Computer Simulation of High-Current Beam Transport

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Summary

Transport of beams with currents large enough to produce significant space charge effects has been the subject of computer simulation studies by various authors in the past years. At a time when real experiments were not yet available, but analytic theory of unphysical beam models had suggested more or less uncomfortable restrictions in betatron phase shift, simulation was the only means of approaching reality. Here we review the main results from simulation work performed so far, with particular emphasis on the question what a stationary beam is and under what conditions its emittance remains constant in a long transport channel. We also discuss the strengths and possible weaknesses of simulation as compared with the real world.

Simulation Codes

A characteristic feature of particle tracking for high current transport is the almost cancellation of the externally applied focusing force due to the defocusing space charge force. In the smooth approximation this can be written as

$$(1)\ \sigma^2/\sigma_o^2 = (F_{ext} - F_{sp.ch})/F_{ext} << 1$$

where σ_o is the betatron tune (phase advance per focusing period) in the presence of the external force only and σ its value with the additional space charge defocusing. The numerical treatment thus requires some effort for computing the space charge force. The principal difference with a real beam lies in the fact that a low number of simulation particles with correspondingly higher charge causes the space charge force to be less smooth (if Poisson's equation is solved) or more collisional (if binary forces between particles are computed) than in a real beam. Demanding for more simulation particles ($10^3 \ldots 10^4$) speaks in favour of codes with space charge force computation by solving Poisson's equation (particle in-cell-method), since the required effort increases practically only linear with the number of particles involved.

Such codes have been used extensively since 1977 by various authors[1-5]. The present status of these codes can be summarized as follows:

(a) orbit tracking [1-3] or matrix transformation [4-5] with ideal or real quadrupoles (to third order)

(b) self-consistent space charge calculation via Poisson's equation assuming a longitudinally uniform beam in a conducting rectangular cross section pipe.

(c) all codes quoted here use different methods but essentially agree in results, which lends credibility to the simulation method.

A fully three-dimensional simulation is still impractical due to the time-consuming field calculations. A compromise has been a 2 ½-dimensional code [6], which imposes rotational symmetry on the beam. This code is, however, not adequate for AG focusing.

The absence of unphysical collisions in particle-in-cell codes has to be examined carefully, if σ/σ_o approaches zero or practically if σ is as small as a few degrees only. It can be shown[7] that in this cold beam limit the collision frequency in a beam consisting of point particles is approximately given by

$$(2)\ \nu_c \simeq \frac{\nu_p}{6N\cdot(\sigma/\sigma_o)^2}$$

where ν_c is the collision frequency, ν_p the frequency of beam plasma oscillations and N the total number of particles (here cylindrical rods). As an example we assume $\sigma_o = 90°$, $\sigma = 5°$ and $N = 10^4$, hence $\nu_c/\nu_p \simeq 10^{-2}$. We thus expect significant collisions effects after 100 plasma periods, which is about 300 focusing periods. Replacing point particles by extended particles of appropriate size or distributing the charges on a mesh (as in the particle-in-cell codes) reduces collision effects by an order of magnitude. In this case we still expect collision effects in the percent range if the beam is simulated over 100 focusing periods. Still smaller σ would make the behaviour worse, according to equ. (2). Clearly, there is some care necessary if very small values of σ are to be treated by simulation. Doubling of the number of simulation particles is a practical way of checking whether artificial collisions could be troublesome. We emphasize that collisions tend to increase the phase space; yet it is also at least in principle conceivable that they damp away collective instabilities, which would be slowly growing in the absence of collisions.

Status of Results

A merit of computer simulation is the flexibility in variation of parameters of the beam and its input distribution, as well as the focusing properties of the transport lattice. In the following we summarize the factors that have been found relevant to the decription of high-current beam transport, in particular the question of conservation of phase space. Quantitative results are contained in table 1 below.

(A) Choice of phase advances σ_o, σ.

By comparision with analytic theory it has been possible to identify σ_o, σ as key parameters for transport of high current. The importance of small σ is seen from the relationship for current [8]

(3) $I \sim a^2 \sim \varepsilon/\sigma$

where a is the aperture of a given channel and ε the emittance of the beam. Hence, for given emittance smaller σ allows for larger current.

