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ESTIMATION OF SPACE CHARGE AND EMITTANCE GROWTH EFFECTS IN A DRIFT REGION*

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Summary

Designing a beam transport system for the EG&G/EM electron linac with the matrix multiplication computer code, TRANSPORT,[1] leaves unresolved questions as to the effects of space charge and emittance growth on beam radius, as the code assumes constant emittance and no space charge. ZFIELD,[2] a trajectory computer code for a linac beam, has been written as a design aid. It includes space charge, plots the emittance ellipse at axial values, and plots beam radius. Comparing ZFIELD and TRANSPORT in drift regions, significant differences in beam radius predictions are found in the 2-MeV region for currents above 200 A and above 400 A in the 4-MeV region. Beam envelope growth for beams with different emittance is compared using ZFIELD. The effect of space charge on emittance growth is shown graphically.

Emittance and Trajectory Simulations

A knowledge of the effects of space charge on beam radius and emittance is needed to design a system to transport an electron linac beam. The TRANSPORT[1] code simulates progress of a beam of constant emittance through a region devoid of space charge, and thus beam envelope radius and emittance growth are underestimated. In the ZFIELD[2] code, rays are propagated through solenoidal fields in the presence of space charge. Choice of a beam of a certain emittance and the turning off of magnetic fields in this code can produce an estimate of the effects of space charge in drift regions, which can be used to supplement TRANSPORT. One can also compare these results with the effect of space charge on a beam of zero emittance. This is achieved by means of the universal curve for space charge spreading of an electron beam in field-free space.

The contribution of space charge and emittance growth to beam radius has been of concern in controlled fusion research in the transport of intense heavy ion beams through magnetic focusing channels. Reiser[3] has confirmed the difficulties of focusing high current beams at low energies, in solving the K-V equations for matched beams of fixed emittance in solenoid and quadrupole systems. Penner[4] has investigated the transport of kiloampere, multi-GeV beams over long distances using initial beams that are uniformly distributed in 2-dimensional phase space. Transport through a particular symmetric focusing-defocusing quadrupole lattice caused the transverse emittance to grow after ten cells. In this case the zero current phase advance per cell was 90° and the phase advance with space charge was 30°.

For the EG&G/EM electron linac, beams up to ½ kiloampere with energies in the 1-5 MeV range will be simulated, with the initial beam uniformly distributed in phase space.

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The universal beam spread curve may be plotted with corrections for relativistic motion (Ref. 5, page 9). This curve plots r/r_0 versus $KZ/2r_0$, where

$$K^2 = \frac{\eta_o}{\pi\,\varepsilon_o}\,\frac{I}{\left(v_z\,\gamma\right)^3} \qquad (1)$$

and

m_o = rest mass of the electron
e = electron charge
η_o = e/m_o in farads/meter
ε_o = permittivity of free space
I = beam current in amperes
c = speed of light in meters/second
$\gamma = 1/(1-[v_z/c]^2)^{1/2}$

For an axially symmetric beam, the emittance ε is defined as $1/\pi$ times the area in rr' space occupied by the points represented by the particles at a given value of z, the distance along the beam. This space may be an irregular shape, but it is convenient to think of an ellipse that encloses the area when the beam is transmitted through the linac system. The acceptance diagram of the system is a contour in rr' space showing the limiting coordinates of the particles that can pass through the system without striking the walls, not usually elliptical. A matching section can be designed, in principle, to transform the shape of the emittance to that of the acceptance. If the emittance and acceptance are approximated by circumscribing ellipses, this transformation may be accomplished by matrix operations, as in the TRANSPORT code.

The equation for calculating ray trajectories, developed by J. W. Beal (Ref. 5, page 7), assumes paraxial motion:

$$r'' = \frac{r'\eta\,\frac{\partial V}{\partial z}}{v_z^2} - \frac{r}{2}\,\frac{\eta}{v_z^2}\,\frac{\partial^2 V}{\partial z^2} - \frac{r}{2}\left(\frac{\eta B}{v_z}\right)^2 + \frac{\eta I\left(1-v_z^2/c^2\right)}{2\pi\varepsilon_o\,v_z^3}\cdot\frac{1}{r} + \frac{C}{r^3\beta^2\gamma^2} \qquad (2)$$

where

$\eta = e/(m_o\,\gamma)$
$\beta = v_z/c$
r = radius of a ray
B_z = externally applied magnetic field
V = electric potential
C = constant

Rays are started with a finite emittance, given by a phase space ellipse of a particular orientation. As an approximation to a uniform phase space distribution, rays were started on the border of the ellipse, on an ellipse of one-half the area, and $1/\sqrt{2}$ times the area of the emittance ellipse. Figure 1 shows a convergent beam with the three sets of rays designated by different markers.

