

# A CORRECTION SCHEME FOR THE QUADRUPOLE MISALIGNMENT ERRORS IN THE ANL-APS POSITRON LINAC\*

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## ABSTRACT

## TRANSVERSE EQUATION OF MOTION

The Argonne Advanced Photon Source (APS) positron linac contains 24 quadrupoles of which 22 are configured as a FODO system and are distributed along the last 7 constant gradient accelerating structures. Errors in quadrupole and waveguide positions deflect the positron beam centroid, contributing to the aperture requirements in the accelerating structures and quadrupoles. A correction scheme using correction dipole magnets is proposed to compensate for the random errors in quadrupoles.

We assume uncoupled x and y motion of the positrons in the linac. The transverse equation of motion in the x-plane is given by ( $' = \frac{d}{dz}$ )

$$x''(z) + \frac{P'}{P}x(z)' + (K_0 + K_{rf})x(z) = 0 \quad (1)$$

where  $P = mc\beta\gamma$ ,  $K_0 = \frac{eB'}{Pc}$  and  $K_{rf} = \frac{\pi eTE_0 \cos\phi_s}{\lambda mc^2(\beta\gamma)^3}$ .  $\phi_s$  is measured from the zero crossing of the electric field.

## INTRODUCTION

The ANL-APS linac systems have been described elsewhere[1]. The main positron linac parameters are listed in Table 1. A block diagram of the positron linac beamline is shown in Figure 1. The positron linac must accelerate twenty-four macro-bunches, each of about 0.24 nC and 30 nsec duration, to 450 MeV for injection into the Positron Accumulator Ring (PAR). The energy variation of the positron beam must be kept below  $\pm 1\%$  to fulfill the PAR energy acceptance requirement. Errors in quadrupole and waveguide positions will produce beam trajectory errors in the linac which will cause the beam to exhibit betatron oscillations in the FODO lattice of the linac and excite transverse wakefields, which contribute to beam degradation and possible beam loss. The use of position monitors and DC correction dipoles are necessary to observe and correct for beam position error due to misalignment. Here, we start with the transverse equation of motion for the positrons and follow the method described by F.E. Mills[2] to derive a general form to describe the rms errors. The error "amplification factors" for each of the seven girders with distributed quadrupoles will be obtained for the case of correlated errors.

Introducing a new variable[3]

$$X = \sqrt{P}x, \quad (2)$$

the transverse equation of motion can be written as

$$X''(z) + (K + K_{acc})X(z) = 0 \quad (3)$$

with

$$K = K_0 + K_{rf}$$

and

$$K_{acc} = -\frac{(\sqrt{P})''}{\sqrt{P}}.$$

$K_{acc}$  is much smaller than  $K$  and will be ignored. The equation of motion becomes

$$X''(z) + KX(z) = 0 \quad (4)$$

which is Hill's equation, and the solution can be expressed in term of amplitude and phase functions as

$$\vec{X} = \begin{pmatrix} X \\ \alpha X + \beta X' \end{pmatrix} \quad (5)$$

where  $\alpha$  and  $\beta$  are the Courant-Snyder parameters satisfying:

$$\gamma X^2(z) + 2\alpha X(z)X'(z) + \beta X'^2(z) = \epsilon. \quad (6)$$

If  $\vec{X}(z_1)$  is a solution of Eq. (3) at positon  $z_1$  in the linac,

then at a new position  $z_2$  ( $z_2 > z_1$ ), the solution  $\vec{X}(z_2)$  is given by

$$\vec{X}(z_2) = \sqrt{\frac{\beta_2}{\beta_1}} R(\mu_{21}) \vec{X}(z_1) \quad (7)$$

where  $R(\mu_{21})$  is the rotation matrix and  $\mu_{21}$  is the phase advance between the two locations  $z_1$  and  $z_2$  along the

Table 1  
Positron linac main parameters

Initial beam energy	$E_i$	8 MeV
Final beam energy	$E_f$	450 MeV
Focusing system		FODO
Operating frequency	$f$	2856 MHz
Peak field gradient	$dE/dz$	18 MV/m

\*Work supported by U.S. Department of Energy, Office of Basic Energy Sciences under Contract No. W-31-109-ENG-38.

