THEORETICAL/NUMERICAL INVESTIGATION OF INDUCTION CAVITY IMPEDANCES FOR MODERATE TO LARGE GAP WIDTHS *

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Abstract

In order to understand the coupling of a charged particle beam to modes in induction cells with gap width – to – beampipe radius ratio $w/b > 1$, the variation of the transverse Z/Q for both axially symmetric and axially asymmetric dipole modes in this regime is investigated. It is found that the gross behavior of the axially symmetric modes when $w/b > 1$ is at least consistent with the approximate analysis of Briggs, et al. [1], although a thorough comparison has not been undertaken. The axially asymmetric modes are found to be unimportant until w/b approaches 2, and they generally exhibit lower values of $Z_\perp/Q$ than the axially symmetric modes.

Introduction

The coupling of a charged particle beam to cavity modes in linac induction modules can pose a significant threat to beam stability in high current accelerators [2]. In particular, the convective beam breakup (BBU) instability has been shown analytically and experimentally to put severe constraints on the cavity mode Qs and gap geometry. For low Q modes, the effects of the interaction between the beam and the induction cell may be characterized using the coupling impedance, and it may be shown that the asymptotic BBU growth rate is proportional to a positive power of the impedance [3]. For high Q modes, the relevant parameter is Z/Q. Briggs, et al. [1], have shown that for fixed b and small ratio of gap width – to – beampipe radius, w/b, the transverse dipole impedance scales linearly with w/b. This finding, coupled with a minimum gap width given by breakdown considerations, yields the requirement of relatively large diameter beampipes in high current accelerators. For a fixed volt-second requirement given by the gap voltage and the pulse length, the amount of ferrite or other magnetic material needed for inductive isolation in an induction cell scales linearly with the beampipe radius. To reduce the cost and weight of an induction machine, one would therefore like to make the beampipe radius smaller and increase w/b.

In this paper we analyze the scaling of the impedance with w/b of the modes in a "Briggs pillbox" when $w/b > 1$. To simplify the interpretation of the results we consider only undamped modes, and therefore focus on the value of $Z_\perp/Q$ instead of $Z_\perp$ itself. This effort is primarily a numerical code exercise, since analytic models for the modes either do not exist in this regime, or are only approximate. The AMOS wakefield code [4] is used to obtain the $Z_\perp/Q$ values for dipole modes in a variety of cases involving different w/b ratios. It is found that the impedances of the traditional BBU modes, i.e., those which are symmetric across the gap, all increase monotonically as w/b increases with b fixed, until $w \approx 2c/\omega$, when a rolloff occurs. Asymmetric gap modes are found to have a very different functional dependence on w/b, and they do not exhibit significant values of $Z_\perp/Q$ until w/b exceeds approximately 1.5.

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Definitions

The radial component of the dipole wake potential in a rotationally symmetric structure has the form [4]

$$W_{r1}(s) = 2\left(\frac{r}{b}\right)\left(\frac{r_Q}{b}\right)\frac{w_{c1}(s)}{r}\cos(\phi), \quad (1)$$

where ϕ is measured from the position of the source charge, r is the position of the test charge, and

$$w_{c1}(s) = \frac{1}{2q_s}\int_{s'=0}^{s}\int_{z=-\infty}^{\infty} E_{z1}(r=b,z,t)\Big|_{t=\frac{z+s'}{c}} dz ds'. \quad (2)$$

q_s is the source charge, which moves axially at radius $r_Q = b$, and E_{z1} is the dipole moment of the axial electric field induced by the source charge [Ref. 4, Eqn. (27)].

