A CORRECTION FOR EMITTANCE-MEASUREMENT ERRORS CAUSED BY FINITE SLIT AND COLLECTOR WIDTHS*

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Abstract

One method of measuring the transverse phase-space distribution of a particle beam is to intercept the beam with a slit and measure the angular distribution of the beam passing through the slit using a parallel-strip collector. Together the finite widths of the slit and each collector strip form an acceptance window in phase space whose size and orientation are determined by the slit width, the strip width, and the slit-collector distance. If a beam is measured using a detector with a finite-size phase-space window, the measured distribution is different from the true distribution. The calculated emittance is larger than the true emittance, and the error depends both on the dimensions of the detector and on the Courant-Snyder parameters of the beam. Specifically, the error gets larger as the beam drifts farther from a waist. This can be important for measurements made on high-brightness beams, since power density considerations require that the beam be intercepted far from a waist. In this paper we calculate the measurement error and we show how the calculated emittance and Courant-Snyder parameters can be corrected for the effects of finite sizes of slit and collector.

Introduction

When the transverse phase space of a beam is measured by scanning the beam with a slit and measuring the angular distribution of the beam passing through the slit with a parallel-channel collector, the measured distribution is the convolution of the true distribution with the acceptance window of the detector. The beam emittance and its Courant-Snyder parameters determined from the measured distribution are different from the true values.

The error caused by the slit-gap width was calculated by Gluckstern [1]. In this paper we extend his calculation to include the error caused by finite-width collector strips.

The next section of this paper calculates the error. Then the size of the effect is evaluated as a function of the slit and collector widths and the slit-collector drift distance for a beam measurement which has been reported [2]. Finally we show how the true beam parameters can be determined from the measured data.

Measurement Theory

The problem is illustrated in Fig. 1. A collector with conducting strips 2e wide is placed a distance L from a slit of width 2b. The slit is positioned a distance X_i from the beam axis (X_o, X'_o) and the centers of the slit and collector strip define a trajectory inclined at an angle X'_i with respect to the beam axis. In the configuration of Fig. 1, the slit and collector have the phase-space acceptance shown in Fig. 2.

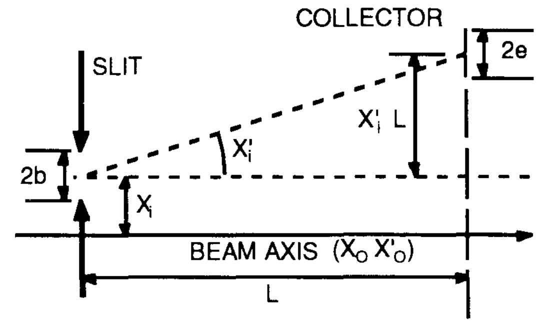

Fig. 1. Geometry of slit-collector detector.

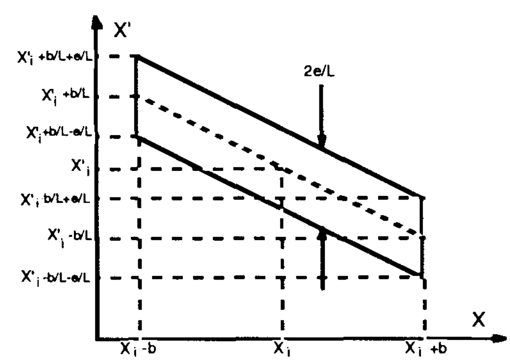

Fig. 2. Phase-space acceptance of slit and collector strip.

The beam to be measured has a true phase-space density distribution of $\rho(X,X')$. When this beam is measured with the slit and collector in Fig. 1, the measured distribution is

$$\rho_m(X_i,X_i') = \frac{L}{4be}\iint \rho(X,X')\,dX\,dX' \ , \qquad (1)$$

where the integral is over the parallelogram-shaped phase window in Fig. 2. If $\rho(X,X')$ is expanded to second order in a Taylor series and substituted into Eq. 1, the result of the integration is

$$\rho_m(X_i,X_i')=\rho(X_i,X_i') + \frac{b^2}{6}\left(\frac{\partial^2\rho}{\partial X^2}\right)_{Xi,X'i} + \frac{e^2+b^2}{6L^2}\left(\frac{\partial^2\rho}{\partial X'^2}\right)_{Xi,X'i} - \frac{b^2}{3L}\left(\frac{\partial^2\rho}{\partial X\partial X'}\right)_{Xi,X'i} . \qquad (2)$$

The rms emittance of a beam is defined as

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$$E = \pi [<(X-X_o)^2><(X'-X'_o)^2> - <(X-X_o)(X'-X'_o)>]^{1/2} \qquad (3)$$

where

$$<(X-X_o)^2> = \frac{\iint (X-X_o)^2 \rho(X,X')dXdX'}{\iint \rho(X,X')dXdX'} \qquad (4)$$

with similar expressions for $<(X'-X'_o)^2>$ and $<(X-X_o)(X'-X'_o)>$.

