

**CALCULATION OF MECHANICAL VIBRATION FREQUENCIES OF STIFFENED SUPERCONDUCTING CAVITIES\***

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**Abstract**

We calculated the frequencies of transverse and longitudinal mechanical-vibration modes of the HEPL-modified, CERN/DESY four-cell superconducting cavity, using finite-element techniques. We compared the results of these calculations, including the stiffening of the cavity with rods, with mode frequencies measured at HEPL. The correlation between data was significant. The same techniques were also used to design and optimize the stiffening scheme for the seven-cell 805-MHz superconducting cavity being developed at Los Alamos. In this report, we describe the final stiffening scheme and the results of our calculations.

**Introduction**

The seven-cell 805-MHz superconducting cavity being developed at Los Alamos will be used to accelerate low-intensity pion beams [1]. Because of the low beam loading, it is planned to drive the cavity with an rf power coupler that has an external Q of  $2 \cdot 10^7$  or higher. Operating with such a high external Q saves rf power only as long as the cavity is not subject to mechanical vibrations (microphonics), which excessively modulate its resonant frequency. If the cavity frequency excursions, however, are of the order of half a bandwidth or greater because of microphonics, they force the control system to provide large amounts of power off resonance to control the cavity's field phase and amplitude to required tolerances. Microphonics can thus prevent rf power savings that would otherwise be possible at high external Q operating conditions.

Most cavity mechanical vibrations are driven by ambient noise sources. These noise sources have large amplitudes at low frequencies ( $< \sim 120$  Hz). Stiffening the cavity to raise its mechanical vibration frequencies above  $\sim 200$  Hz decouples it from these noise sources.

Before designing the stiffening scheme using finite-element techniques to estimate the mechanical vibration frequencies of the seven-cell cavity, we felt we needed to validate these techniques. To do this, we calculated and compared the frequencies for a stiffened (and unstiffened) HEPL-modified CERN/DESY four-cell cavity to the actual frequencies measured at HEPL [2].

After validating the finite-element techniques, we analyzed and optimized the stiffening structure for the seven-cell cavity at Los Alamos.

**HEPL FOUR-CELL CAVITY CALCULATIONS**

Figure 1 shows the four-cell CERN/DESY structure that was modified at HEPL for 1300-MHz operation. The cavity is

approximately 68.7 cm long and nominally 0.3175 cm thick. It has 20.5-cm-diam cell equators and 6.5-cm-diam cell irises (approximately).

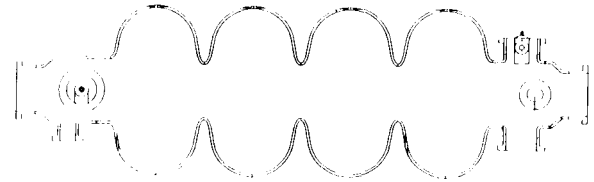


Fig. 1 Four-cell CERN/DESY structure.

We built a three-dimensional PATRAN® model of the cavity which consisted of 792 shell elements and 2400 nodes. We used ABAQUS®, which relies on the subspace iteration technique [3], to extract the eigenvalues so that we could determine the fundamental modes. We modeled the rf coupler ports and end flanges as masses distributed on rings around the beam tube extensions at the proper longitudinal positions.

We first ran the model for the unstiffened cavity, using the same end conditions used at HEPL (free-free). Figures 2-3 show some of the exaggerated mode shapes. The lowest mode is a 76-Hz transverse-vibration mode.

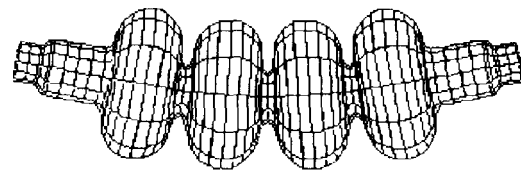


Fig. 2 First transverse mode.

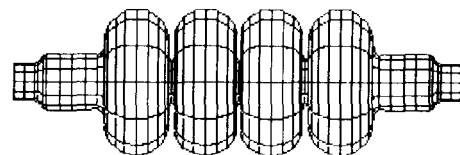


Fig. 3 First longitudinal mode.

The results for the unstiffened cavity are compared with the HEPL test data in Table 1. As can be seen, the correlation between data is significant.

