STABILITY OF FIELD DISTRIBUTION IN COUPLED CAVITY STRUCTURES

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Abstract

The general properties of coupled cavities structures are studied and the conditions of maximal stability of accelerating field distribution in respect to the random errors of manufacturing and tuning are determined. To describe the biperiodic structures with strong coupling such as DAW structure a coupled oscillator model with couplings of two types - elastic and inertial - is used. It is shown that the field distribution stability in the multicoupled cavity structures depends not only on the frequency separation between modes of structure's spectrum, but also on the field distribution and amplitudes of these modes.

Introduction

In order to increase the field distribution stability in the high energy proton linac the biperiodic structures operated in $\pi/2$ standing wave mode were developed[1,2]. The main instrument for investigation of the field distribution stability in such structures is a coupled oscillator model. The biperiodic chain of oscillators with nearest and next neighbor couplings was succesfully used in many cases [1,3, for example] . But this model does not give a good agreement between calculated spectrum of mode frequencies and experimental spectrum of the multicoupled biperiodic structure (DAW structure for example) and does not adequately describe the field distribution in such structures in full[4] . The chain of the oscillators with the two types of couplings - elastic and inertial - is the solution of this problem. The conditions of the maximal field distribution stability in the accelerating structures were specified using the developed oscillator model. The results were used and proved experimentally during the linac accelerator structure of Moscow meson factory (MMF) tuning procedure.

Coupled oscillator model

The small free oscillations of conservative system of coupled harmonic oscillators are described by a system of equations:

$$\sum_{j=1}^{N} (g_{ij} - \lambda m_{ij}) x_j = 0, \quad i = 1, \dots, N \qquad (1)$$

or in a matrix form:

$$G\vec{x} - \lambda M\vec{x} = 0, \qquad (2)$$

This matrix equation is considered here as a model describing the standing wave structure - a chain of multiply-coupled resonators. The rigity g_{ii} and the mass m_{ii} of the i-th oscillator determine the eigenfrequency of the i-th resonator $\omega_{ii}^2 = g_{ii}/m_{ii}$. The elastic couplings and the inertial ones (nondiagonal elements of G and M) characterize the couplings between magnetic and electrical fields of the resonators. The amplitude x_{ik} (the i-th component of the vector $\vec{x}_i$) is some integral characteristic of the field in the i-th resonator in the k-th mode of the chain. It together with g_{ii} and m_{ii} determines the stored energies of the magnetic and electrical components of the field in the i-th resonator. The solution of the equation (2) is N eigenfrequencies of the oscillator system $\Omega_k^2 = \lambda_k$, representing the frequency spectrum of the accelerating structure and N M-orthogonal eigenvectors $\vec{x}_k$, describing the field distribution at frequency Ω_k .

The model parameters (the elements of matrixes G and M) may be always determined if the full spectrum of the accelerating structure and the field distributions of all modes are known. Usually only the accelerator spectrum is used for the model parameter determination. But basing on the spectral theorem [5] and using any orthogonal system of vectors we can always find a matrix which has a given frequency spectrum. In such approach we have uncertainty in the describing of the field distribution and loose the physical meaning of the model parameters. To avoid this disadvantage the proper structure of the coupled oscillator system must be choosed. For periodic accelerating structures with half-cell termination there is a simple and reliable sign of the proper choice of the model structure - the eigenfrequencies of the model with number of oscillators (N+1) must be equal to odd eigenfrequencies of the model with number of oscillators (2N+1). The known model [1] is a canonical form of the equation (2) and doesn't satisfy this condition in every case because of the properties of the matrix $M^{-1/2} \cdot G \cdot M^{-1/2}$ or matrix $M^{-1} \cdot G$.