The apparent emittance from a beam measurement is found by using the distribution given by Eq. 2 in Eq. 3. If the beam is symmetric about (X_o,X'_o), and the lowest-order terms are kept, the apparent emittance is,

$$(E)_m{}^2 = E^2 + \frac{b^2}{3}\left[\frac{<(X-X_o)^2>}{L^2}\left(1+\frac{e^2}{b^2}\right) + <(X'-X'_o)^2> + \frac{2<(X-X_o)(X'-X'_o)>}{L}\right] \qquad (5)$$

The measured emittance is larger than the true beam emittance. For a given beam the error increases as b or e become larger or as L becomes shorter. If the geometry of the slit-collector detector is fixed, the error increases as the drift distance from a waist is increased. As a beam drifts both $<X^2>$ and $<XX'>$ get larger.

The formalism outlined above can be put into a useful form by the use of the Courant-Snyder parameters defined as

$$\beta = \frac{\pi<(X-X_o)^2>}{E} \qquad \gamma = \frac{\pi<(X'-X_o')^2>}{E} \qquad \alpha = -\frac{\pi<(X-X_o)(X'-X_o')>}{E}.$$

Equation 5 becomes

$$(E)_m = E\left(1+\frac{\Delta}{E}\right)^{1/2} \qquad (6)$$

where,

$$\Delta = \frac{\pi b^2}{3}\left(\frac{\beta}{L^2}\left(1+\frac{e^2}{b^2}\right) + \gamma - \frac{2\alpha}{L}\right) \qquad (7)$$

To apply Eq. 6 to the design of a slit and collector system, it is convenient to consider the dimensions of the measured beam at the last waist. When the phase ellipse is upright, it has an rms extent along X of A and along X' of θ. The emittance and Courant-Snyder parameters at the waist are $E = \pi A\theta$, $\beta(0) = \frac{A}{\theta}$, $\gamma(0) = \frac{\theta}{A}$, and $\alpha(0) = 0$. If a measurement is made a distance Z downstream from this waist the C-S parameters are

$$\beta(Z) = \beta(0) + Z^2\gamma(0) \qquad \alpha(Z) = -Z\gamma(0) \qquad \gamma(Z) = \gamma(0)\,. \qquad (8)$$

Equations 8 can be substituted into Eqs. 7 and 6 to give

$$E_m = E\left(1+\frac{1}{3}\left(\frac{b^2}{A^2}+\frac{b^2+e^2}{L^2\theta^2}+Z\left(\frac{2b^2}{LA^2}\right)+Z^2\left(\frac{b^2+e^2}{L^2A^2}\right)\right)\right)^{1/2}. \qquad (9)$$

Application of Measurement Theory

We previously reported a transverse phase-space measurement made on the accelerator test stand at Los Alamos National Laboratory [2]. The proton beam had rms dimensions of ~0.32 mm and ~5.8 mrad at the waist and the measurement was made ~235 mm beyond the waist. The slit and collector were spaced 625 mm apart and their half widths were; slit = b = 0.05 mm and collector = e = 0.25 mm. Using these values, Eq. 9 estimates that the emittance calculated from the measurement was 2.1% larger than the true emittance. The actual error is calculated from the measured beam parameters in the next section.

Figure 3 plots contours of constant measurement error on the (e,b) plane for the specific beam and slit-collector spacing given above. The concave-down contours give (e,b) combinations which produce measurement errors of 2, 4, 6, 8, and 10%. The acceptance window shown in Fig. 2 has an area of 4eb/L so, for a given L, the size of the measured signal is proportional to the product eb. The concave-up contours are those for which the product eb is constant. Combinations of e and b which fall on the diagonal line produce the largest signal for a given measurement error. The circle represents the slit and collector combination we used for our measurement.

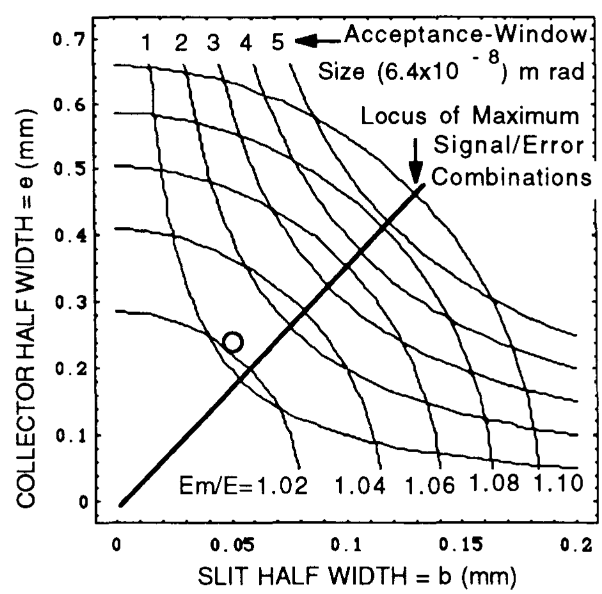

Fig. 3. Contours of constant measurement errors and constant signal sizes on the e,b plane.

