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A TWO-DIMENSIONAL CALCULATIONAL MODEL FOR AN ELECTRON BEAM PREBUNCHER*

S. M. Kocimski and J. Lewis Detch
EG&G Energy Measurements, Inc.
Santa Barbara Operations
Goleta, California 93116

Abstract

A new, two-dimensional model for the computer simulation of an electron beam prebuncher has been developed to aid modification of the DOE EG&G/EM electron linac. The model uses the particle-in-cell approach with the coordinate system moving with the beam center. The calculations take into account both radial and axial components of an electron space charge field, external bunching fields, and magnetic confining and focusing fields. A paraxial approximation is used to calculate the magnetic field focusing. The model accommodates both single gap and traveling wave prebunchers. The results of calculations indicate a degradation in electron phase focusing as compared with the parameters obtained from the one-dimensional model.

Introduction

One consideration in the linac injection system design is determination of the number and placement of beam bunching elements: the subharmonic bunching cavities, fundamental traveling wave buncher, and the tapered buncher. Several computational models were developed to simulate the action of these bunching elements on the electron beam.[1-4]

The original model[1] treats the electron charge as a system of 'charge disks', while in the most recent model[4] the electron charge consists of number of concentric 'rings'. All the above models are one-dimensional; that is, the electrons are allowed to move only in the longitudinal direction.

In some cases the application of the ring model[4] produces effects which cannot be explained physically. In these cases, the electrons from near the center of beam packet are suddenly decelerated and 'knocked out' from the packet, while the neighboring electrons on both sides continue to be bunched,[5] effects which are sometimes called 'hernias'.

In our linac redesign task we intended to use the 'ring' model,[4] after the causes for the 'hernias' were removed. In the course of model examination, new ideas were developed which lead to a new, two-dimensional buncher model. In this, the electron charge consists of 'smoke rings', which expand or contract radially under the influence of space charge or magnetic fields, as well as longitudinally. The model was successfully applied in calculating the linac design parameters.

Model Description

Assume that the injected electron charge is moving inside a smooth grounded pipe with a radius a, and that the system under consideration has a cylindrical symmetry. Define the coordinate system moving with the velocity of injected beam. Thus, the electromagnetic field acting on electrons in the charge packet in the moving coordinate system may be decomposed into two parts. One is the static electric field due to the electrons own space charge; the other consists of the 'external' fields, transformed from the stationary coordinate system to the moving system by the Lorentz transformation. First, we perform the calculations in the moving coordinate system and then transform the selected quantities of interest, such as charge density distribution or electron phase versus drift distance, back to the stationary system using the Lorentz transformation. Since the system has cylindrical symmetry, we represent it by a two-dimensional system with r and z coordinates.

The calculations will consist of the following steps:

1. At time T_i, calculate the space charge field from the current distribution of charges.
2. Calculate the values of 'external' fields at T_i and transform them to the moving coordinate system.
3. Calculate the Lorentz forces on all the charges in the moving system.
4. Calculate the charge displacements and the velocity increments.
5. Obtain the new charge distribution for T_{i+1} and update the charge velocity values.
6. Go to Step 1 with $i = i+1$.

For simplicity, we assume that the charge distribution at time $T = 0$ is stationary in the moving coordinate system. This condition is equivalent to zero emittance in the stationary system.

Space Charge Fields

To calculate the space charge effects, we consider a point charge q_{ik} inside a grounded pipe with radius a, and calculate the potential at point j,k.

The coordinates of the charge in the cylindrical coordinate system are (b_i,ϕ,z_k) and similarly, the coordinates of the test point are (c_j,ϕ,z_ℓ). Thus,[6]

$$\psi_{pt}\left(b_i,\phi,z_k;c_j,\theta,z_\ell\right) = \frac{q_{ik}}{\pi\,\varepsilon_0\,a}\sum_{m=0}\sum_{n=1}\frac{1}{1+\delta_{m0}} \times \exp\left(-\nu_{mn}\,|\,z_\ell - z_k\,|/a\right) \times \frac{J_m\left(\nu_{mn}\,b_i/a\right)\,J_m\left(\nu_{mn}\,c_j/a\right)}{\nu_{mn}\,J_1^2\left(\nu_{mn}\right)}\cos m\,(\phi-\theta) \quad (1)$$