In searching for stable transport regimes simulation has shown that for $\sigma_o = 90°$ or larger there is a coherent instability for rather extended bands of σ. A third-order instability[9,10] is found for $\sigma_o = 90°$ and $\sigma \simeq 45°$ as a result of locking the period of a sextupole-type deformation of the beam to the periodic focusing, as is shown in Fig. 1. The strength of the instability is somewhat dependent on the initial distribution as will be shown under (B). If σ_o is raised above 90° there is an increasing tendency for unstable behaviour at small σ values. This can be explained by the following argument: for very small σ the beam behaves like a plasma (kept together by the focusing force rather than by charged particles of opposite sign). In this cold beam limit the frequency of plasma oscillations (here transverse deformations of the beam) is known to be independent of the wavenumber and one can show analytically that for a weakly modulated focusing force all plasma oscillations are in parametric resonance with the periodically modulated focusing force provided that $\sigma_o = 180°/\sqrt{2} \simeq 127°$ (see Ref. 11,12). Extending this argument to a strong focusing channel we expect that small σ's are unstable in a broad band of σ_o around the critical $\sigma_o \simeq 127°$. This conjecture is well confirmed by numerous simulation examples with σ_o near or above 100°, which have indicated all kinds of unstable modes for small σ values (see Fig. 2).

An important step towards defining an optimum channel has been the discovery by simulation [12] of stable and emittance conserving transport for even rather small values of σ (10° and below), if σ_o was chosen 60°. This theoretical result has been recently confirmed by experiment[14]. It can be understood if one realizes that the most unstable modes found for $\sigma_o = 90°$ or above are now stable due to the smaller σ_o, which helps to avoid parametric resonance.

(B) Initial Distribution and Matching

Matching of an intense beam to a transport channel is straightforward only for a KV-distribution employing uniform charge density. In this case one obtains a strictly periodic solution by adapting the entrance phase ellipses properly (r.m.s or envelope matching). A statistical set of particle positions and momenta is then generated on the four-dimensional hyperellipsoid in transverse phase space defined by the maching.

For non-KV distributions we distinguish between three procedures:

(a) intrinsically matched beam:

for continuous focusing exactly stationary solutions have been determined mathematically and numerically for rather arbitrary distribution functions of the Hamiltonian, which has to include the self-consistent space charge potential[15,16]. Using a waterbag-distribution in phase space (step function distribution) one thus obtains the Bessel-function distribution in real space [16]. We have formally generalized this procedure (see also Ref. 17) for periodic focusing by using the round beam distribution from continuous focusing as input at a position where the periodically focused beam is expected also round. Simulation has shown that this input is practically perfectly matched. The matching is intrinsic, i.e. the interior density profile repeats itself periodically (and not only does the beam envelope so). This is shown in Fig. 3 for a case with $\sigma_o = 60°$ and $\sigma = 5°$. We emphasize that the real density for this small σ is very close to uniform. The r.m.s. emittance growth after one cell is below 1 %, i.e. no mismatch within statistical errors. After 50 cells we find only 5 % r.m.s. emittance growth which agrees with the results for a KV-distribution.

(b) thermal beam

for small σ we approximate the nearly uniform self-consistent charge density by a uniform one and employ a Gaussian velocity distribution. The deviation from an intrinsically matched distribution is expected to be small for $\sigma \ll \sigma_o$.

This is confirmed by simulation, which gives only about 1 % emittance growth in the first cell, apparently as the result of only slight mismatch (Fig. 4).

(c) r.m.s. matched beam
here the actual profile of the self-consistent potential at start is ignored, i.e. replaced by a parabolic total potential and only r.m.s. quantities are matched. Hence, the real space and velocity space distributions are similar, for instance both Gaussian for a Gaussian distribution of the Hamiltonian. While this approximation is reasonably good for moderate space charge ($\sigma/\sigma_o \geq 0.5$), it results in large emittance growth for small σ due to self-matching of the beam in the very first transport cell[4]. Obviously, the r.m.s. matching - successfully used in linear accelerators - is inappropriate for currents very close to the space charge limit due to the lack of intrinsic force belance. In this procedure non-KV distributions lead to over-density of space charge near the beam axis, which immediately pushes particles into a larger velocity space.

(d) Nonlinear lens optics
Up to now we have considered quadrupole lenses as ideal lenses with only linear focusing forces. For given emittance it is necessary according to equ. (3) to allow for large aperture if σ becomes small (i.e. for large current). In case of quadrupole apertures comparable with their length third order aberrations due to the fringe fields can be significant. This is shown in Fig. 5 for an aperture: length ratio of 1:3 (aperture = radius) The effect of aberrations was found negligible for a ratio of 1:10 (practically negligible growth of the emittance over 50 cells).