## POSITRON LINAC FODO SYSTEM

linac. For a given quadrupole displacement error ( $\delta x_i$ ), the change in  $X'$  is

$$\delta X'_i = \theta_i = -\frac{B'L}{B\rho} \delta x_i \quad (8)$$

and the change in  $\vec{X}_i(z)$  is given by

$$\delta \vec{X}_i = \begin{pmatrix} 0 \\ \beta_i \theta_i \end{pmatrix}. \quad (9)$$

For a given girder with  $n$  quadrupoles, the solution  $\vec{X}_f$  at the end of a girder is

$$\vec{X}_f = \sum_{i=1}^n \sqrt{\frac{\beta_f}{\beta_i}} R_{fi}(\delta \vec{X}_i) \quad (10)$$

where  $\beta_f$  is the value of the  $\beta$ -function at the end of a given girder and  $R_{fi}$  is the rotation matrix from the position of the  $i^{\text{th}}$  magnet to the end of a given girder. The average of  $\vec{X}_f^T \vec{X}_f$  over all the errors per girder is the expectation value of the mean square of  $\vec{X}$

$$\sigma_x^2 = \frac{\langle X_f^2 \rangle}{P_f} \quad (11)$$

where

$$X_f^2 = \sum_{i,j} (\beta_f \sqrt{\beta_i \beta_j} \sin \mu_{fi} \sin \mu_{fj}) \theta_i \theta_j. \quad (12)$$

So

$$\sigma_x^2 = \frac{1}{P_f} \langle \sum_{i,j} (\beta_f \sqrt{\beta_i \beta_j} \sin \mu_{fi} \sin \mu_{fj}) \theta_i \theta_j \rangle. \quad (13)$$

Each  $\theta_i$  can be written as

$$\theta_i = \delta \theta_i + \Delta \theta_g \quad (14)$$

where  $\delta \theta_i$  is the random quadrupole error on a girder and  $\Delta \theta_g$  is the girder displacement error (this error is the same for all the quadrupoles on a girder). Thus one can write

$$\begin{aligned} \sigma_{x(\text{uncorrelated})}^2 &= \frac{1}{P_f} \sum_i (\beta_f \beta_i \sin^2 \mu_{fi}) \delta \theta_{rms}^2 \\ &\equiv \frac{1}{P_f} A \delta \theta_{rms}^2 \end{aligned} \quad (15)$$

and

$$\begin{aligned} \sigma_{x(\text{correlated})}^2 &= \frac{1}{P_f} \sum_{i,j} (\beta_f \sqrt{\beta_i \beta_j} \sin \mu_{fi} \sin \mu_{fj}) \Delta \theta_g^2 \\ &\equiv \frac{1}{P_f} A_g \Delta \theta_g^2. \end{aligned} \quad (16)$$

Where  $A$  and  $A_g$  are the uncorrelated and correlated amplification factors, respectively. So

$$\sigma_{x(\text{tot})}^2 = \sigma_{x(\text{uncorrelated})}^2 + \sigma_{x(\text{correlated})}^2. \quad (17)$$

Positrons are produced by bombarding a thin tungsten target with a 250 MeV electron beam. The nominal positron beam energy is 8 MeV at the production target. The positron bunches are accelerated to about 120 MeV through the first two accelerating structures. Two long continuous solenoids surround the first two accelerating structures with a nominal magnetic field of 0.35 tesla to keep the beam size within the bore of the traveling waveguides. The beam then passes through a matching quadrupole doublet before being accepted by the FODO array. The FODO array consists of 22 quadrupoles which are distributed over 7 accelerating structures. All the quadrupole magnets have been designed for a maximum field gradient of 4 T/m and an effective magnetic length of 30.0 cm.

The positron focusing system is optimized for maximum positron beam transmission. Beam simulation indicates that the positron beam size will clear the bore of the accelerating structures by a comfortable margin. Because of the traveling waveguide aperture limitation, any mechanical misalignment of the beamline components (in particular, the quadrupoles) results in an offset of the beam centroid from the reference beam axis. If these errors are not compensated for by proper steerings, beam degradation will occur.

The distribution of the magnetic quadrupoles<sup>1</sup> among 7 girders is given in Table 2. If the accelerating structures and the quadrupoles are mounted on girders, then one can ascribe to each girder a position error. Here, it is assumed that all the elements on the same girder have the same error. Eq. (17) is used to determine the errors for the cases listed in Table 2.