The impedance that corresponds to the wake potential in Eqn. (1) is given by

$$Z_{r1} = \frac{-i}{c}\int_{-\infty}^{\infty} W_{r1}(s)e^{i\omega s/c}ds. \quad (3)$$

It can be shown that the single-mode response of the rota-

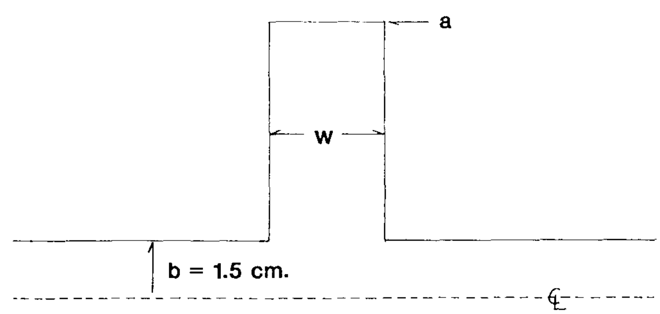

Fig. 1. Rotationally symmetric "Briggs pillbox" used in all calculations presented in this paper. Pillbox is terminated with a perfect conductor. Beampipe was long enough so that the beampipe terminations did not affect calculated results (considered only modes with resonant frequencies less than the beampipe cutoff). Pillbox radius $a = 6.7b = 10.05$ cm.

tionally symmetric geometry under consideration (Fig. 1) has the form [5]

$$W_{r1}^{(i)}(s) = \frac{\Re\left(Z_{r1}(\omega_i)\right)}{Q_i}\omega_i e^{-\frac{\omega_i s}{2Q_i c}} \sin\left(\frac{\omega_i s}{c}\right), \qquad (4)$$

where ω_i and Q_i are the resonant frequency and quality factor of the ith mode, respectively. With reference to the impedance spectrum of the cavity as it would be computed by AMOS, the value of ω_i may be obtained from the position of the local maxima corresponding to the ith mode. When Q_i is small enough, the values of Q_i and $Z_{r1}(\omega_i)$ can also be obtained directly from the impedance spectrum. However, as $Q_i \longrightarrow \infty$, $\Re\left(Z_{r1}(\omega_i)\right) \longrightarrow \infty$, and the ratio $\Re\left(Z_{r1}(\omega_i)\right)/Q_i$ must be evaluated independently. One method is to use an eigenmode solver such as URMEL-T [6], which will calculate the ratio directly. In the event that the impedance spectrum is available, and the modes are reasonably isolated from one another in frequency, a less computationally intensive approach may be used. For each mode of interest, the ratio can be computed by windowing the impedance function to remove the other resonances, then applying an inverse Fourier transform to the windowed impedance spectrum to obtain the component of the wake potential which is due to that mode. The value of $Z_\perp/Q$ is related to the amplitude of the derived wake potential using the expression

$$Z_\perp/Q = \frac{\Re\left(Z_{r1}(\omega_i)\right)}{Q_i} = \frac{\left|W_{r1}^{(i)}\right|}{\omega_i}, \qquad (5)$$

where $\left|W_{r1}^{(i)}\right|$ is the amplitude of the sinusoidal oscillation in $W_{r1}^{(i)}$.

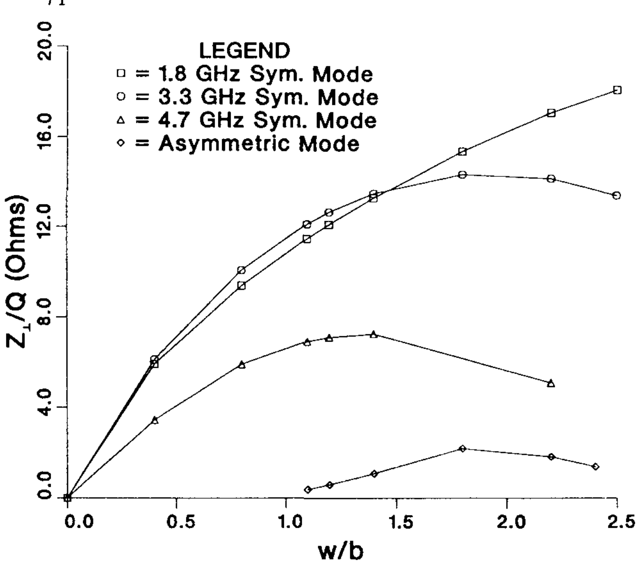

Fig. 2. Variation in $Z_\perp/Q$ with w for first (lowest frequency) three axially symmetric dipole modes and the first asymmetric dipole mode. Result is for $r = r_Q = b$, and $\phi = 0$.

