

CALCULATION OF TIME DELAY LINES WITH H_{0n} MODES
FOR RF - ENERGY UPGRADE SYSTEMS

B. Yu. Bogdanovich, Yu. K. Majorov, V. V. Rassadin, and V. A. Senyukov

Moscow Engineering Physics Institute
Kashirskoe sh., 31, 115409, Moscow, USSR

Abstract

A calculation method for time delay lines based on circular iris - loaded waveguide with H_{0n} modes is discussed in present paper. The lines are intended for use in RF - energy upgrade systems as an energy storing elements. Some practical results of delay lines calculation are represented and discussed also. This results show that a satisfactory parameters of delay lines can be obtained when using circular iris - loaded waveguides excited at H_{0n} modes.

Time delay lines intended for RF - energy storage are among the most important elements of certain RF - energy upgrade systems [1]. Upgrade systems characteristics, e. g. power multiplying coefficient and and efficiency, are in strong correlation with the time delay lines used parameters. The main requirements for this lines parameters are as following : minimum possible RF - energy losses and dimensions. Rather small losses one can achieve using a superconducting device. But superconductivity usage is inconvenient in most cases because of considerable growing of the device cost. Also this concerned the delay line dimensions (especially its length) that shouldn't be extremely big. That's why an actual problem is a compact time delay line with small energy losses construction. Calculations show that an acceptable value of energy losses is not higher than 1 dB.

RF - energy losses in delay lines can be decreased by using a circular waveguides with H_{0n} modes. Because of their anormally small losses they are widely used in RF - communication lines. Existance of some parasitic modes is perhaps the only but essential disadvantage of this modes using. In particular singularity of H_{0n} and E_{1n} modes takes place for all frequencies. It leads for additional energy losses. The main goal of this paper is to show that low enough values of energy losses can be achieved in circular iris - loaded waveguides (CIW) excited at H modes for relatively small values of group velocity β_{gr} (about 0.01 - 0.03). This gives a possibility to construct a compact delay line at submicrosecond puis width band. Moreover in CIW the E_{1n} and H_{0n} modes natural frequencies are separated and so their singularity eliminated [2].

A partial regions method was used to calculate a CIW

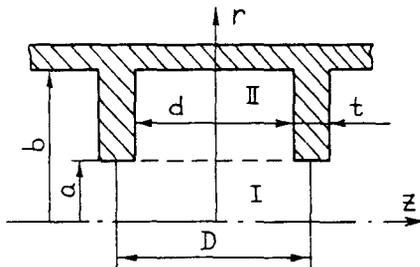


Fig.1 Iris - loaded waveguide cell.

electromagnetic characteristics for H_{0n} modes. A not big memory volume and not high operation speed computer is acceptable for this method using.

To carry out the calculations a waveguide is divided by a cylinder surface with radius r = a into two parts as it is shown at fig. 1. For magnetic - type modes discription Here magnetic vector Π_μ consistent with a Helmholtz equation can be used [3].

$$\frac{\partial^2 \Pi_\mu}{\partial r^2} + \frac{1}{r} \frac{\partial \Pi_\mu}{\partial r} + \frac{\partial^2 \Pi_\mu}{\partial z^2} + \kappa^2 \Pi_\mu = 0 \quad (1)$$

Solution of this equation can be represented as following form for both partial regions :

$$\Pi_\mu^I = - \sum_{n=-\infty}^{+\infty} A_n \frac{I_0(p_n r)}{p_n^2 I_0(p_n a)} e^{ih_n z} \quad (2)$$

$$\Pi_\mu^{II} = \sum_{s=1}^{\infty} B_s \frac{\Phi_0(\alpha_s r)}{\alpha_s^3 a} \sin \frac{\pi s}{2} \left(1 - \frac{2z}{d}\right)$$

Here I₀(x) is a modified Bessel function :

$$\Phi_0(\alpha_s r) = \frac{J_1(\alpha_s b) N_0(\alpha_s r) - N_1(\alpha_s b) J_0(\alpha_s r)}{N_1(\alpha_s b) J_1(\alpha_s a) - J_1(\alpha_s b) N_1(\alpha_s a)}$$

$$h_n = \frac{\beta + 2\pi n}{D}; \quad p_n = \sqrt{h_n^2 - \kappa^2}; \quad \alpha_s = \sqrt{\kappa^2 - \eta_s^2}; \quad \eta_s = \frac{\pi}{d} S;$$

β is a phase shift for one period.