For DAW structure the coupled oscillator system shown in Fig.1 had been suggested. The oscillators "a" and "c" have

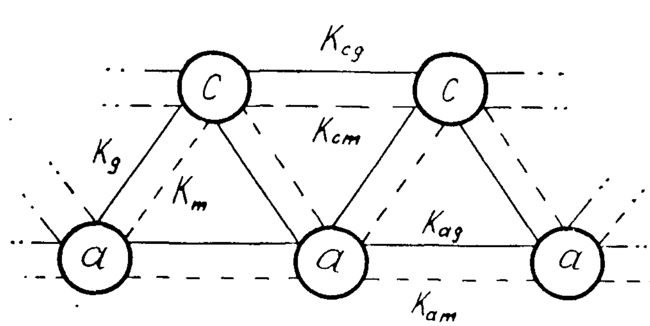

Fig.1

the eigenfrequencies $\omega_a^2 = g_a/m_a$ and $\omega_c^2 = g_c/m_c$ respectively. The subscript "g" signs the elastic coupling coefficient and the subscript "m" - the inertial ones. In the case of half-cell termination the end oscillators "a" have the rigity $g_a/2$ and mass $m_a/2$. The neighbouring oscillators "c" - ($g_c + K_{cg}$) and ($m_c + K_{cm}$) respectively. The numerical values of the model parameters are determined in general case by means of a fitting procedure so that the rms difference between the experimental spectrum and the calculated one (or between the parts of them) is minimal. The procedures of the parameter determination for the different models, including nonperiodic ones, are given in detail in [6]. The eigenvalues and the eigenvectors of the oscillator model are found by means of the numerical solution of the equation (2). The differencies between the calculated and experimental frequencies don't exceed 0.05%.

It is convinient to represent the spectrum of the biperiodic structure model as the solution of the sequence or the equations [7] :

$$det[\lambda^{\pm} \cdot M(\theta) - G(\theta)] = 0 \qquad (3)$$

for every intresting $\theta_k = 0 - \pi/2$. The eigenvalues λ_k^+ and λ_k^- are above and below the operating frequency respectively. At the operating point a phase shift of a spatial harmonic between the oscillators is equal to π and between the oscillators "a" - $\pi/2$.

In the equation (3) the matrix

$$M(\theta) = \begin{pmatrix} m_a + 2K_{am} \cdot \cos 2\theta_k & 2K_m \cos\theta_k \\ 2K_m \cdot \cos\theta_k & m_c + 2K_{cm} \cdot \cos 2\theta_k \end{pmatrix}$$

and the matrix

$$G(\theta) = \begin{pmatrix} g_a + 2K_{ag} \cdot \cos 2\theta_k & 2K_g \cos\theta_k \\ 2K_m \cos\theta_k & g_c + 2K_{cg} \cdot \cos 2\theta_k \end{pmatrix}$$

In other words the biperiodic chain of oscillators for every θ_k is considered as two oscillators with the eigenvalues

$\mu_k = M(\theta)_{11}/M(\theta)_{22}$; $\nu_k = G(\theta)_{11}/G(\theta)_{22}$

and coupled with the coefficients $2K_g \cos\theta_k$ and $2K_m \cos\theta_k$. The Fig.2 shows the frequency spectrum with a stopband of one variant of the DAW structure.

