TRANSIENT WAVE ANALYSIS PROGRAM USING WAVE EQUATION OF VECTOR POTENTIAL

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Summary

To simulate time dependent electromagnetic field, a computer code named ' TWA-program ' (Transient Wave Analysis program) was developed. The TWA-program solves the wave equation of the vector potential, and shows field lines on a computer graphic display. It was applied to solve problems such as the traveling microwave in a rectangular waveguide, the dipole radiation and beam induced field in a cavity structure.

Introduction

In recent accelerator technology, it becomes important to understand the time dependent phenomena of the electromagnetic field; for example, the transient response of an accelerating structure against the pulsed microwave and the short time beam loading, i.e., the wake field loss in a cavity.

In case of the two-dimensional field or the axi-symmetrical field, the vector potential has only one component which gives all field parameters. We call this vector potential ' wave potential ' in this paper. The wave potential propagates in a space according to the wave equation. The field lines are given by the equi-potential lines of the wave potential. TWA-program solves the wave equation of the wave potential using the finite difference method, and draws the equipotential lines on a computer graphic display.

The BCI-program[1] has been used to calculate the beam induced field in a cavity. The memory size of TWA-program is smaller than that of BCI-program; about one third, because TWA-program treates only one field parameter, i.e., the wave potential, on the other hand BCI-program solves three field parameters of Er, Ez and H_θ . In addition, the process of calculation is simpler than that of BCI-program.

Wave Equation of Vector Potential

In a charge-less region, we can define the electric vector potential $\mathbf{G}$ as follows.

$$\mathbf{E} = \nabla \times \mathbf{G} \qquad (1)$$

The wave equation of the electric vector potential is given by the Maxwell equations

$$\left(\nabla^2 - \frac{\partial^2}{c^2 \partial t^2} \right) \mathbf{G} = 0 \qquad (2)$$

The relation between the electric vector potential $\mathbf{G}$ and the magnetic field $\mathbf{B}$ is

$$\mathbf{B} = \frac{\partial \mathbf{G}}{c^2 \partial t} \qquad (3)$$

For the magnetic vector potential, similar expressions are given as follows

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (4)$$

$$\left(\nabla^2 - \frac{\partial^2}{c^2 \partial t^2} \right) \mathbf{A} = -\mu_0 \mathbf{J} \qquad (5)$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \qquad (6)$$

In case of the two-dimensional field, there are TE and TM-modes as illustrated in fig. 1. For the axi-symmetrical field, there are also TE and TM-modes as Fig. 2. The field components and the wave potentials of these modes are listed in Table. 1.

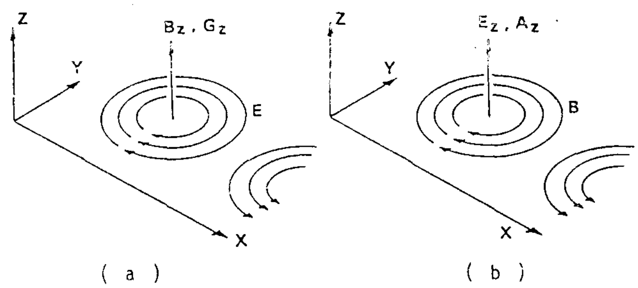


Fig. 1. Modes of the two-dimensional field.
(a) TM-mode. The magnetic field and the electric vector potential have only the z-component and smooth in z-direction.
(b) TE-mode. The electric field and the magnetic vector potential have only the z-component and smooth in z-direction.

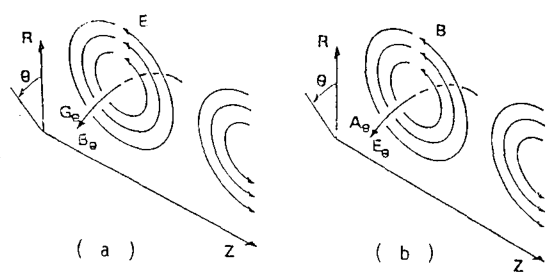


Fig. 2. Modes of the axi-symmetrical field.
(a) TM-mode. The magnetic field and the electric vector potential have only the θ-component and smooth in θ-direction. The beam induced field is also TM-mode.
(b) TE-mode. The electric field and the magnetic vector potential have only the θ-component and smooth in θ-direction.

TABLE I

FIELD PARAMETERS OF THE MODES

Coordinate	Two-dimensional		Axi-symmetrical	
Mode Name	TM	TE	TM	TE
Wave Potential (U)	Gz	Az	rG_θ	rA_θ
Equipotential Lines	$\mathbf{E}$	$\mathbf{B}$	$r\mathbf{E}$	$r\mathbf{B}$
Transverse Field	Bz	Ez	B_θ	E_θ

The wave equations (2) and (5) have the same form, so that the TE and TM-mode can be solved by the same program except the boundary conditions. For the two-dmensional field, the wave equation becomes

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{c^2\partial t^2}\right)U=0, \quad (7)$$

where U is the wave potential listed in Table 1.