Equations for fields in partial regions as a sums of space harmonics (I region) and standing waves (II region) one can get using a definition for Π_μ :

$$E_\varphi^I = \sum_{n=-\infty}^{+\infty} A_n \frac{\kappa}{p_n} \frac{I_1(p_n r)}{I_0(p_n r)} e^{ih_n z};$$

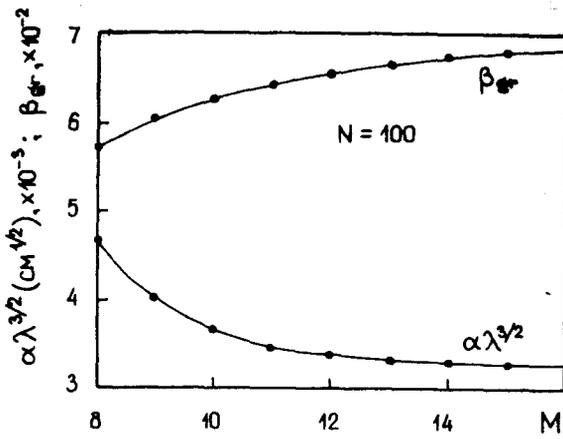
$$E_\varphi^{II} = -i \sum_{s=1}^{+\infty} B_s \frac{\kappa}{\alpha_s^2 a} \Phi_0'(\alpha_s r) \sin \frac{\pi s}{2} \left(1 - \frac{2z}{d}\right);$$

$$H_r^I = -i \sum_{n=-\infty}^{+\infty} A_n \frac{h_n}{p_n} \frac{I_1(p_n r)}{I_0(p_n r)} e^{ih_n z};$$

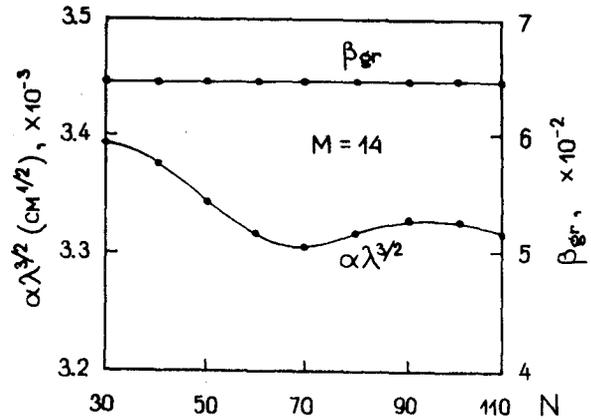
$$H_r^{II} = - \sum_{s=1}^{+\infty} B_s \frac{\eta_s}{\alpha_s^3 a} \Phi_0'(\alpha_s r) \cos \frac{\pi s}{2} \left(1 - \frac{2z}{d}\right); \quad (3)$$

$$H_z^I = \sum_{n=-\infty}^{+\infty} A_n \frac{I_0(p_n r)}{I_0(p_n a)} e^{ih_n z};$$

$$H_z^{II} = \sum_{s=1}^{+\infty} B_s \frac{\Phi_0(\alpha_s r)}{\alpha_s a} \sin \frac{\pi s}{2} \left(1 - \frac{2z}{d}\right).$$



a)



b)

Fig. 2. Group velocity and damping convergence against parameters M (a) and N (b) for H_{04} mode.

Here marked function Φ'_0 is the derivative with respect to argument.

Such a representation satisfied a boundary conditions in resonant region and Floke conditions in axial region. To satisfy a boundary condition at iris walls and a tangential components continuity condition at a regions boundary one may represent field E_φ for $r = a$ as following [3]:

$$E_\varphi|_{r=a} = \sum_{m=-\infty}^{+\infty} F_m \sqrt{1 - \left(\frac{2z}{d}\right)^2} e^{ih_m z} \times \begin{cases} 1, & |z| < d/2 \\ 0, & d/2 < |z| < D/2 \end{cases} \quad (4)$$

Expressed fields amplitudes in partial regions by F_m and used the magnetic field tangential components continuity at regions boundary a system of simultaneous linear algebraic homogeneous equation in unknown F_m can be set up. To set a dispersion equation one should equate the systems determinant to zero. When equation roots are found out it is possible to determine unknown F_m and sequently a field decomposition components for partial regions. Then a CIW characteristics (group velocity β_{gr} and a wave damping coefficient $\alpha\lambda^{3/2}$ where λ is a free-space wavelength) can be determined.