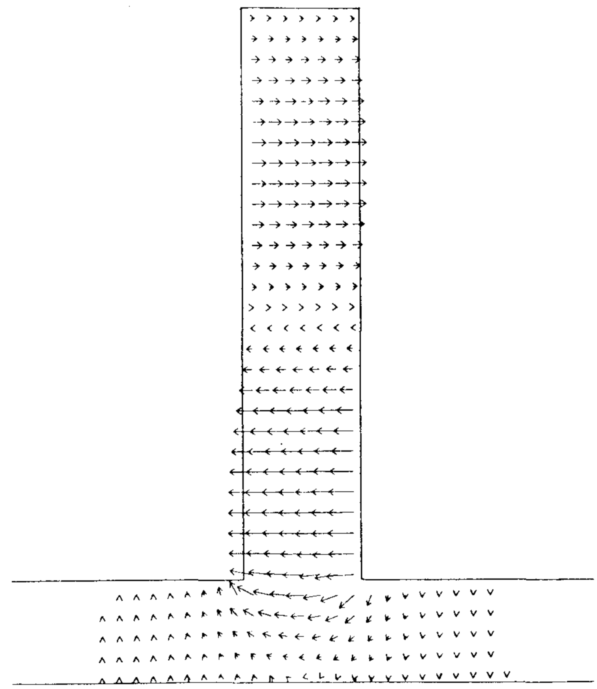

Fig. 3. Electric field pattern for 3.3 GHz axially symmetric mode when $w/b = 1.4$.

Simulation Results

The geometry under consideration is shown in Fig. 1. The series of calculations that will be shown all involve the dipole ($m = 1$) modes in the structure. The value of w/b was changed by altering w while leaving all other dimensions fixed. For each case we calculated the quantity $Z_\perp/Q$ for all resonances below the beampipe cutoff.

The results of this survey are depicted graphically in Fig. 2, which shows the calculated $Z_\perp/Q$ values for dipole modes in the Briggs pillbox when it is terminated by a short. In this configuration all of the modes below the beampipe cutoff are undamped, and Eqn. (5) is used to obtain the mode parameters. Shown are two types of modes: those which have a unidirectional axial electric field across the gap (axially symmetric modes) (Fig. 3), and one in which the axial electric field changes sign across the gap (axially asymmetric mode) (Fig. 4). For small values of w/b, the $Z_\perp/Q$ for the symmetric modes varies linearly with w/b, and the asymmetric mode is negligible. For larger values of w/b the impedances of the symmetric modes peak and fall off, while the asymmetric mode becomes evident for $w/b > 1$.

If it is assumed that the axial electric field doesn't vary across the gap for the symmetric mode, then the analysis of Briggs, et al., shows that the impedance should vary as $w\left(\frac{\omega w}{2c}\right)^{-2}\sin^2\left(\frac{\omega w}{2c}\right)$, which yields a maximum at $w \approx \frac{1.1c}{\pi f}$. For the three symmetric modes being considered, we find that $Z_\perp/Q$ should peak at $w/b \approx 4.0$, 2.2, and 1.5 for the 1.8 GHz mode, the 3.3 GHz mode, and the 4.7 GHz mode, respectively. These values are within 10% of those observed (see Fig. 2).

