o\Delta^2_{oc} = -\frac{x^2}{ln(1-p)} \qquad (10)$$

and from equation (9) for $N_o\Delta^2_{oc}$.

If one assumes that the initial displacement is the displacement of drift tube center Δ_c, then $\overline{\Delta^2} = 2\cdot\Delta_c^2$.

The aforecited equations be valid for a focusing channel with an arbitrary periodic structure. They demonstrate the relation between the effective emittance growth, the initial errors, the number of perturbed elements and the channel parameters (the mean refractive force and mean beam envelope).

Analyzing formulas defining the beam radius growth caused by random errors, as well as the coherent oscillations amplitude growth, we found that the lens refractive force $P = G\cdot\varepsilon/\nu$, where ε is the relative lens length, is the universal parameter which, on the one hand, determines the beam growth caused by random errors, and which, on the other hand, may be varied in a rather wide range harmlessly to the channel admittance capacity in order to reduce its error susceptibility.

For simplicity we confine our investigation to the most widely used focusing period structures FODO and FDO, rectangular focusing field distribution in the absence of RF field. We designate the relative distance between lenses centers in focusing period length as ε_o and relative lenses length as ε ($\xi \le \xi_o \le 0.5$). For the FDO structure $\xi_o < 0.5$, for FODO $\xi_o = 0.5$.

It is necessary to find the minimum of the function $P(\xi,\xi_o)$ for a given channel admittance capacity. For this purpose we consider the equation

$$\nu_{min}(P, \xi, \xi_o) = const$$

where ν_{min} is the transverse oscillations relative frequency minimum. This transcendental equation may be solved using a computer. The solution shows that function P depends weakly on the parameter ξ/ξ_o which changed in a wide range (from 0.01 up to 1). But the point of peculiar interest for linac focusing systems designers is the following. There is strong nonlinear dependence of lens refractive force on relative inter-lenses distance. For example in Fig.1 are shown these curves with ν_{min} = 0.6 and $\varepsilon = \varepsilon_o$. As ε_o diminishes the required lens refractive strength increases. While ε_o is rather big its change in a wide range does not result in a considerable refractive strength growth and only for $\xi_o \le 0.06$ parameter P grows considerably.

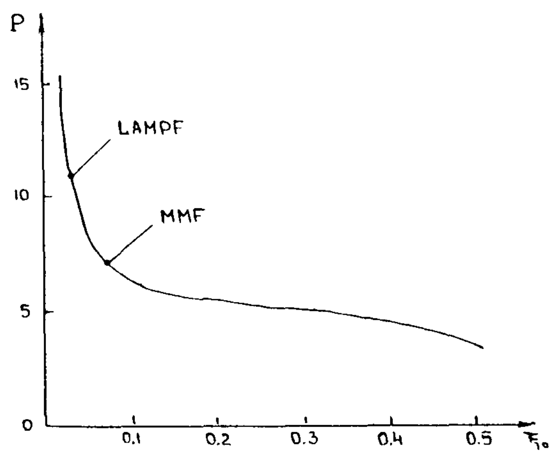

Fig.1.Plot of refractive lens force $P = G\varepsilon/\nu$ versus distance between lenses (in units of focusing period length) for the $\varepsilon = \varepsilon_o$, ν_{min} = 0.6 case

The following recommendations might be of use for designers:

1) The choice of small ξ_o for which $P \gg P_{min}$ is inadmissible.

2) It is advisable to increase ξ_o, and for chosen ξ_o to reduce ξ as far as technology allows.

As an example we may cite the high energy LAMPF part. In its first eight resonators ξ_o = 0.05, on the average, (the focusing period length is 370...425 cm, distance between lenses centers in doublets is 20 cm) and in other 36 resonators ξ_o is even smaller ξ_o = 0.025 (the focusing period length is 760...860 cm with the same distance between lenses centers). The too small value of ξ_o resulted in a big coherent oscillations amplitude and considerable growth of the beam radius. Had the designers increased the value of ξ_o up to 0.08, the coherent oscillations amplitude growth as well as the beam radius growth would have been 1.5...2 times smaller with the same tolerances and the same admittance capacity.

The aforenamed recommendations were taken into consideration when the Moscow Meson Facility focusing channel was designed as a result of which the average value of ξ_o was chosen to be 0.085 (the focusing period length is 220...350 cm, the distance between lenses centers is 27.5 cm).

References

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2. B.I.Bondarev, A.P.Durkin, L.Yu.Soloviev. Strong Focusing Fields Perturbations and Their Influence upon Particle Motion in Linac. J. Techn. Phys., 1979, vol.49, No.8, pp.1662-1672.