NUMERICAL MODELS OF BEAM DYNAMICS IN ION INDUCTION LINACS*

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Abstract

We have written a numerical simulation code to study beam dynamics in ion induction linear accelerators. The code is used to study axial beam expansion for a particular example. Voltage ramping of the gap reduces the expansion. The code is also used to study the longitudinal instability. The instability grows and saturates after increasing the axial divergence.

Introduction

High current linear induction accelerators have been proposed for many applications, including heavy ion fusion. Axial beam expansion and the longitudinal instability are two important issues for these accelerators. We have written a simulation code named FASTINS to study these problems.

Analytical scaling estimates and computational models of the longitudinal instability have been published by Stanley Humphries.¹ Humphries' simulation code, named LIDIA, used a 1-D multiple small radius disc dynamics model. The discs' space charge was used to compute an E, that modified the discs' velocities. The gaps were described by a circuit equation. A laboratory frame periodic simulation box with a single gap was employed.

The FASTINS code discussed here was based heavily on the BUCKSHOT code.² BUCKSHOT is a gridless particle simulation code written to study relativistic electron beam propagation in the low pressure ion focused regime. The basic beam element (particle) in BUCKSHOT is a finite length cylinder. In BUCKSHOT, beam particles are constrained to stay in the same axial (z) beam slice they were born in. In FASTINS, beam particles are allowed to overtake each other. Both codes treat transverse motion identically. A 2-D Poisson solver is used to compute the E_z electric field. FASTINS thus generalizes the models of Humphries Lidia code.

For the first application studied, namely space charge driven expansion of the ends of the beam, the code was written in the beam frame. Accelerating gaps approach from the positive-z direction and pass through the beam. As they pass, the β_{z} of adjacent particles is instantaneously increased consistent with the gap voltage.

The second application was the study of the longitudinal instability. Both beam frame and lab frame simulations were carried out. The Humphries gap model was used, in which the gap was modeled by a simple resistance. For the lab frame periodic simulations, the average or dc voltage was subtracted from the total circuit based voltage.

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Numerical Model

The transverse force model used in FASTINS is a generalization of the gridless BUCKSHOT model. The force between two cylinders moving paraxially is given by.

$$F = \frac{2Q_1Q_2}{r} (1 - \beta_1\beta_2) (1 - \frac{|z_1 - z_2|}{ds})$$
(1)

which includes both the electric and magnetic forces. In Eq. 1, Q is the charge, β is the velocity normalized to the speed of light, and ds is the cylinder length. The z-dependent factor allows the force between two cylinders to be proportional to the amount that they overlap. If $|z_i - z_2|$ is greater than ds, then F is set to zero, since remote cylinders should not be included in the local force.

FASTINS uses an axial mesh with a cell-size equal to the cylinder length. This mesh is needed to accumulate the currents required as input to the gap model. It also provides axial resolution of diagnostics, such as the beam radius, divergence, and rotation. Another use of the mesh is pre-sorting of the cylinders before computing the selfconsistent transverse force. The pre-sorting compiles a list for each mesh of cylinders within \pm ds of any of its own cylinders. The transverse force calculation scales with N², where N is the number of interacting particles. Sorting reduces N by excluding particles too far apart to interact. The overhead associated with sorting is small compared to the time saved by reducing N on the interaction list. This technique, which is vectorized on the Cray XMP, about doubled the speed of the transverse force computation.

The use of an analytic transverse force law and finite length cylinders requires that their length be much longer than the beam radius. In addition, they also must be much shorter than the gap spacing and the wavelengths to be resolved for the longitudinal instability. Typical values were 3-5 times the beam radius.

FASTINS requires an axial electric field $E_z(r)$ to allow the ends of the beam to expand axially. It is also needed to allow the beam to respond to gap-induced charge bunching. The radial electric field is already included in the gridless BUCKSHOT model.

The LIDIA E, model used an axial mesh and an FFT solution. It was basically 1-D and had no radial resolution. For many accelerators, where the beam radius is only slightly less than the drift tube radius, this approximation is not adequate.

A 2-D iterative Poisson solver is used in FASTINS to compute the axially and radially resolved E_{z} . The individual beam cylinders are linearly weighted onto an axisymmetric r-z mesh at each time step. A finite difference solution for the electrostatic potential is differentiated to give E_{z} .

The initial beam transverse profile assumed was a uniform current density cylinder of radius 10 cm contained in a 16 cm radius vacuum drift tube. The transverse focusing force was assumed to be produced by a simple 35 kG uniform solenoid. A 10 MeV, 2.4 kA proton beam was injected into the linac. The current linear rise and fall times were 50 ns. Typical beam lengths were assumed to be 40 m. Since the diode was assumed to be non-immersed, the beam had an average rms rotation velocity, normalized to the speed of light, of 0.037. An additional transverse thermal $\beta_{th} = 0.032$ was added to achieve an approximate transverse radial equilibrium.