Discussion of Results
(see Table 1)

From the point of view of optimum current transmission through a periodic channel we find that a system with $\sigma_o = 60°$ is preferable for quite arbitrary distribution functions, provided that the matching into the channel is sufficiently intrinsic i.e. better than r.m.s. matching. For practical purposes a uniform spatial density ("thermal"-beam) appears to be acceptable. For σ of a few degrees artificial collisions in the simulation have been estimated to be possibly troublesome, hence some care is necessary in interpreting numerical results. Examples with $\sigma_o = 90°$ have shown formation of a halo in phase space as well as in real space, in case where emittance growth due to instability was absent. There is no clear understanding yet for this observation, also why there is no such halo in examples with $\sigma_o = 60°$. This may be of practical importance, if one is worried about small fractions of beam loss. Ignoring the halo and assuming a thermal (Gaussian) distribution makes the $\sigma_o = 90°$ system look quite as good as the $\sigma_o = 60°$ system. The absence of a strong third order instability for $\sigma \approx 40°$ could be a consequence of the presence of phase mixing (Landau damping) in the Gaussian distribution. The kV order waterbag distributions have essentially no phase mixing due to the absence of a spread in σ (kV) or a gradient in the distribution function (waterbag).

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Beam Distribution:		KV	Waterbag intrinsically matched	thermal (uniform density Gaussian velocity)	r.m.s. matched (double Gaussian or waterbag)
Phase Advance: $\sigma_0 = 60^0$	$\sigma = 15^0$	≤ 1 %	≤ 1 %		≤ 50 % in first cell due to mismatch
	$\sigma = 5^0$	≤ 5 %	≤ 5 %	≤ 5 %	≤ 100 % in first cell due to mismatch
$\sigma_0 = 90^0$	$\sigma = 40^0$	≤ 100 % due to third order instability		20 % slowly growing, due to halo	20 % rapidly growing, due to halo
	$\sigma = 10^0$	≥ 100 % r.m.s. growth due to halo, core density practically unchanged			≥ 100 % in first cell
$\sigma_0 = 110^0$ (similar for other values of σ except near σ_0)	$\sigma = 10^0$	violent instability ("plasma oscillations" periodically excited)			

Table 1: R.m.s. emittance growth after 100 focusing cells.

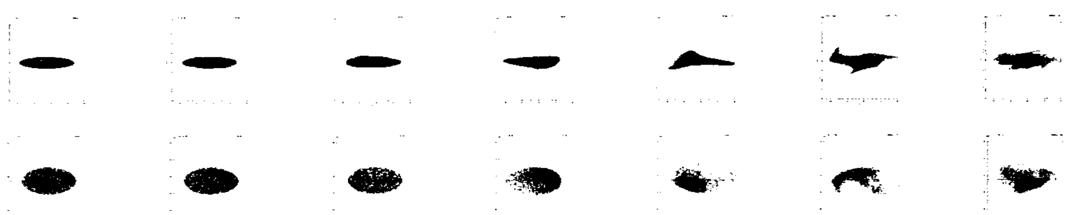

Fig. 1 Third order instability for periodic focusing with $\sigma_0 = 90^0$, $\sigma = 40^0$. Projections into x, x' plane (upper row) and x, y plane (lower row) at start (KV-distribution) and in steps of 5 cells.

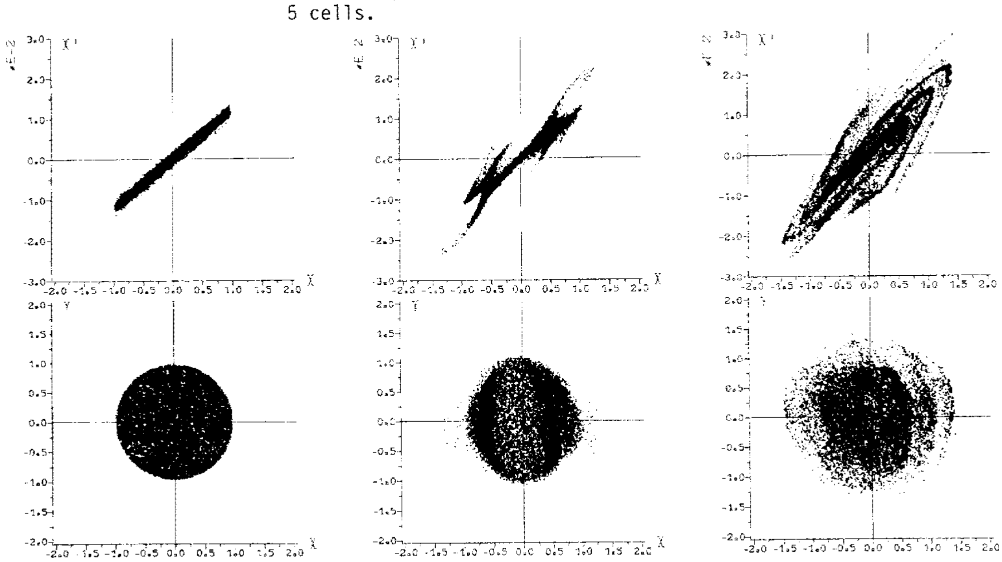

Fig. 2 Instability (of "plasma oscillations") for periodic focusing with $\sigma_0 = 110^0$, $\sigma = 10^0$. Projections into x, x' and x, y planes at start ("thermal" distribution with Gaussian velocity distr.) and after 5 and 10 cells.

