OSCAR2D, A COMPUTER CODE FOR THE DESIGN OF RF CAVITIES AND STRUCTURES

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Abstract: The OSCAR2D code for computation of field and frequencies of resonant cavities and structures is presented.

The code is mainly oriented to the computation of RF fields, of cavities and structures, with high precision in order to use the computed fields for simulation of electric vacuum discharges (resonant or not) close to the cavity boundary.

The NEWTRAJ postprocessor allows for the computation of electron trajectories on monopolar TM modes of the R.F. structures, taking into account the multiplication of electrons due to secondary emission and back-scattering.

Introduction

Designers of RF accelerators need everyday more powerful codes in order to simulate the behaviour of accelerating structure for particles accelerators.

In designing superconducting (S/C) accelerating structures, but also normal conducting ones, a careful prediction of electron discharges like multipactoring (MP) is very helpful in order to avoid severe malfunctions of the accelerating device, and harmful overloading of the RF power generators.

Dealing with the feasibility study of a S/C linac at 4.5 GHz, in order to design a MF-free accelerating structure, we developed the OSCAR2D codes package in order to simulate the RF vacuum discharge of accelerating cavities and structures.

The OSCAR2D package is composed by:

i) OSCAR2D, the bidimensional (2D) solver for the resonant
   a) TE and TM monopolar modes of cylindrically symmetric structures;
   b) TE and TM modes of constant cross section structures.

ii) NEWTRAJ the postprocessor for the computation of the electron trajectories in the RF fields and electron multiplication via secondary effect and back-scattering of TM monopolar cylindrically symmetrical cavities. This postprocessor descends from the TRAJECT code developed at KFK Karlsruhe by J. Halbritter.

The OSCAR2D code

This code is an improvement of the old LALA code by Hoyt of Los Alamos National Laboratories. The implemented new features are

i) improved accuracy of fields obtained by a better treatment of the boundary conditions
ii) possibility of calculation of all the modes of the TM_{01} like band pass of an accelerating structure for a Linac.

iii) a better convergence of the iterative process, resulting in a better accuracy of the final field distribution, by using the "Rayleigh quotients" of the discretized problem, instead of the continuous one, to compute the eigenvalue during the iteration

iv) computation of azimuthally symmetrical TE modes of a cylindrically symmetrical cavity including computation of relevant RF properties of the resonator (e.g. quality factor Q, stored energy, geometry factor, etc.)

v) computation of TM and TE modes of finite length constant cross section cavities including relevant RF properties and computation of TM and TE modes of waveguides

vi) computation of resonant modes higher than the fundamental one by starting from suitables initial approximations and/or by subtracting the components of previously computed lower modes.

The OSCAR2D code solves the eigenvalue problem for a resonant cavity by using a discretization of the continuous problem on a regular square grid. This choice allows for a very small memory occupancy of the code because the coefficients for the interior points of the problem need not be memorized as for an irregular mesh discretization. Only we need to store the coefficients for the boundary points; such coefficients are computed by a "second order" bi-dimensional fit that force the value of the solution on the boundary point in such a way that the solution on the cavity wall satisfies the physical condition of a metallic wall, for the mode wanted (TE or TM mode), on the chosen geometry.

The code uses an overrelaxation method to solve the algebraic system for the discretized eigenfunction of the problem by using to start a guessed eigenvalue and a guessed eigenfunction. Every twenty iteration a new approximation for the eigenvalue is computed by using the Rayleigh quotient of the discretized problem; this choice allows for a better convergence of the problem. The computation is stopped when the convergence criteria for the solution (eigenfrequency and eigenvalue) are satisfied. Because the solution of the problem tends to converge to the eigenfunction nearest in shape to the guessed solution used to start the iteration, some modes of a resonator can be computed just by running the code with a suitable choice of the guessed function.

This method of computation may fail due to numerical instabilities in the solution of the discretized problem. Anyway the wanted mode can be computed by using an orthogonalization method which finds the modes in an ascending order. To this aim during the iteration the components of the previously computed lower modes are periodically subtracted from the current approximation of the mode under computation.
This orthogonalization method always converges to the wanted solution but we have to pay the price of an higher memory occupancy, to store the lower mode eigenfunctions, and an higher computation time needed to find all the lower modes.

