INVESTIGATION OF THE EFFECTS OF SPACE CHARGE AND IMAGE CHARGE FORCES ON BEAM LOSS IN AN RF QUADRUPOLE

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Summary

The radiofrequency quadrupole (RFQ) has become a favoured method for providing the initial acceleration and bunching of ion beams prior to injection into a drift tube linear accelerator. For proton beams of several hundred milliamperes cw it will be essential to minimize the beam loss to the walls. The effects on beam emittance caused by space charge and image charge forces may be significant, particularly at the lower energy end of the RFQ. In this study the space and image charge potentials have been calculated using a three dimensional finite element model of two $\beta\lambda/2$ cells, taking snapshots in time for a bunch of particles traversing the cells. By varying the total charge in the bunch an estimate of the change of the beam emittance is obtained. Extension of this work will enable a better beam loss estimate to be made for a complete RFQ.

Introduction

The use of radiofrequency quadrupole structures to accelerate intense ion beams is being studied in a number of laboratories. Computer programs for the study of beam dynamics in an RFQ have either neglected the space charge effects, or have allowed for these effects in a way that does not take account of the proximity of the poles to the beam and the resultant image charges. A program RFQTRAK has been developed which uses a differential finite element method[1] to give a better representation of space charge and, because of the boundary conditions inherent in the finite element method, also takes account of image charge forces. This paper describes the program structure and reports results of a study of space charge effects on the emittance of beams of varying intensity passing through two RFQ cells (360°). Further development of the program will enable a longer length accelerator to be studied.

Method

The program solves the Poisson Equation $k\varepsilon_0 \nabla^2 \phi = -\rho$ using the finite element method[1]. Macroparticles, each with charge e_p referred to as particles in the following discussion, are traced by the program through the two RFQ cells and are regarded for the purposes of space charge calculation as if they were point charges. The matrix equation for the space charge potentials is

$$\int [B]^T[D][B]\phi dv + \{e_p\}[N]^T = 0$$

where [B] is a matrix of shape function derivatives with respect to the global coordinates, x, y and z, taken at the nodes. [D] is a material constants matrix, here equal to a single constant, the permittivity of free space ε_0. N is a matrix of shape functions, taken at the positions of the particles within the two cells.

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Analytic coefficients previously obtained from the RFQCOEF[2] program are used to calculate the rf components of electric field for particle tracking. The program tracks particles through the two cells with equal time steps and initially no space charge until the first particles reach the output plane. At this time step the coordinates of the particles are used to calculate the contribution of the charges to the matrix right hand side. The nodes at the exit plane are slaved to those at the input plane to give the semblance of continuity of both accelerator and beam. The matrix equations are then solved to evaluate the space charge potentials.

The particles are tracked through the next time step and the space charge potentials are recalculated, using the previous values as a first approximation for the conjugate gradient solver. The above process is repeated for each time step, the output parameters are recorded as each particle crosses the output plane and the file of output particles is updated. Thus, once a complete bunch has been accepted into the cells, there will be particles emerging from the exit plane of the double cell. These particles will have their parameters logged by the program and will then be discarded. During each time step a further set of particles will be accepted at the input plane.

By the time the beam has been tracked for a second rf cycle the emerging particles will have had the space charge forces applied for the whole of their traversal through the double cell. The parameters for these particles which emerge during the third rf cycle are used for the graphical plots.

In order to economize in computing time and core, particles are tracked in one quadrant only. Those particles that cross the z-y and/or z-x planes are regarded by the program as if they were reflected in that plane, i.e., their x coordinate and x component of velocity and/or y coordinate and y component of velocity, are reversed in sign. To represent off-axis beams it would be necessary to use a mesh covering all four quadrants.

Program Structure

A flow diagram of RFQTRAK is shown in Figure 1. The package is built on a modular basis for core economy. The mesh generation and matrix assembly stages are identical to that used in the RFQCOEF program. The mesh used for these tests comprised 36 three-dimensional 'brick' elements in each xy plane, with 8 planes of elements in the axial (z) direction, giving a total of 288 elements.