Next, we calculate the potential and field of charge annulus with the inner radius $r_1^{(i)}$ and outer radius $r_2^{(i)}$ at the test point. The average electric fields at the test annulus from the source at (i,k):

$$\bar{E}^z_{i,j,k,\ell} = \text{sign}\,(\ell-k)\;Q_{i,k}\;M^z_{1,|\ell-k|,j} \quad (2)$$

$$\bar{E}^r_{i,j,k,\ell} = Q_{i,k}\;M^r_{1,|\ell-k|,j} \quad (3)$$

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where

$$M^{z}_{\alpha,\beta,\gamma} = \operatorname{sign}(\beta) \sum_{n} \frac{2 \exp\left(-\nu_{0n} L\beta / a N_D\right)}{\pi \varepsilon_0 a^2} \left(\frac{N_R^2}{a^2 J_1\left(\nu_{0n}\right)}\right)^2 \times \frac{1}{(2\alpha-1)(2\gamma-1)} \int_{2(\alpha-1)/N_R}^{a\alpha/N_R} J_0\left(\nu_{0n} b/a\right) b\, db \times \int_{a(\gamma-1)/N_R}^{a\gamma/N_R} J_0\left(\nu_{0n} c/a\right) c\, dc \qquad (4)$$

$$M^{r}_{\alpha,\beta,\gamma} = \sum_{n} \frac{2 \exp\left(-\nu_{0n} L\beta / a N_D\right)}{\pi \varepsilon_0 a^2} \left(\frac{N_R^2}{a^2 J_1\left(\nu_{0n}\right)}\right)^2 \times \frac{1}{(2\alpha-1)(2\gamma-1)} \int_{a(\alpha-1)/N_R}^{a\alpha/N_R} J_0\left(\nu_{0n} b/a\right) b\, db \times \int_{a(\gamma-1)/N_R}^{a\gamma/N_R} J_1\left(\nu_{0n} c/a\right) c\, dc \qquad (5)$$

The matrices M^r and M^z depend only on the geometry of the problem and need to be calculated only once.

Now, we use the above equations to further define the 'charge tracking' model. The moving coordinate system consists of N_D 'disks' and N_R 'rings' or $N_C = N_D N_R$ 'cells'. The space charge contribution of electric field in each cell is given by the eqs. (2 and 3). These fields, plus any other external fields, will act on a 'test' charge occupying each cell. We represent each 'test' charge by a point charge Q_α with coordinates (R^a_α, Z^a_α) and velocities (V^r_α, V^z_α). We also assume that the 'source' charges used to calculate the field values in eqs. (3 and 4) are uniformly distributed in each cell occupied by their representative 'test' point charges. The accuracy of this approximation increases as the cell size decreases.

Equation of Motion in the Electromagnetic Field

Using the Lagrange's equation and the relations $E_z = -\partial\phi/\partial z$ and $E_r = -\partial\phi/\partial r$, we obtain:

$$m \frac{\partial^2 z}{\partial t^2} + e\, r\, \dot{\phi} \frac{\partial A_\phi}{\partial z} + e\, E_z = 0$$

$$m\, r^2 \dot{\phi} - e\, A_\phi\, r = \text{const} = p_\phi$$

$$m \frac{\partial^2 r}{\partial t^2} - m\, r\, \dot{\phi}^2 + e\, \dot{\phi} \frac{\partial}{\partial r}\left(r\, A_\phi\right) + e\, E_r = 0 \qquad (6)$$

If we consider only the meridional electron trajectories at the initial time, T = 0, then the quantity $p_\phi = 0$ gives:

$$\dot{\phi} = \frac{e\, A_\phi}{mr} \qquad (7)$$

In the paraxial approximation[7] $(r \approx 0)$ we obtain

$$A_\phi \approx r B_z/2 \qquad B_r \approx -\, r/2\, \frac{\partial B_z}{\partial z} \qquad (8)$$

Using only the first order terms in B field, we obtain the equations of motion for the electrons:

$$\ddot{r} + \left(\frac{-\,3B_z}{2m}\right)^2 + e/m\, E_r = 0 \qquad \ddot{z} + e/m\, E_z = 0 \qquad (9)$$

These are the nonrelativistic equations of motion which will be used to calculate the displacement of electrons in the moving coordinate system. The quantities E_r, E_z, and B_r are the electric and magnetic fields from space charge, buncher, and the confining coils transformed to the moving coordinate system.