Table 2  
FODO system quadrupole distribution

Girder no.	No. of quads	Type designation
Three	Six	D0,F1,D1,F2,D2,F3
Four	Four	D3,F4,D4,F5
Five	Three	D5,F6,D6
Six	Three	F7,D7,F8
Seven	Three	D8,F9,D9
Eight	Two	F10,D10
Nine	Two	F11,D11

## CORRECTION SCHEME

The waveguide aperture in the positron linac is the main limiting factor in the overall beam transmission. In order to keep the beam centroid within the aperture and to

<sup>1</sup>D0 is a matching quadrupole.

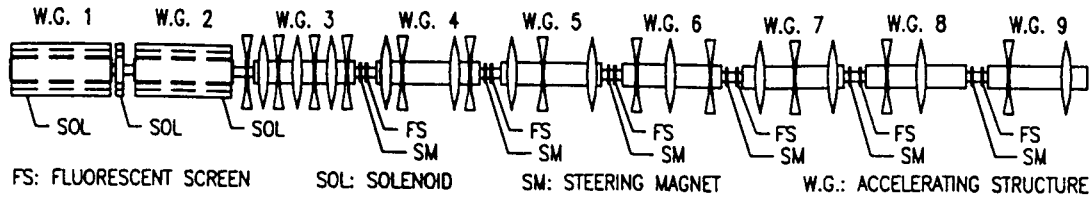


Figure 1  
e<sup>+</sup> linac beam-line layout

minimize possible beam loss due to other effects discussed above, beam positions along the positron linac will be measured. Correction magnets are provided to compensate for any transverse errors. At least four beam profile monitors and correcting magnets are necessary to reconstruct the beam phase-space ellipses in both horizontal and vertical planes. We have provided 7 beam profile monitors (fluorescent screens + BPMs) and 7 correction magnets located after each of the last 7 accelerating structures. The correcting dipole magnets are specially designed to be compact and to provide both horizontal and vertical steering at a given location. The magnet has a length of 10 cm and can produce a maximum field of 1.2 kG[4]. The beam profile monitors are approximately 90° apart in phase.

### HOW MUCH CORRECTION IS NEEDED ?

In order to keep the beam within the aperture of the accelerating structure, beam positions will be measured and correction dipoles will be installed between each girder. An angle  $\theta_f$  produced by a corrector dipole at  $f$  will give a displacement

$$\delta X_{f+1} = (\sqrt{\beta_{f+1}\beta_f} \sin \mu_{f+1,f}) \theta_f \quad (18)$$

at  $f+1$ .

Typically the correction dipole should be strong enough to produce more than twice the value of  $\sigma_x$  to cover the probability distribution. We shall take

$$\delta X_{f+1} = 3 \times [\sigma_{x(tot)}(f+1)]. \quad (19)$$

This then specifies the strength

$$\theta_f = \frac{(BL)_f}{B\rho} \quad (20)$$

of the correction dipole at  $f$ .

For the positron linac the achievable alignment precisions are  $\pm 0.15$  mm for quadrupoles on the girders and  $\pm 0.20$  mm for the girders. Column 3 in Table 3 shows the required deflection angle for each corrector dipole. The corresponding required strength of each correction dipole (BL) is given in Column 4.

### SUMMARY

The study presented in this paper is a continuation of a previous work which addressed the effects of misalignment

Table 3

Displacement error and correction strength required

Girder	$\sigma_{x(tot)}$ (mm)	$\theta_f$ (mrad)	$(BL)_f$ (T.m)
3	3.45	2.20	$1.3 \times 10^{-3}$
4	2.40	1.85	$1.4 \times 10^{-3}$
5	1.62	1.80	$1.6 \times 10^{-3}$
6	1.50	1.34	$1.4 \times 10^{-3}$
7	1.25	1.67	$2.0 \times 10^{-3}$
8	1.50	1.32	$1.9 \times 10^{-3}$
9	1.20	1.12	$1.7 \times 10^{-3}$

of the APS electron linac[5]. Here, we derived an expression for the correlated and the uncorrelated errors for each girder. Since, on the average, each girder contributes about 1.0 mrad, we conclude that a 2.0 mrad correction for each plane will be adequate. The correction magnets are designed to provide a maximum correction of 5.0 mrad (at 450 MeV) for both planes.

### ACKNOWLEDGMENTS

I am very grateful to G. Mavrogenes for his support. I wish to thank Fred Mills for many valuable discussions on misalignment errors and tolerances of the linac focusing system. Finally, I would like to thank Lee Teng for his valuable suggestions and comments.

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