Limitations of core and running time demanded that the mesh size be kept to a minimum. It was felt that even with a 10% accuracy for the space charge fields the general picture would be adequate to demonstrate the viability of the method. (The space charge fields are about 10% of the total fields, so the minimum error in relation to the total focusing field would then be about 1%.)

In allocating the contributions made to the right hand side of the matrix equations by the particle charges it is necessary to determine in which element

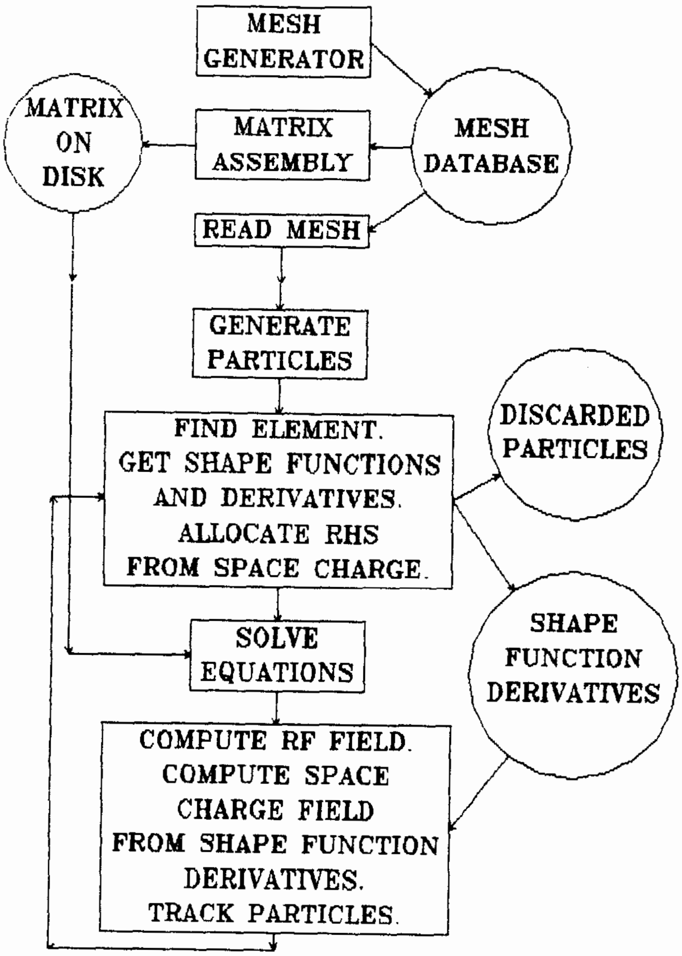

Fig. 1 Flow diagram.

the particle was situated and also its local coordinates. Initially the program makes a guess at which element contains the particle by comparing distances from the element centroids. Evaluation of the local (normalized) coordinates of the particle with respect to an element indicates whether a particle is inside or outside the chosen element. If the particle is outside then at least one of the local coordinates will lie outside the range -1 to +1.

The shape function routines that are used to map an element in real space onto a normalized 'brick' are generally only useful for computing the global (x,y,z) coordinates from the local (u,v,w) coordinates. If the local coordinates are known then the shape functions may be directly evaluated. It is thus necessary to use a Newton-Raphson iterative method to get from the (x,y,z) coordinates to the local system. This generally takes about 4 iterations.

For the RFQTRAK program the local w and global z directions are always the same, as all the elements are of the same length, so it is trivial to compute w from z. Thus w is evaluated first and u and v must be sought in the plane w. This means that to get u and v it is necessary to set up and invert a 2 x 2 Jacobian matrix for the Newton-Raphson scheme.