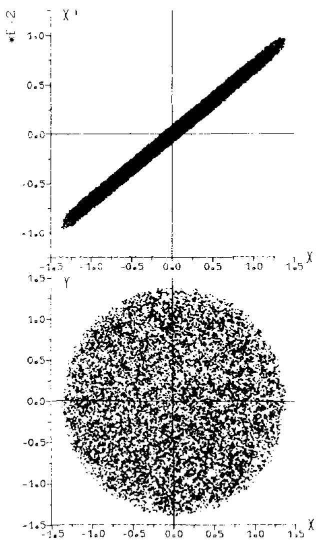

Fig. 3 Intrinsically matched waterbag distribution for periodic focusing with $\sigma_0 = 60^0$, $\sigma = 5^0$ at start and after 50 cells.

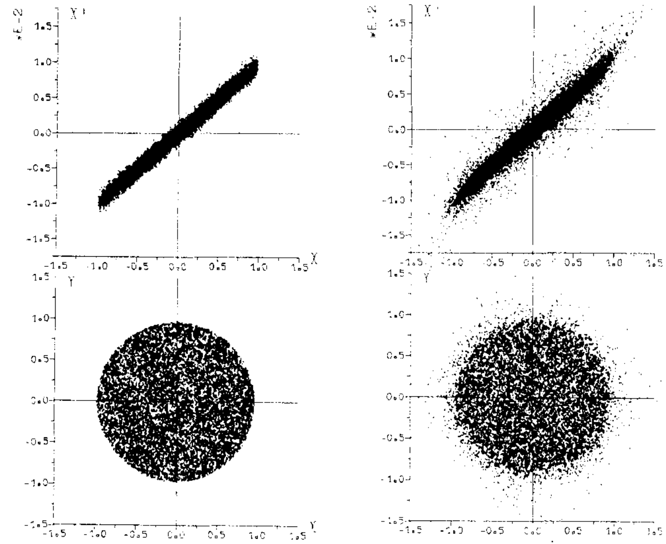

Fig. 4 "Thermal" distribution (uniform in space, Gaussian in velocity) for periodic focusing with $\sigma_0 = 90^0$, $\sigma = 10^0$ at start and after 50 and 100 cells.

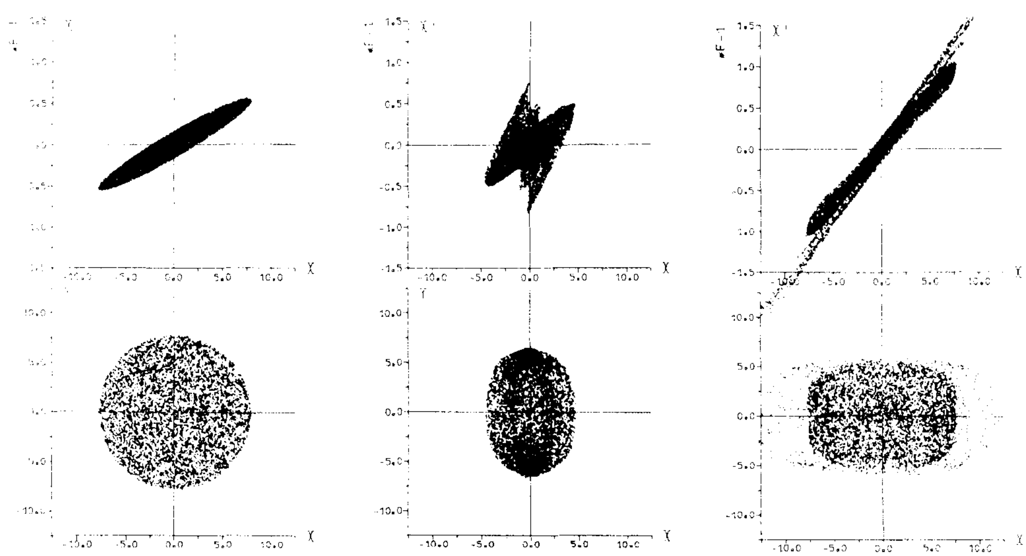

Fig. 5 Effect of nonlinear lens optics due to fringe fields up to third order in too large aperture quadrupoles (aperture: length = 1:3) for $\sigma_0 = 60^0$, $\sigma = 15^0$ at start (KV-distribution) and after 1 and 2 cells.