For the axi-symmetricl field

$$\left(\frac{\partial^2}{\partial r^2}-\frac{1}{r}\cdot\frac{\partial}{\partial r}+\frac{1}{r^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{c^2\partial t^2}\right)U=0, \quad (8)$$

where U is rA_θ or rG_θ.

The beam induced field in a cavity structure is a kind of the axi-symmetrical TM-mode, and the wave potential is rG_θ. If there is a charge in the calculating region, the divergence of the electric field is not zero and the electric vector potential can not be given uniquely by eq. (1). To avoid this difficulty, the beam is assumed to be a line charge with zero diameter. In this case the beam is considered as flux source on the axis. The boundary condition at the axis is given from eq. (3)

$$\left[rG_\theta\right]_{r=0} = \frac{1}{\varepsilon_0}\int rH_\theta\cdot dt = \frac{1}{2\pi\varepsilon_0}\int I\cdot dt = \frac{1}{2\pi\varepsilon_0}\int \xi(z)\cdot dz, \quad (9)$$

where the Ampere's law was used and the beam is assumed to be running with constant velocity. $\xi(z)$ is the line charge density.

Numerical Calculation

The calculating region is divided into the meshes as shown in fig. 3. The difference equations of the potential equations are given as follows.

For the two-dimensional field,

$$U^{N+1}_{I,J} = \frac{\Delta T^2}{\Delta x^2}\left(U^N_{I,J+1}+U^N_{I,J-1}-2U^N_{I,J}\right) + \frac{\Delta T^2}{\Delta y^2}\left(U^N_{I+1,J}+U^N_{I-1,J}-2U^N_{I,J}\right)+2U^N_{I,J}-U^{N-1}_{I,J} \quad (10)$$

For the axi-symmetrical field,

$$U^{N+1}_{I,J} = \frac{\Delta T^2}{\Delta r^2}\left(U^N_{I+1,J}+U^N_{I-1,J}-2U^N_{I,J}\right) - \frac{\Delta T^2}{\Delta r^2}\cdot\frac{(2I-1)U^N_{I+1,J}+2U^N_{I,J}-(2I+1)U^N_{I-1,J}}{4I^2-1} + \frac{\Delta T^2}{\Delta r^2}\left(U^N_{I,J+1}+U^N_{I,J-1}-2U^N_{I,J}\right)+2U^N_{I,J}-U^{N-1}_{I,J} \quad (11)$$

where T is the normalized time ct, and N is the number of the time steps.

The wave potenatial U should satisfy the boundary conditions listed in table 2, where the free boundary means that the wave can propagate the boundary without any reflection. n is the unit vector which is normal to the boundary.

The integral time step ΔT must be smaller than convergence limit given by the following equations. In case of the two-dimwnsional field,

$$\Delta T^2\cdot\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right) < 1. \quad (12)$$

For the axi-symmetrical field,

$$\Delta T^2\cdot\left(\frac{1}{\Delta r^2}+\frac{1}{\Delta z^2}\right) < 1. \quad (13)$$

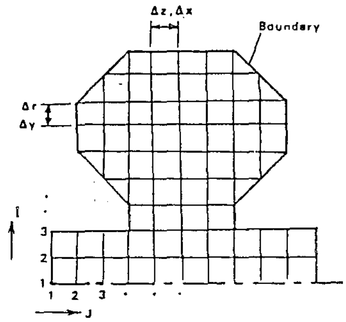


Fig. 3. The rectangular mesh for the finite difference method.

TABLE II

BOUNDARY CONDITIONS

Coordinate	Two-dimensional		Axi-symmetrical	
Mode	TM	TE	TM	TE
Conducting Boundary	$\partial G_z/\partial n = 0$,	$Az = 0$,	$\partial(rG_\theta)/\partial n = 0$,	$rA_\theta = 0$
Symmetric Boundary	$Gz = 0$,	$\partial Az/\partial n = 0$,	$rG_\theta = 0$,	$\partial(rA_\theta)/\partial n = 0$
Free Boundary	$-\frac{\partial G_z}{\partial n}=\frac{\partial G_z}{c\partial t}$,	$-\frac{\partial A_z}{\partial n}=\frac{\partial A_z}{c\partial t}$,	$-\frac{\partial(rG_\theta)}{\partial n}=\frac{\partial(rG_\theta)}{c\partial t}$,	$-\frac{\partial(rA_\theta)}{\partial n}=\frac{\partial(rA_\theta)}{c\partial t}$

Applications

Traveling Microwave

The field of the traveling TE-mode in a rectangular waveguide does not have spatial dependence along the direction of the electric field. Hence, the field is two dimensional TE-mode.