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**Table 1 Comparison of HEPL Test Data and Finite Element Model—Four-Cell Cavity (Unstiffened)**

Model	HEPL Test Data	Analysis
First Transverse	72 Hz	76 Hz
Second Transverse	172 Hz	185 Hz
Third Transverse	288 Hz	309 Hz
First Longitudinal	236 Hz	232 Hz
Second Longitudinal	460 Hz	467 Hz

Next, we modified the analytical model to include the three rods used at HEPL to stiffen the cavity. These rods, made of 1.27-cm-diam threaded steel, were attached at each cell to Nb tabs welded to the cell wall and then modeled with 33 three-noded bar elements. Nodes on the cavity shell elements and on the rods were constrained to move together in all six degrees of freedom.

Results of the modified analysis are compared to the HEPL test data in Table 2. Again, the correlation between data was significant, thus proving the validity of the finite-element techniques.

**Table 2 Comparison of HEPL Test Data and Finite Element Model—Four-Cell Cavity (Stiffened)**

Model	HEPL Test Data	Analysis
First Transverse	440 Hz	423 Hz
Second Transverse	532 Hz	536 Hz
Third Transverse	No Data	982 Hz
First Longitudinal	744 Hz	720 Hz
Second Longitudinal	800 Hz	862 Hz

**SEVEN-CELL 805-MHz CAVITY CALCULATIONS**

The basic unstiffened seven-cell 805-MHz cavity [4] is 159.4 cm long and is nominally 0.3175 cm thick. The cavity's seven cells have an equatorial diameter of 34.1 cm and an iris diameter of 13.0 cm.

Figure 4 shows the analytical model of the seven-cell cavity. Shell elements were also used to build this model (912 elements and 2760 nodes) and the end conditions for this calculation were also "free-free".

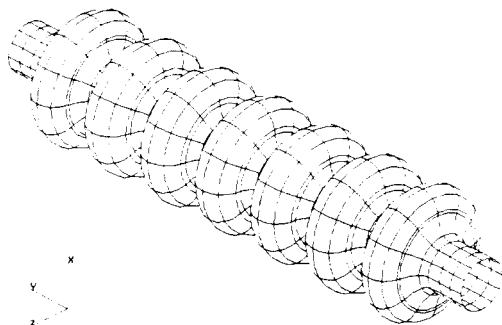


Fig. 4 Seven-cell cavity model.

Table 3 shows the results of the unstiffened cavity analysis. The mode shapes are not presented here, but they are quite similar to those of the four-cell cavity.

**Table 3 Finite Element Model Results—Seven-Cell 805-MHz Cavity (Unstiffened)**

Mode	Resonant Frequency
First Transverse	37 Hz
Second Transverse	91 Hz
Third Transverse	160 Hz
First Longitudinal	115 Hz
Second Longitudinal	231 Hz

We first tried raising the vibrational frequencies by increasing the nominal wall thickness. The analysis, however, showed that the vibrational frequencies were actually lowered, indicating to us that the mass increase actually offset the modest gain in stiffness caused by the increase in wall thickness.

Our next attempt to increase the vibrational frequencies was to add stiffening rods to the cavity. Because of the cavity's larger size, we used a total of six 2.54-cm-diam rods. The addition of the rods raised the mechanical vibration frequencies significantly. For example, the first transverse mode frequency increased to 263 Hz.

To determine the optimum rod diameter, we performed calculations using different rod diameters. The results of these calculations are summarized in Table 4. As before, the increased mass of the rods offset the gain in stiffness with increased diameter.

**Table 4 Parametric Analysis—Effect of Increasing Rod Diameter**

Rod Diameter (cm)	First Transverse Mode
2.54	263 Hz
3.81	271 Hz
5.08	250 Hz

The results of the rod-stiffened seven-cell cavity were encouraging, but we could see from the deformed-geometry plots that the unsupported rod ends and the cavity end half cells with the attached beam tubes were deflecting excessively. To lessen the problem and provide an attachment point for the cavity's frequency tuning mechanism, we shortened the rods and tied them together with a 2.54-cm thick titanium bulkhead [4]. The results of this calculation are shown in Table 5.