The proper model structure is more important for adequate description of the

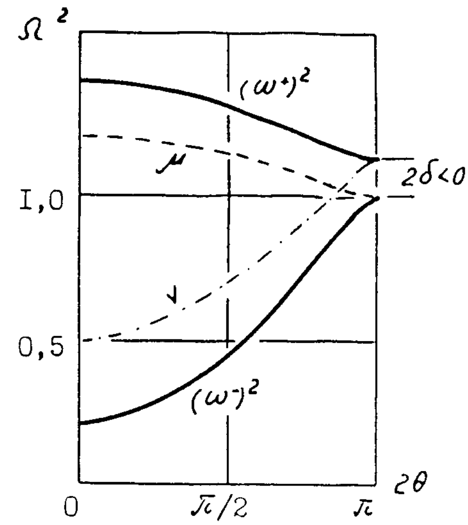

Fig.2

field distribution then for the exact reproducing of the experimental spectrum. In this sense the developed model gives an intresting result. Let the eigenvectors of the model is expressed in form $x_k^{\pm} = a_k^{\pm} \cdot v_k^{\pm}$. For an ideal structure with half-cell termination $x_k^{\pm} = a_k^{\pm} \cdot \cos 2(j-1)\theta_k$ for the oscillators "a" (j is odd). In general case of the multicoupled biperiodic structure the model shows that $a_k^+ \neq a_k^-$ and $\vec{v}_k^+ \neq \vec{v}_k^-$ and there is no complete compensation of the nonoperating modes in the perturbated accelerating structure [7].

Field distribution stability

Basing on the small perturbation theorem it may be shown that in the case of the frequency detuning of the accelerator structure resonators the perturbed field distribution of the operating mode $\tilde{\vec{x}}_k$ is expressed in the terms of the unperturbated structure as

$$(\tilde{\vec{x}}_0 - \vec{x}_0) = \sum_{\lambda_0 \neq \lambda_k} \lambda_0 a_0 \left[\frac{(\Delta \vec{v}_0, \vec{v}_k^+)(a_k^+)^2}{\lambda_0 - \lambda_k^+} \vec{v}_k^+ + \frac{(\Delta \vec{v}_0, \vec{v}_k^-)(a_k^-)^2}{\lambda_0 - \lambda^-} \vec{v}_k^- \right] \quad (4)$$

where Δ is the matrix small perturbations. The expression (4) characterizes a level of mutual compensation of the upper and lower (with respect to λ_0) nonoperating modes in perturbated accelerator structure. It is clear from (4) that the worsening of the compensation is possible a) because of an assymetry of spectrum - $|\lambda_0 - \lambda_k^+| \neq |\lambda_0 - \lambda_k^-|$; b) because of a disbalance of the nonoperating mode amplitudes - $a_k^+ \neq a_k^-$; c) because of an inequality of the field distribution shapes - $\vec{v}_k^+ \neq \vec{v}_k^-$.

For the periodic and biperiodic structures without the mixed and nextneighbouring couplings a stopband and a separation of mode frequencies play a main role in the field distribution stability. The increasing of the coupling between the resonators of an accelerating structure in order to increase the stability leads to an appearence of the mixed couplings and the nextneighbouring couplings. They cause the amplitude disbalance and the asymmetry of the spectrum (simultaneosly or separately). For instance, in the DAW structure tanks of MMF linac the amplitude disbalance reaches the value $a^+/a^- \approx 1.4$ and the width of the upper part of spectrum is smaller then the width of the lower part approximately by a factor 4. So, the DAW structure basically hasn't the complete mutual compensation of the nonoperating modes in the first order approximation.

An influence that or another factor on the stability depends also on an accelerating structure length. For the long structures a stopband is a main factor because the value $|\lambda_0 - \lambda_k^{\pm}|$ is very small and $a^+ \approx a^-$ for the nearest to λ_0 mode frequencies. For the short structures the asymmetry and the amplitude disbalance may be important. In some case the stopband may play a positive role compensating the disbalance and the asymmetry. In the real structures the inequality of the field distribution shapes $\vec{v}_k^+ \neq \vec{v}_k^-$ because of the local perturbations (detuned end half-cells, for example) may worse the stability. For the systems, consisting of small number of resonators, the resonant frequencies of which are different (four-tank accelerating module of MMF) this factor may be important [8] .

Conclusions

The considered factors effecting upon the accelerating field distribution stability were observed experimentally and taken into account during the accelerator system tuning at MMF. Inspite of the absence of the complete compensation in the DAW structure, this structure is much less sensitive to the tuning and fabrication errors then other biperiodic structures [9]. The strong couplings of the DAW structure provided good frequency separation of the spectrum of the 4-tank modules. But in order to avoid the instability due to the ineqality of the field distribution the special tuning procedure was developed [10] .

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