The effect of the slit-collector spacing on the measurement error is shown in Fig. 4. This is E_m/E plotted as a function of L with all the parameters except L set equal to their actual values. The dotted lines indicate the drift distance and error of the actual measurement. Since the signal size is proportional to L^{-1}, a shorter spacing of 420 mm would have increased the signal by 50% and would have doubled the error.

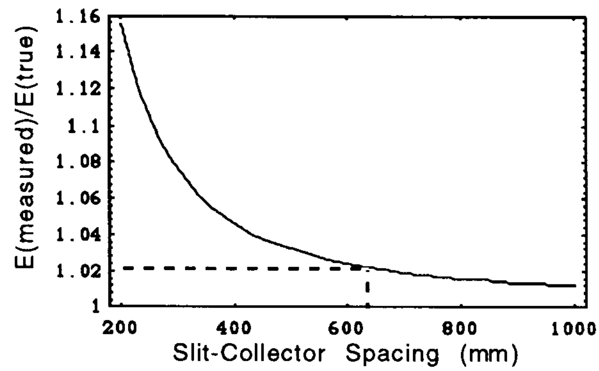

Fig. 4. Measurement error vs. slit-collector spacing.

In Fig. 5 the measurement error is plotted as a function of the slit distance from the waist. Using the same detector, the error would have been 1% at 100 mm and 5% at 400 mm.

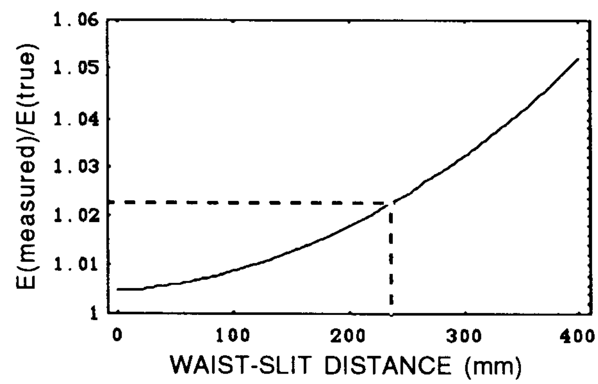

Fig. 5. Measurement error vs. beam waist-slit spacing.

Correcting Emittance Measurements

After a measurement has been made and the emittance and Courant-Snyder parameters have been calculated from the second moments of the distribution, the effect of the slit-collector error can be evaluated and corrected. The emittance correction is

$$E = E_m \left(1 - \frac{\Delta_m}{E_m} \right)^{1/2}, \qquad (10)$$

$$\Delta_m = \frac{\pi\, b^2}{3} \left(\frac{\beta_m}{L^2} \left(1 + \frac{e^2}{b^2} \right) + \gamma_m - \frac{2\alpha_m}{L} \right)$$

where the subscripts m indicate values calculated from the measured distribution. In making this correction the uncertainty of the calculated true emittance value is larger than the uncertainty of the measured emittance. If the measurement gives a value of $E_m \pm \Delta E_m$, then the true value is $E \pm \Delta E$, where

$$\Delta E = \Delta E_m \left(1 + \frac{\Delta_m{}^2}{4E_m(E_m - \Delta_m)} \right)^{1/2}. \qquad (11)$$

After the true emittance has been calculated, the Courant-Snyder parameters can be corrected by using

$$\beta = \beta_m \frac{E_m}{E} - \frac{\pi b^2}{3E} \quad \alpha = \alpha_m \frac{E_m}{E} - \frac{\pi b^2}{3LE} \quad \gamma = \gamma_m \frac{E_m}{E} - \frac{\pi b^2 (e^2 + b^2)}{3L^2 E}. \qquad (12)$$

The beam measurement reported in [2] has been corrected using these equations. The results are

Parameter	Measured	Corrected
Emittance	$(1.93 \pm 0.02) \times 10^{-4}\ \pi$ m rad	$(1.87 \pm 0.02) \times 10^{-4}$
α	-5.22	-5.38
β	1.55 m	1.60 m
γ	18.2 m^{-1}	18.7 m^{-1}

Conclusion

We have calculated the error caused by the finite sizes of the slit and collector in emittance measurements. These results can be used to design a system which is appropriate for the beam to be measured. An example was given from one of our previous measurements, for which it showed a 3% error. We could have increased both the slit gap and the collector widths and decreased the slit-collector distance and kept the error under 10%. Also the slit could have been moved downstream reducing the power loading on the slit or giving more space for a deflector magnet in laser-neutralization measurements [2].

It is important for measurements using laser neutralization of H- beams to have the slit-waist distance large and to have large signals. Using the equations of the last section measurement errors can be corrected. This calculation does not consider the step sizes used in taking the data. It has been shown [3] that at least five measurements should be made across the beam section or granularity errors arise. For this reason if large slits and collectors are used they might have to stepped in increments smaller than their widths.

References

[1] R. L. Gluckstern, "Notes on Beam Dynamics in Linear Accelerators," Los Alamos National Laboratory report LA-8526-MS, pp. 48-52.

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[3] J. Guyard and M. Weiss, "Use of Beam Emittance Measurements in Matching Problems," Proc. 1976 Linear Accel. Comf., Atomic Energy of Canada Limited report AECL-5677, 254 (1976).