After the local coordinates have been found, the shape functions with respect to all 20 nodes of the element are evaluated. The charge from the particle is then assigned to the nodal right hand side in proportion to these shape function values. Derivatives of the shape functions with respect to the x, y and z coordinates are also computed for the particle positions and stored for computing the space charge field components later. If the particle is outside the element structure then it is discarded from the beam and sent to the output file. It is flagged to indicate whether it was lost to the cell boundaries or passed through the exit plane.

Particle Generation

The input particle distribution for RFQTRAK was generated by adding a modified output routine to the PARMTEQ[3] package. The input to the PARMTEQ program comprised one particle per degree of phase with random distribution within the acceptance ellipse of the design known as RFQ1-B[4]. The output from the PARMTEQ program contained particle coordinates, velocity components and time at the end of the time step during which they enter the cells under investigation. These were then stored in a file and used in a 'closed loop' to represent continuous particle generation at the input plane.

For each time step RFQTRAK scans the file of particles and accepts only those within the relevant phase interval.

Results

Preliminary results have now been obtained for cells 19 and 20 of the RFQ1-B proton accelerator design.

The cell parameters were as follows:

minimum pole distance from axis	a = 0.409 cm
modulation factor	m = 1.02
mean transverse pole radius	r_0 = 0.348 cm
cell length	cl = 0.58 cm
half width at root of vane	HW = 1.1 cm
distance from axis to root of vane taper	RS = 2.82 cm

Figures 2 and 3 display the results for beam currents of 100 mA and 1000 mA, respectively, for particles emerging from the 20th cell. The same input particle parameters were used for both current densities. The 1000 mA result was used to make the effects of increased emittance sufficiently pronounced to serve as an illustration. The top two pictures of each figure give the plots of ∂x/∂z against x and ∂y/∂z against y respectively, for emerging particles. A small increase in emittance with increasing current is apparent. The third member of each set shows a distribution of the beam over the exit plane. To present a more complete picture the plots for the one quadrant are shown reflected in the other three quadrants. The fourth member of the set shows beam profiles. Here again, some broadening of the beam from Fig. 2 to Fig. 3 is apparent.

Because the emerging particles are distributed over the complete rf cycle the emittance plots do not show the narrow ellipse which would be characteristic of particles arriving during a single step.

In order to arrive at relative numerical values for the emittances the following procedure was adopted: for each time step the particles were analyzed to determine the eccentricity of the emittance ellipse and its α, β and γ parameters. The emittance of each particle was then calculated using the formula

$$\text{emittance} = \gamma v^2 + 2\alpha v v' + \beta v'^2$$

where v is the spacial coordinate (x or y) and v' is ∂x/∂z or ∂y/∂z. For each direction of the transverse plane the emittance for each particle was calculated by this method. The x' and y' maximum values and also the values containing 80% of the beam are shown in Table 1 for different beam currents.

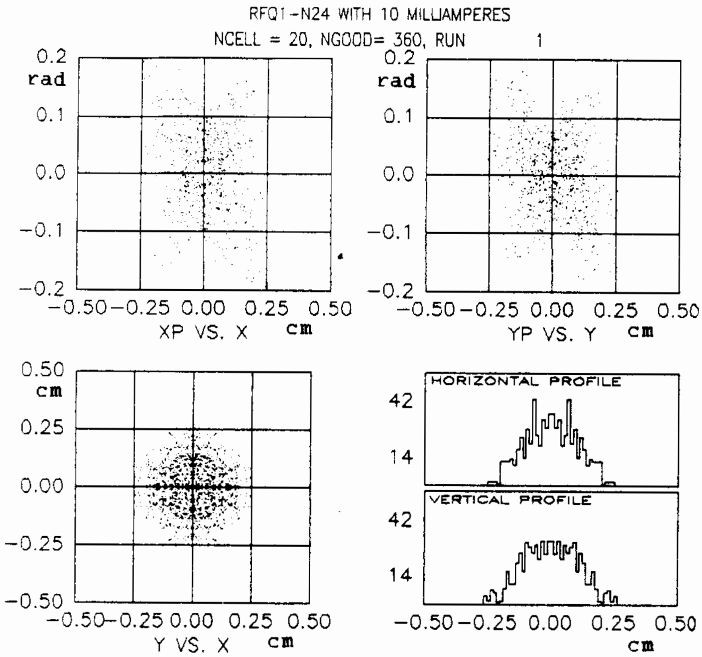

Fig. 2 Particle distribution at output plane of cell 20, with 10 milliamperes beam current.