Frequency comparison was used for equation solving program precision control. When H_{04} mode is excited in CIW which parameters are close to a smooth circular waveguide ($r = 4.838 \text{ cm}$; $\Phi = 0.3 \text{ rad}$; $a/b = 0.991$) relative calculation error is about 1.54% and for a cylinder cavity with H_{044} mode ($r = 4.838 \text{ cm}$ and its length $l = 3.89 \text{ cm}$) this error drops to 0.95%.

Solution convergence investigation is very important for a partial regions method use. A special investigations were carried out to determine an optimal values of parameters M (according to (3) and (4) $m = \pm 1, \pm 2, \dots, \pm M$; $s = 1, 2, \dots, 2M$) that defines the combined equations order and N (according to (3) $n = 0, \pm 1, \pm 2, \dots, \pm N$) that defines a sum turns number in the determinant elements. Values of this parameters should be chosen in such a way that one can obtain a satisfactory precision with acceptable calculating time. A convergence of β_{gr} and $\alpha\lambda^{3/2}$ can be illustrated by fig. 2 where this parameters are given as a functions of M (a) and N (b) for copper CIW with following parameters: $\alpha/\lambda = 0.402$; $t/\lambda = 0.0927$; $\Phi = \pi/2$; $\beta_{ph} = 3.88$; $a/b = 0.4729$. All further calculations are carried out for $M = 13 \dots 15$ and $N = 80 \dots 100$.

Calculation results of damping coefficient $\alpha\lambda^{3/2}$ and group velocity β_{gr} for time delay line based on CIW with H_{04} modes are plotted. Normalized damping $A\lambda^{3/2}/\tau$ as a function of phase velocity β_{ph} for H_{04} mode is shown at fig. 3. Normalized damping $A\lambda^{3/2}/\tau$ is the whole damping in the line with length L and delay time τ (and so $A = 20L\alpha(\text{dB})$). A plot of group velocity β_{gr} vs β_{ph} is given at fig. 3 also. Within considered phase velocities band a minimum energy losses are for $\beta_{ph} \approx 3.6$. For example at 10-cm wavelength band a lowest value of damping is about 1.6 dB/ μs and so less than 0.6 μs delay time is permitted in order not to exceed 1 dB damping value. At this point $\beta_{gr} \approx 0.965$ and the delay line would be 12 m long.

Much more interesting results can be obtained for H_{02} mode in CIW. Plots of $A\lambda^{3/2}/\tau$ and β_{gr} against β_{ph} and α/λ for H_{02} mode are shown at fig. 4 and fig. 5 respectively. A group velocity sign changing is observed within considered values of β_{ph} and α/λ . In the sign change point a $A\lambda^{3/2}/\tau$ function has a singularity. At the

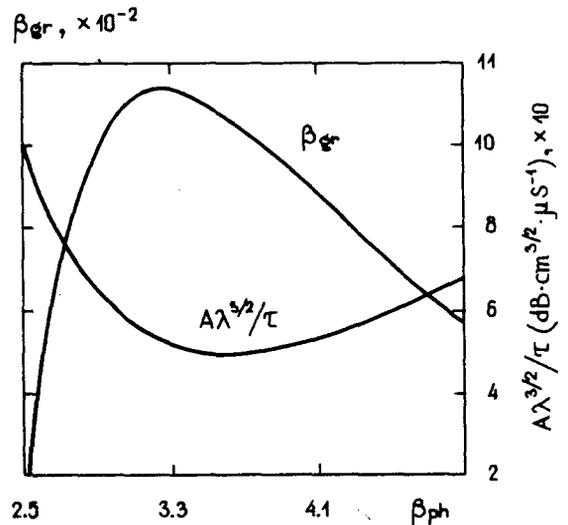


Fig. 3. Normalized damping $A\lambda^{3/2}/\tau$ and group velocity β_{gr} as a function of phase velocity β_{ph} for H_{04} mode.