The character of the axially asymmetric modes in the

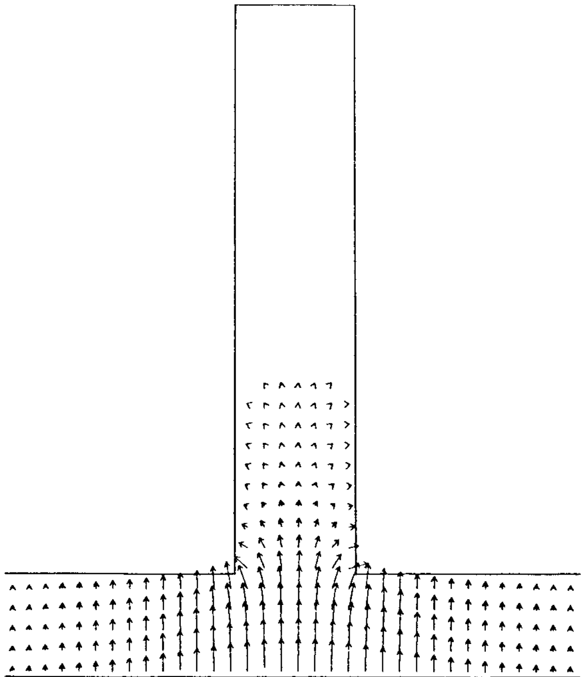

Fig. 4. Electric field pattern for lowest frequency axially asymmetric mode when $w/b = 1.4$. Note that the mode is evanescent into the beampipe and into the radial line. Resonant frequency of this mode is 5.42 GHz.

Briggs pillbox changes significantly when the frequency at which a half wavelength will fit in the radial line falls below the cutoff of the beampipe, which for dipole modes is when

$$\frac{w}{b} > \frac{\pi}{1.841}. \tag{6}$$

Below this limit the asymmetric modes must evanesce into the radial line (Fig. 4) and can only be damped by means applied in the vicinity of the gap. For $w/b > 1.7$ the impedance spectrum rapidly becomes more complex as the familiar $p = 1$ (1/2 wavelength axially) pillbox modes are supported. These modes extend to the pillbox termination (Fig. 5), and in principle are subject to absorption by the ferrite load in the cavity, or by other damping mechanisms not supported at the location of the gap. It is interesting to note, however, that the lowest frequency asymmetric mode persists, and remains weakly evanescent into the radial line, even when $w/b > 1.7$. This mode is neither a $p = 1$ or $p = 0$ pillbox mode, but rather a mode which exists only because there is a joint between the beampipe and the radial line. More detailed discussions of this mode have been made by by Godfrey [7], and by Craig [8].

Conclusions

In induction cavities with gap width – to – beampipe radius ratios that are greater than unity, both axially symmetric and asymmetric dipole modes may be expected. The values of $Z_\perp/Q$ for the axially symmetric modes are seen to peak at values of w/b near those suggested by the Briggs model. For the range of w/b investigated in this work the axially asymmetric modes generally have lower

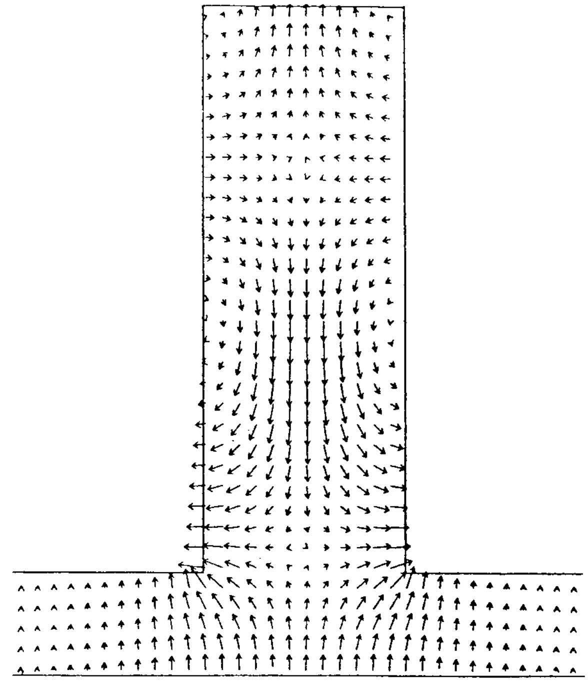

Fig. 5. Electric field pattern for axially asymmetric (p=1) pillbox mode when $w/b = 2.4$. Note that this mode is *not* evanescent into the radial line, but is evanescent into the beampipe. Resonant frequency of this mode is 5.18 GHz.

values of $Z_\perp/Q$, as expected, but they nevertheless may pose a BBU threat if the geometry is such that they are localized to the gap.

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