

## A SCHEME TO COMPENSATE THE TRANSIENT BEAM LOADING IN TW ELECTRON LINACS

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### Abstract

A scheme to compensate the transient beam loading of the TW electron linac operated in multi-bunch mode is described. The proposed method is to excite an auxiliary cavity located at the end of the accelerator section with the residual rf power, which comes out of the section after one filling time, with properly adjusted phase and amplitude of the field. The transient energy gain of the electron beam in passing through the cavity will compensate the transient energy loss due to beam loading in the accelerator section. The parameters of ATF of Brookhaven National Laboratory are used for calculation as an example to illustrate the result of computation.

### Introduction

Many scientific applications of the electron beam produced by linac require low energy spread and the subject of energy spread reduction has been studied by many authors. Among the various factors causing energy spread in a multi-bunch linac beam, such as space charge, rf phase spread, wake-field, modulator voltage ripple and jitter, temputure variation, etc., the transient beam loading which lasts one fill time before arriving at steady state is generally the most serious one. A well known method to alleviate it is to turn on the beam before the accelerator section is completely filled<sup>1</sup>. But, it should be noted that from the beam point of view, it still takes about one fill time to settle down. The transient beam loading is not important if the beam pulse is of several fill time long. However, in practice, most electron linacs operate in short beam pulse mode because of BBU, klystron and gun pulse width limit. Therefore, it seems worthwhile to explore some way that could compensate the transient beam loading of the linac and improve the energy spectrum for short pulse operation. Even though it is not always possible to find a scheme that gives perfect compensation, any method that could significantly reduce the energy fluctuation of the output electron beam will facilitate further compensation by other means.

### Scheme of Compensation

The essence of the proposed scheme is to excite an auxiliary cavity (or cavities) located at the end of the linac with the residual rf power which comes out of the

linac (or with another rf power source driven by the same oscillator) after one filling time. The transient energy gain of the electron beam passing through a cavity with the amplitude and the phase of the field properly adjusted by the coupling waveguide, will tend to compensate the transient energy loss due to beam loading in the linac. Fig.1 shows the schematical arrangement of the system. Fig.2 shows a typical transient beam loading in the linac and the build-up process in the cavity to illustrate the principle of operation. Actually, it can be envisioned that the auxiliary cavity be made as an integrated part of the accelerator section. The electric field variation with time at the end of accelerator wave guide and the energy gain of the electrons injected exactly after one filling time are given by the following expressions<sup>2</sup>:

$$E_a(t, t) = \left[ E_0 - \frac{\omega r i_0}{2Q} (t - t_f) \right] U(t - t_f) + \frac{\omega r i_0}{2Q} (t - 2t_f) U(t - 2t_f) \quad (1)$$

$$U_a(t) = \frac{E_0 l}{1 - e^{-2\tau}} \left\{ \left( 1 - e^{-\frac{\omega_0}{Q} t} \right) U(t) - e^{-2\tau} \left( 1 - e^{-\frac{\omega_0}{Q} (t - t_f)} \right) U(t - t_f) \right\} - \frac{r i_0 l}{2(1 - e^{-2\tau})} \left\{ \left( 1 - e^{-\frac{\omega_0}{Q} (t - t_f)} \right) - \frac{\omega_0}{Q} e^{-2\tau} (t - t_f) \right\} U(t - t_f) + \frac{r i_0 l e^{-2\tau}}{2(1 - e^{-2\tau})} \left\{ \left( 1 - e^{-\frac{\omega_0}{Q} (t - 2t_f)} \right) - \frac{\omega_0}{Q} (t - 2t_f) \right\} U(t - 2t_f) \quad (2)$$

where  $E_0 l = (1 - e^{-2\tau})^{1/2} (P_{0l} r l)^{1/2}$ ,  $E_0$  is the average electric field amplitude along the accelerator axis,  $l$ , the linac length,  $P_{0l}$  is the rf power at the entrance of the linac,  $r$ , shunt impedance of the accelerator wave guide,  $i_0$ , the average accelerated current,  $\tau = \omega_0 t_f / (2Q)$ , the attenuation factor,  $t_f$ , the filling time of the linac, and  $U(t)$ , the unit step function. These expressions are for the constant gradient structure which will be taken in our calculation example while similar results hold for constant impedance case. The exit power from the linac is also a function of time and is given by:

$$P_e(t) = \frac{E_a(t, t)^2}{r(e^{2\tau} - 1)} \quad (3)$$

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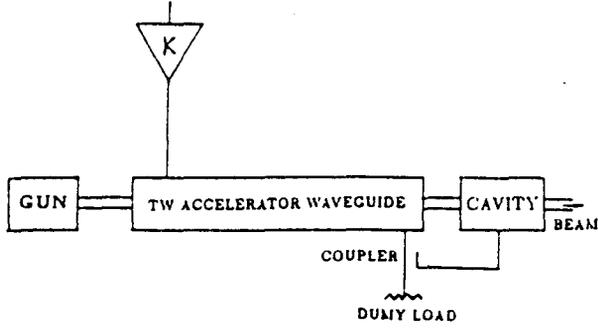


Figure 1: Schematic Diagram of the proposed scheme.