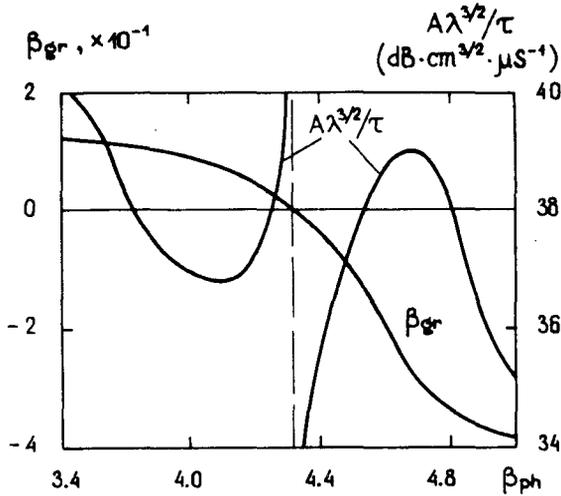


Fig.4. Normalized damping and group velocity as a function of phase velocity for H_{0z} mode.

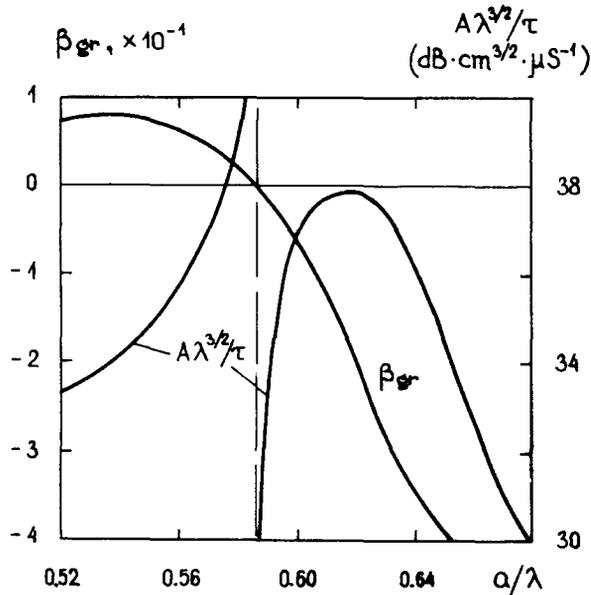


Fig.5. The plots of normalized damping and group velocity against parameter α/λ for H_{0z} mode.

right side from this point (see fig. 4 and fig. 5) damping is tended to zero. A small enough values of normalized damping and group velocity both can be achieved in this region. For example when $\alpha/\lambda = 0.585$ ($\beta_{ph} = 4.4$) we are having $\beta_{gr} = 0.0182$ and $A\lambda^{3/2}/\tau \approx 2 \cdot 10^{-2}$ dB·cm^{3/2}·μs⁻¹. Delay line energy losses would be about 0.7 dB/ at 10 - cm wavelength band and about 7.5 dB/μs at 2 - cm band. So in this case a compact time delay line with 1 dB energy losses and about 0.5...1.0 m long (it's depended on the t/λ parameter) can be constructed for 1.5 μs delay time at 10 - cm wavelength band.

However it's important to note that given calculated results don't take account of the CIW inner surface conditions (surface treatment clearness, impurities etc.). Real values of energy losses are about 10 % to 70 % higher.

The results obtained show that the time delay line for RF - energy storage can be constructed based on CIW with H_{0n} modes. An acceptable values of energy losses and line dimensions can be obtained for wavelength bands from 10 - cm to 2 - cm. The lines parameters can be determined by the method described.

References

1. P.B.Wilson, " Linear Accelerator for TeV Colliders ", SLAC-PUB-3674, May 1985.
2. A.S.Ilinisky, G.Ya.Slepyan, " Kolebaniya i volni v elektrodinamicheskikh sistemach s poteryami ", M.: Isd. MGU, 1983 (Russian).
3. E.L.Burshtain, G.V.Voskresenskiy, " Lineynie uskoriteli elektronov s intensivnyy puchkami ", M.: Atomisdat, 1977 (Russian).