Fig. 4 shows the magnetic field liens of the traveling pulse microwave. The wave source of TE_{10}-mode is located at the left boundary. The head wave packet deminishes gradually. This is due to the fact that the group velocity is smaller than the phase velocity (dispersion), and the head wave packet loses its energy.

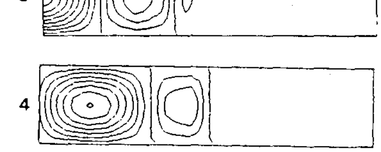
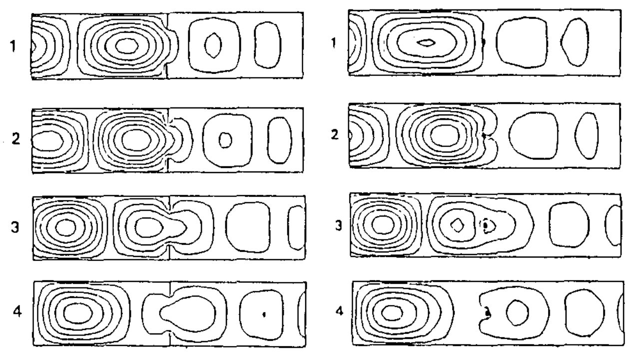


Fig. 4. The propagation of the pulsed microwave. The mesh size is 10 x 40. The width of the wave guide is 10 cm and the frequency is 2600 MHz.

Fig. 5 shows pulsed microwave traveling in the rectangular wave guide with an iris (a) and a rod (b). The microwave is partially reflected at the iris and the rod, so that the field density at the left hand side becomes greater than the right hand side. A comparison of these field densities gives the reflection coefficient and the transmission coefficient.



(a) (b)

Fig. 5. Poropagation of microwave in a rectangular wave guide with (a) an iris, (b) a rod. The right end line is the free boundary. The mesh size is 10 x 40.

Beam Induced Field in a Cavity

Fig. 6 shows the beam induced field in a disk loaded accelerating structure. The electric field on the axis is derived from eq. (1):

$$E_z = \frac{1}{r}\cdot\frac{\partial}{\partial r}(rG_\theta) \qquad (14)$$

The wake field potential is given by integrating eq. (14) about the time along the beam propagation.

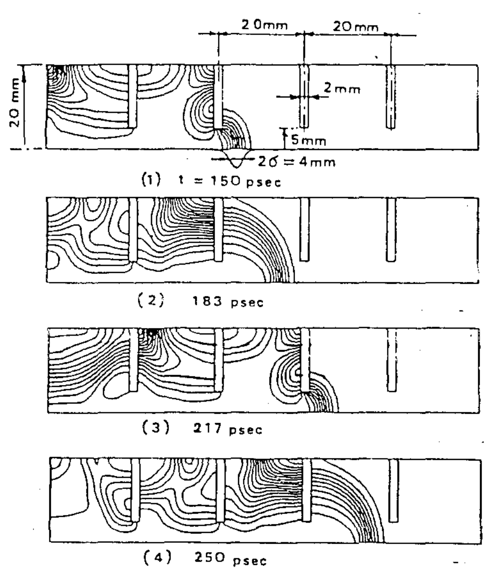
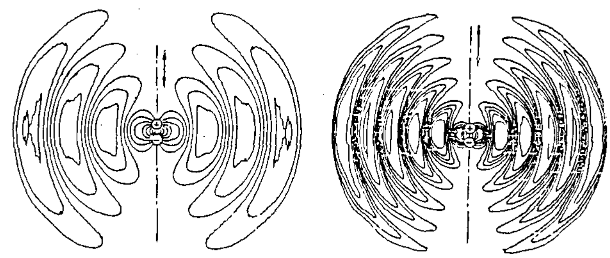


Fig. 6. Beam induced field in a disk loaded accelerating structure. The mesh size is 20 x 100.

Dipole Radiation

The field of the dipole radiation is the axi-symmetrical TM-mode. The moving charges are approximated by the line beams on the axis with bunch length equal to the diameter of the charge.

Fig. 7 (a) shows the dipole radiation in case of the maximum velocity βmax is equal to 0.5. The direction of the motion is shown by the arrows. Fig. 7 (b) is the case of βmax = 0.8. It is not necessary to say about the frequency and the amplitude of the oscillation, because the radiation pattern is determined only by the maximum velocity βmax , and there is a scaling law about the pattern dimension and the frequency.



(a) (b)

Fig. 7. The electric field pattern of the dipole radiation. The maximum velocity of the oscillation is (a) βmax = 0.5, (b) βmax = 0.8.

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REFERENCE

1) T Weiland, "On the computation of electromagnetic fields excited by relativistic bunches of the charged particles in accelerating structures" CERN/ISR-TH/80-07, (1980)