**Table 5 Finite Element Model Results—Seven-Cell Cavity (Stiffened with Rods and Bulkheads)**

Mode	Resonant Frequency
First Transverse	257 Hz
Second Transverse	464 Hz
Third Transverse	673 Hz
First Longitudinal	530 Hz
Second Longitudinal	>670 Hz

Rather than raise the first transverse mode frequencies, the addition of the bulkheads actually resulted in a drop of 6 Hz in the resonant frequency because of the bulkheads' weight (2 x 13.3 kg) that was added to each end of the cavity. The solid titanium rods were replaced with 3.8-cm- and OD/3.3-cm-ID hollow rods, and the solid bulkhead was changed to an "efficient bulkhead" that had an equivalent stiffness with only one third the mass. Implementing these two changes increased the first transverse mode frequency to 280 Hz.

At this point, we added 17.6 kg to the model's bulkheads to represent the approximate added mass of the tuning mechanism. Other hardware, such as flanges and coupler ports, was simulated by adding masses to the model at appropriate locations.

Figure 5 shows a representative sample of the different models we analyzed. The terms "efficient bulkhead" and "infinitely efficient bulkhead" refer to a lightweight bulkhead and a zero weight bulkhead, respectively. Because the vibration frequency of the zero weight bulkhead case is not substantially higher than that of the lightweight bulkhead case, we made no further attempt to optimize the bulkhead design and settled on a design that has a first transverse mode frequency of ~241 Hz. Figure 6 shows the cavity with the selected stiffening structure and tuning mechanism.

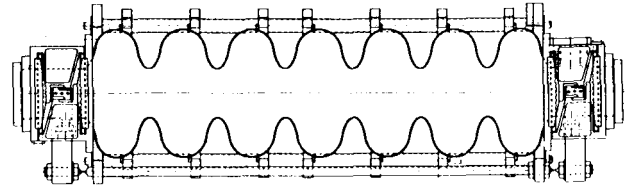


Fig. 6 Seven-cell cavity with stiffeners and tuners.

We also looked at torsional modes close in frequency to the transverse modes. For these modes the stiffening rods transformed torsional motion into a cavity length change that could dramatically change the cavity's frequency. We did not consider these modes to be detrimental to the developmental cavity because its symmetry will prevent the excitation of such modes. However, there can be configurations in which such modes can be excited, as in the case of cavities with a flange cantilevered off to one side. In such cases, even if the torsional mode amplitude is small, the longitudinal contraction amplitude and, therefore, the frequency variation could be large.

**Conclusions**

The significant correlation between the analytical models and the test data validates the use of finite-element techniques to investigate mechanical vibration problems affecting multiple cell superconducting cavities. This technique was used to optimize the stiffening scheme for the seven-cell 805-MHz superconducting cavity at Los Alamos. This cavity, when clamped to six hollow stiffening rods tied together with a 2.54-cm-thick ring bulkhead, is expected to have a lowest transverse-mode mechanical-vibration frequency of approximately 241 Hz.

**References**

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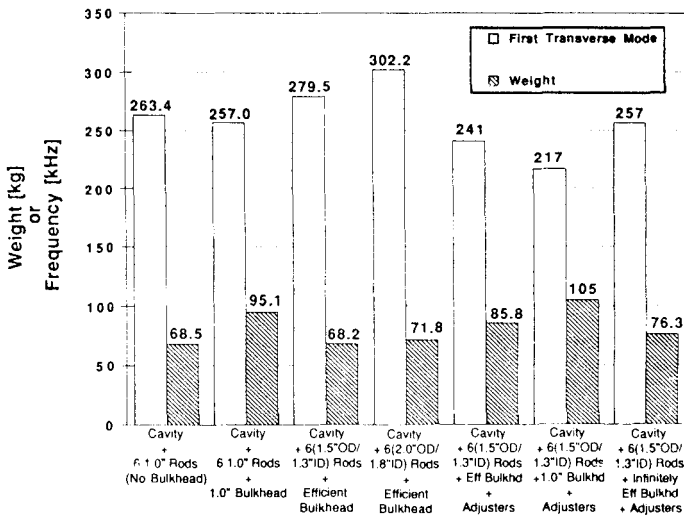


Fig. 5 Summary of Pilac Vibration Analysis.