Results of Calculations

The calculations were conducted to obtain the optimal values for the buncher positions and the amplitudes and phases of RF fields. The existing linac configuration put certain constraints on the parameters in the model calculations. Thus, the calculations were performed for pipe radius a = 2.54 cm, injection energy of electrons = 120 keV, and the length of the drift region between the injector gun and the first buncher = 2 m. The fundamental RF frequency was 1300 MHz.

The calculations were conducted in two phases. In the first phase, the effects of radial forces were disabled, thus making it the one-dimensional case. After the optimization of parameters, the radial forces were 'turned on' and the calculations were repeated for the same set of parameters.

Figure 1 shows the plot of the electron phase versus drift distance for the resulting optimal set of parameters, and fig. 2 gives the plot of current versus time at the exit of traveling wave buncher.

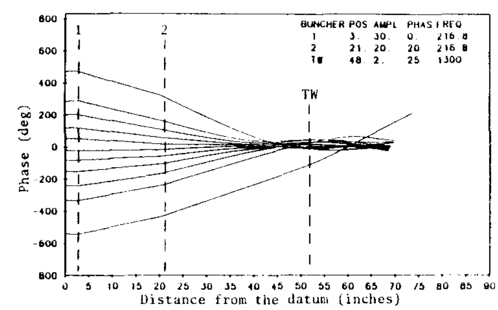

Fig. 1. Beam phase versus distance for buncher configuration

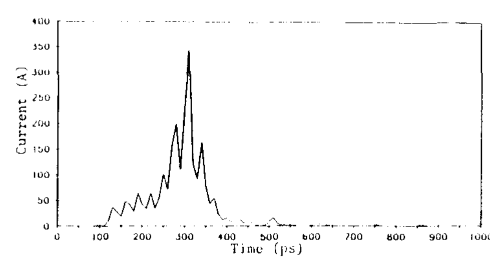

Fig. 2. Beam current after bunching

In the course of our calculations, we confirmed earlier findings[4,8] that the use of single subharmonic buncher results in highly asymetric bunching, with the back of the pulse lagging in phase. Addition of the second buncher improves the bunching significantly. We also found that the RF phase on each buncher has a significant effect on bunching symmetry. As a result, we plan to control the RF phases and amplitudes on all bunchers independently. We expect that this will be an improvement over the fixed phase-fixed amplitude system.[9]

In the second phase, we repeated the calculations with the same parameters, but with the radial forces 'turned on'. Figure 3 shows the resulting plot of electron phase versus drift distance, and fig. 4 gives the current versus time at the exit of traveling wave buncher. Note that the bunching has degraded considerably.

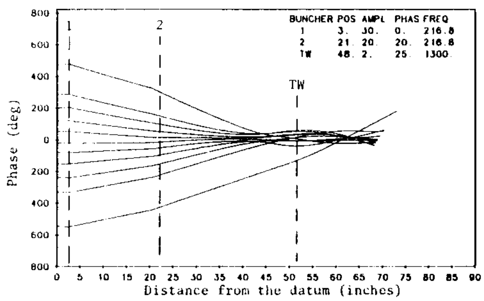

Fig. 3. Electron phase versus drift distance with radial forces

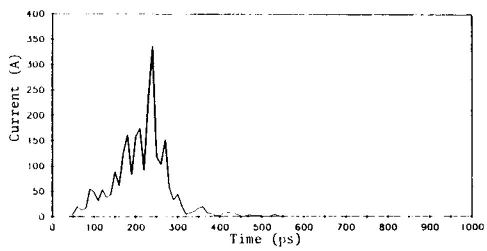

Fig. 4. Current versus time at exit of traveling wave buncher with radial forces

The design parameters for the second stage of linac upgrade call for 3-ns injected beam pulse and up to 40 pC of injected charge. Preliminary results of the calculations indicate that we have to use two twelfth subharmonic bunchers and one sixth subharmonic buncher in the beam injection system.

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