The energy gain of the electrons passing through the auxiliary cavity which is building up with a time varying driven power is given from Appendix A:

$$\begin{aligned}
 U_c(t) = U_{c0} \left\{ \left( 1 - i \sqrt{\frac{RQ_e}{Q_0 P_{01}}} e^{\tau} \right) \left( 1 - e^{-\frac{\omega_0}{3Q_L} t} \right) U(t) \right. \\
 + \frac{Q_L r_{i0}}{Q E_0} \left( 1 - e^{-\frac{\omega_0}{3Q_L} t} \right) U(t) \\
 - \frac{\omega_0 r_{i0}}{2Q E_0} [tU(t) - (t - t_f)U(t - t_f)] \\
 \left. - \frac{r_{i0} Q_L}{Q E_0} \left( 1 - e^{-\frac{\omega_0}{3Q_L} (t - t_f)} \right) U(t - t_f) \right\} \quad (4)
 \end{aligned}$$

where  $U_{c0} = 2Q_L \sqrt{\frac{P_{01} R}{Q_0 Q_e}} e^{-\tau}$ ,  $Q_0$ ,  $Q_L$ , the unloaded and loaded Q-value of the cavity separately,  $R$ , the effective shunt impedance, and  $i$ , the average beam current passing through the cavity.

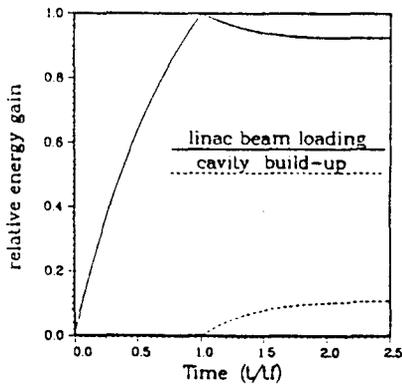


Figure 2: Diagram showing the transient beam loading in a linac and the build-up processes in the auxiliary cavity.

In order to have the energy loss of the accelerated beam due to beam loading completely compensated by

the extra acceleration from the auxiliary cavity, the requirement is obvious:

$$U_a(t) + U_c(t) = const \quad (5)$$

Because the time dependence of  $U_a(t)$  and  $U_c(t)$  are not the same, this condition can hardly be realized in practice. However, we can use the following equations as a guide in the choice of cavity parameters.

$$U_c(\infty) + U_a(\infty) = U_a(0) \quad (6)$$

$$\left. \frac{dU_c(t)}{dt} \right|_{t=0} = - \left. \frac{dU_a(t)}{dt} \right|_{t=0} \quad (7)$$

which gives:

$$U_{c0} \left( 1 - i \sqrt{\frac{RQ_e}{Q_0 P_{01}}} e^{2\tau} \right) = \frac{r_{i0} l}{2} \left( 1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}} \right) \quad (8)$$

and

$$Q_L = \frac{Q}{2} \left( 1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}} \right) \quad (9)$$

It should be mentioned here that eq.(8) and eq.(9) give good compensation for light beam loading only. Practically, the choice of cavity parameters should be proceeded by making the energy variation less than the allowable energy spread and using least square method for fitting.

### Numerical Results

In order to illustrate how the proposed scheme works out, we take the parameters of the Accelerator Test Facility (ATF) of Center of Accelerator Physics (CAP), Brookhaven National Laboratory<sup>3</sup> and perform numerical computation. The ATF consists of two 3.05m SLAC constant gradient accelerator sections with 5MW rf power input to each section and 48mA accelerated current generated from a photo-cathode microwave gun. The beam pulse length is 1.25μs, the shunt impedance of the accelerator section  $r = 58 M\Omega/m$  and attenuation factor  $\tau = 0.57$ .

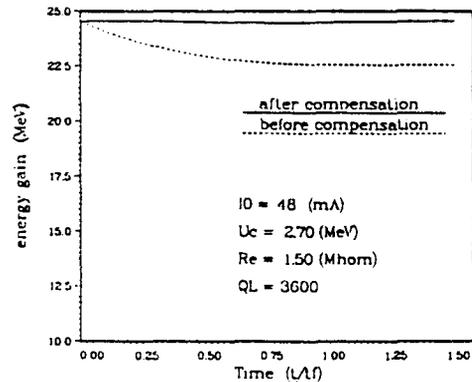


Figure 3: The energy gain variation with time after compensation for the ATF case.