**Examples**

We report some examples of the results obtained by the OSCAR2D code.

A comparison between the frequencies and the fields of a pill box cavity as computed by OSCAR2D and the analytical solution is reported in Table I.

| \# of points | $|f_a - f'_a|/f_a$ | $\text{Max } |V_a - V'_a|/\text{max } V_a$ |
|--------------|--------------------|---------------------------------|
| 36           | $3.9 \times 10^{-3}$ | $1.9 \times 10^{-2}$            |
| 121          | $9.4 \times 10^{-4}$ | $3.7 \times 10^{-3}$            |
| 441          | $2.4 \times 10^{-4}$ | $8.0 \times 10^{-4}$            |

where $f_a$ = analytically computed frequency  
$f'_a$ = numerically computed frequency  
$V_a$ = analytically computed field  
$V'_a$ = numerically computed field.

In figure 1 the field distribution for a five equal cells structure operating on the $\bar{n}$ mode is shown.

![Field distribution of a five cells accelerating structure](image1)

Fig. 1 - Field distribution of a five cells accelerating structure; a) compensated, b) uncompensated.

Figure 2 shows the effect of shrinking the radius of the end cells of the same structure of .65% in order to achieve a flat distribution of the axial accelerating field; we notice that the variation in radius of the end cells is a fifth of the mesh size.

![Axial field distribution for the five cell structure of fig. 1](image2)

Fig. 2 - Axial field distribution for the five cell structure of fig. 1.

In figure 3 the field distributions of some modes of a TE elliptic waveguide is shown.

![Four modes of an elliptic waveguide](image3)

Fig. 3 - Four modes of an elliptic waveguide.

**The NEWTRAJ postprocessor**

The NEWTRAJ postprocessor descends from an early program called TRAJECT developed at KFK Karlsruhe by J. Halbritter for single cell TM010 cavities. We are fully indebted to J. Halbritter for the possibility of using that code as a starting point for our NEWTRAJ.

This postprocessor uses the RF fields of cylindrically symmetrical cavities, as computed by OSCAR2D, in order to simulate the RF induced electron discharge in an accelerating cavity. The program computes the trajectories of an electron starting from a point on the cavity wall. The trajectories are computed by a "step by step" integration of the relativistic equations of the motion of the electron in the RF fields of the cavity.

Due to the symmetry of the problem and our needs for simulating RF discharges generated by electrons emitted by the cavity walls we restricted ourselves to compute the motion of the electrons in a cross section of the cavity through the symmetry axis.

The field emitted electron is followed until it strikes a cavity wall. At the impact point a secondary electron is generated. The energy and direction of the new electron are randomly generated accordingly to the distributions of the secondary emission of the cavity wall and of the back-scattering process. At each impact the electron yield is stored; the tracking of the electron trajectories is stopped when the yield of the whole process is too low to sustain the simulated discharge.

As an example we report a comparison between the 2 point multipactoring levels in a pill box cavity. For such cavity the impact energy and the fields of the resonant discharge could be easily computed in analytical way leading to

$$E_n = \frac{4\pi}{e} m c^2 \frac{L}{\lambda^2} (2n-1) \quad E_n = 8 m c^2 \frac{1}{(2n^2-1)^2}$$

where $m c^2$ is the electron rest mass, $L$ is the cavity length and $\lambda$ the vacuum wavelength of the RF field.

Table II reports the impact energies and electric field levels as computed by our NEWTRAJ for a pill box
cavity with $L = 1$ cm and $\lambda = 10$ cm.

<table>
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<th>$n$</th>
<th>$n\text{ MV/m}$</th>
<th>$E\text{ KeV}$</th>
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</thead>
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<td>6.4</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>4.5</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
<td>1.6</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
<td>0.8</td>
</tr>
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</table>

The resonant discharge trajectories in the empty cell of an accelerating structure operating on the $2\pi/3$ mode are shown in fig. 4.

Fig. 4 - Empty cell multipactoring discharge of a $2\pi/3$ accelerating structure.

References