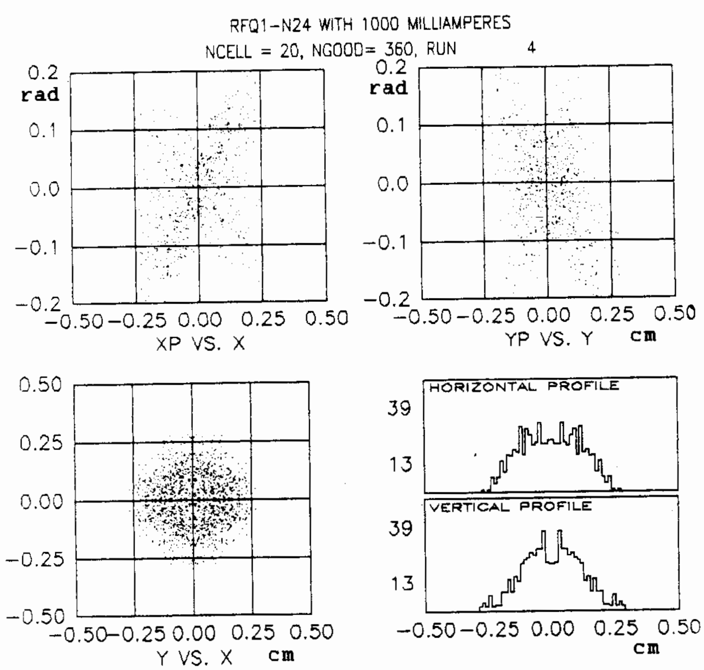

Fig. 3 Particle distribution at output plane of cell 20, with 1000 milliamperes beam current.

Table 1

Variation of Emittance with Beam current

Current (mA)	Emittance in π cm mrad $E_x(80\%)$	$E_x(100\%)$	$E_y(80\%)$	$E_y(100\%)$
10	1.16	3.82	1.00	5.73
100	1.15	3.59	1.05	6.09
400	1.13	3.69	1.13	7.53
1000	2.06	5.72	2.28	8.43

The y direction emittance increases much more than that of the x direction because within the two cells the beam is not symmetric, having a waist in the zx plane halfway through the first cell and one in the zy plane halfway through the second cell. It is not known whether the initial decrease of E_x, with increasing current, is real. With only 360 particles the error in computation is likely to be of this magnitude.

To get a more complete analysis of the effects of space and image charges a longer length RFQ must be represented. However the results are sufficient to show that this is a powerful method which could be extended to other rf devices such as drift tube linacs.

Computing Time

At present the mesh generation and matrix assembly take about 20 seconds CPU on a CDC Cyber 175. Thereafter, each time step with space charge takes about 3.5 seconds, the total CPU time being about 3 minutes for a run with graphical output.

Further Development

The program will be extended to cover a larger number of RFQ cells using the following procedure: The beam will be cycled for three rf periods through each pair of cells. (Tests on a fourth rf cycle showed that the output particle parameters are almost identical to those of the third cycle.) The particle parameters for the output of the third cycle will be stored and used as input to the next pair of cells. This process will be expensive in computing time. Therefore it is intended to try recalculating the space charge potentials only once in several time steps. The space charge field components will then be computed at each time step taking account of the mean movement of the bunch since the potential distribution was obtained. Further development work will show to what extent the program can be speeded up without too much loss of accuracy.

Preliminary results have indicated that this program should be useful in the investigation of dynamics of high current beams in rf quadrupoles.

Acknowledgements

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