Fig.3 gives the numerical result of exit electron energy gain of the ATF case after compensation. The cavity loaded Q-value  $Q_L$  is assumed to be 3060, which can be controlled by adjusting the coupling. It can be seen the maximum energy spread ( $U_{max} - U_{min}$ ) during the macro-pulse is reduced from about 8% to about 0.4%. A 20 times reduction could be achieved in principle.

### Acknowledgement

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### Appendix A:

#### Energy Gain of Electrons in Passing Through a Cavity Driven by Time Varying Power

The energy conservation law in a cavity can be expressed as<sup>4</sup>:

$$P_{in} = \frac{dW}{dt} + P_s + P_b \quad (A.1)$$

where  $P_{in}$  is the power into the cavity,  $W$ , the energy stored in the cavity,  $P_s$ , the power dissipated in the cavity walls, and  $P_b$ , the power carried away by the beam.

$$P_s = \frac{\omega_0 W}{Q_0} \quad (A.2)$$

$$P_b = i(P_s R)^{1/2} \quad (A.3)$$

where  $Q_0$  is the unloaded Q-value,  $R$ , the effective shunt impedance of the cavity, and  $i$ , the average beam current. In eq.(A.3), proper phasing has been assumed.

When the transient building up process is considered, the input admittance at resonance is:

$$\frac{Y_{in}}{Y_0} = \frac{Q_e}{Q_0'} + \frac{Q_e}{\omega_0 W} \frac{dW}{dt} \quad (A.4)$$

where  $Q_0'$  is defined as:

$$Q_0' = \frac{\omega_0 W}{P_s + P_b} \quad (A.5)$$

Therefore, the ratio of power fed into the cavity  $P_{in}$  to the incident power  $P_0$  is:

$$\frac{P_{in}}{P_0} = \frac{4(Y_{in}/Y_0)}{[1 + (Y_{in}/Y_0)]^2} \quad (A.6)$$

Substituting the eq.(A.2) — eq.(A.6) into eq.(A.1), one has:

$$\frac{dW}{dt} + \frac{\omega_0 W}{Q_L} = 2\sqrt{\frac{P_0 \omega_0 W}{Q_e}} - i\sqrt{\frac{R \omega_0 W}{Q_0}} \quad (A.7)$$

For the sake of convenience, letting  $W = y^2$  and taking the Laplace transform with respect to time, we obtain:

$$sY(s) + \frac{\omega_0}{2Q_L} Y(s) = \sqrt{\frac{\omega_0}{Q_e}} \sqrt{P_0(s)} - \frac{i}{2} \sqrt{\frac{R \omega_0}{Q_0}} \quad (A.8)$$

where  $Y(s)$  is the Laplace transform of  $y(t)$ , and  $\sqrt{P_0(s)}$  is the Laplace transform of the rf power fed into the cavity,  $\sqrt{P_0(t)}$ . Assuming the beam is injected into the linac exactly after one filling time and choosing the time at which the beam is turned on as zero,  $\sqrt{P_0(s)}$  can, from eq.(3), be expressed as:

$$\sqrt{P_0(s)} = \sqrt{\frac{P_{0l} e^{-s\tau}}{s}} \left[ 1 - \frac{C}{s} + C \frac{e^{-ts}}{s} \right] \quad (A.9)$$

where  $P_{0l} = \frac{E_0 e^{-2\tau}}{\tau(1-e^{-2\tau})}$  and  $C = \frac{\omega_0 r i_0}{2Q E_0}$ . Substituting eq.(A.10) into eq.(A.8), taking the inverse transform and using eq.(A.3), we obtain the energy gain of electrons in passing through the cavity as follows:

$$U_c(t) = U_{c0} \left\{ \left( 1 - i\sqrt{\frac{RQ_e}{Q_0 P_0}} e^\tau \right) \left( 1 - e^{-\frac{\omega_0}{2Q_L} t} \right) U(t) + \frac{Q_L r i_0}{Q E_0} \left( 1 - e^{-\frac{\omega_0}{2Q_L} t} \right) U(t) - \frac{\omega_0 r i_0}{2Q E_0} [tU(t) - (t-t_f)U(t-t_f)] - \frac{r i_0 Q_L}{Q E_0} \left( 1 - e^{-\frac{\omega_0}{2Q_L} (t-t_f)} \right) U(t-t_f) \right\} \quad (A.10)$$

where  $U_{c0} = 2Q_L \sqrt{\frac{P_{0l} R}{Q_0 Q_e}} e^{-\tau}$ .

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