The axial velocity distribution was computed by assuming that all of the protons had the same total energy γ_0 , so that

$$\beta_{z} = \sqrt{1 - \frac{1}{\gamma_{0}^{2}} - \beta_{\perp}^{2}}.$$
 (2)

This produces the initial thermal spread in β_z shown in Fig. 1.



FIG. 1. Initial axial β_r distribution for a 144 particle/slice loading.

Longitudinal Instability

The longitudinal instability is triggered by beam loading of the gaps. High current perturbations receive less than the average acceleration. This portion of the beam slows, increasing the perturbation in an unstable manner.

One possible circuit equation describing the gap loading is given by¹

$$V_{g} = \frac{2V_{0}(1 + \alpha)}{(2 + \alpha)} - \frac{I_{B}R_{0}\alpha}{(2 + \alpha)}, \qquad (3)$$

where V_0 is the voltage applied by the pulsed power, V_g is the actual accelerating voltage, R_0 is the cavity impedance. I_b is the instantaneous beam current, and αR_0 is the shunt impedance. If the pulsed power is matched to the beam, $R_0 = V_0/I_0$, where I_0 is the average beam current in an unperturbed system.

The growth distance for an initially cold beam is given by¹

$$L_{g} = 2 \left[\frac{T_{O}DL_{B}(2 + \alpha)}{2\pi e V_{O} \alpha} \right]^{1/2} , \qquad (4)$$

where L_{η} is the beam length. T_0 is its initial energy, and D is the gap spacing.

Initial axial velocity spreads have been predicted to stabilize the instability. The required velocity spread in terms of the normalized rms divergence is given by

$$\frac{\Delta \beta_z}{\beta_0} = \left[\frac{\alpha V_0}{(2 + \alpha) 2DT_0 k_0}\right]^{1/2} .$$
 (5)

Space charge electric fields have been predicted to provide a stabilizing factor, causing the beam clumps to reexpand. A measure of the importance of space charge effects is given by¹

$$A = 2 \left(\frac{T_0}{eV_0}\right) \ln \frac{r_{wall}}{r_B} k_0 DK \frac{(\alpha + 2)}{\alpha},$$
(6)
where $K = \frac{eI_0}{2\pi\epsilon_0 m_p (\gamma\beta c)^3}$

Space charge effects should be important if A is greater than unity.

For α = infinity, the present test case parameters yield A = 2 k₀. For the shortest wavelengths, λ = 2.0 m and A = 6.3. For the full beam length of 40 m, A = .31. The growth distance should be increased by L_g/FI(A), where FI(A) = 2(1 + A²)^{1/4} sin [tan⁻¹ (1/A)/2]. For A = .32, FI(A) = 0.86, but for A = 1.5. FI(A) = 0.52. Space charge should be effective at damping short wavelengths, but should have a smaller effect on the longer wavelengths that would characterize full beam transport.

A series of runs used a periodic box of 40 m, a length equal to typical beam lengths. This allowed the simulation of the longest wavelengths that would be of concern. Figs. 2 shows the initial conditions. At z = 190 m, Fig. 3, the growth has reached non-linear levels, but the instability is still growing strongly. At z = 570 m, the phase space is beginning to become isotropic. This isotropic distribution then persists until the end of the accelerator with no re-growth. The final configuration is shown in Fig. 4 at z = 1140 m.





FIG. 2. Initial axial phase space distribution for 40 m beam length example.









FIG. 4. Axial phase space after beam front has traveled 1140 m. Initial loading from Fig. 2.



FIG. 5. Evolution of normalized axial divergence.

The growth and saturation of the instability for the full beam case can be quantified by the evolution of the normalize axial divergence, plotted in Fig. 5. The final value of the divergence has grown three to four times greater than the initial value. For some ion linac applications, this is allowable, but for others it would be excessive. The divergence required to stabilize the beam, given by Eq. 5 modified for space charge, is 0.32 for this example. It is interesting to note that the final saturated divergence is only 0.19. The computed results for this example are thus more optimistic about final saturation levels and what constitutes a stable equilibrium than the analytic theory.

Summary

A code for simulating beam transport in ion linear induction accelerators has been written. The code, named FASTINS, is based on the BUCKSHOT beam propagation code. The basic physical element in FASTINS is a long cylinder of charge. FASTINS couples the accelerating gaps to the beam cylinders by means of a circuit equation.

FASTINS has been used to investigate two problems of interest for ion induction linacs. The first was space charge driven expansion of the ends of the beams. The effect is quite strong at the 2.4 kA, 50 ns risetime level for an example using protons. Voltage ramping of the gaps helps, but large ramp rates would be required. (A discussion of this problem is contained in Ref. 3.)

The second problem studied was the longitudinal instability. Analytic linear growth rates are only available for cold beams. The natural initial beam axial divergence, combined with the stabilizing effect of space charge, significantly reduced the growth in the simulations. The instability grew to moderate amplitudes and saturated.

Preliminary beam frame simulations showed that the instability grew up at a rate similar to the lab frame simulations. This occurred even though the beam average energy was allowed to increase in the beam frame simulations.

References

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