As a given axial position, the radius and divergence, r', of each ray can be used to calculate a quantity

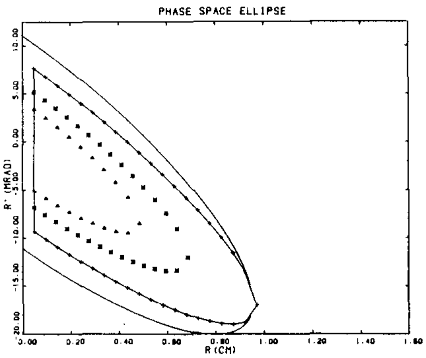

Fig. 1. Initial convergent beam with emittance of 10.01 cm-mrad. Three sets of rays are designated by different markers.

called the rms emittance, defined in Septier (Ref. 6, page 199). Equation (3) gives the rms emittance:

$$\bar{\varepsilon} = 4\left(\overline{r^2}\;\overline{r'^2} - \left(\overline{r\,r'}\right)^2\right)^{\frac{1}{2}} \qquad (3)$$

where

$$\overline{r^2} = \sum_1 \left(\frac{r_i^2}{n}\right)^{\frac{1}{2}} \qquad (4)$$

Using $r_{max} = 2\left(\overline{r^2}\right)$ and $r'_{max} = 2\left(\overline{r'^2}\right)$, a phase space ellipse is plotted in fig. 2 for rays that have drifted 70 cm, with the continuous line designating the rms emittance and the markers showing to where in phase space the individual initial rays have drifted.

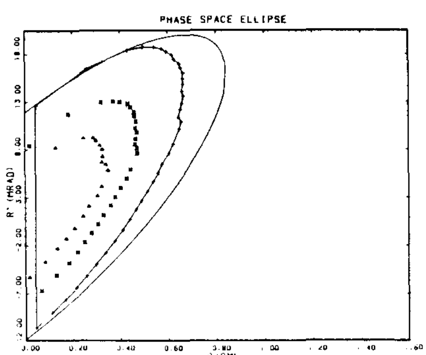

Fig. 2. Beam drifted 70 cm at 5.0 MeV and 500 A with emittance of 10.01 cm-mrad.

If we assume the electrons are far apart, emittance should not grow due to space charge (Ref. 6, page 170). But it does for purposes of transporting the beam through a linac. This is apparent if we examine fig. 1, 2, and 3. In fig. 2, we see the beam of fig. 1 has become divergent and the points designating the rays have been slightly skewed in the r' direction, due to space charge. Figure 3 shows additional distortion of the ellipse at higher current and lower energy. With increased space charge, in addition to skewing of the ellipse, distortion of the phase space area is introduced by disorder in the ray crossings. The rms ellipse is enlarged to contain these variations. The actual area in phase space has not changed, but this area requires a larger circumscribing ellipse, and this ellipse is the relevant design quantity.

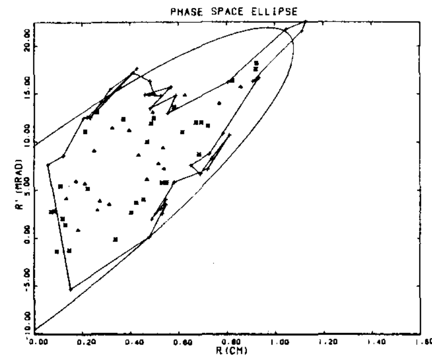

Fig. 3. Beam drifted 70 cm at 3.0 MeV and 500 A with emittance of 10.33 cm-mrad.

Results

We can determine the rms emittance and maximum beam radius for a range of beam currents and energies using the ZFIELD trajectory code. Figure 4 shows the trajectory for a current of 100 A for an energy of 5.0 MeV. The starting beam rms emittance is 10.01 cm-mrad. We can compare the maximum beam radius after a 70-cm drift with the radius of a zero emittance beam that has drifted from a waist at a radius and axial position determined by the trajectory code. Table 1 shows results of the beam current varied from 100 to 500 A for beam energies between 2.0 and 5.0 MeV.

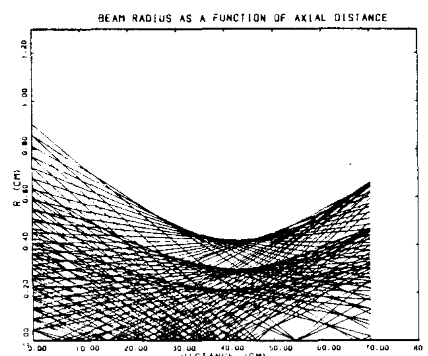

Fig. 4. Trajectory of rays drifted 70 cm at 5.0 MeV and 100 A

The differences in final beam radius produced by TRANSPORT and ZFIELD are significant in the 2.0 MeV energy row of table I. If the TRANSPORT code is used to design beam elements assuming a fixed emittance of 10 cm-mrad, table I indicates transport problems in currents above 200-300 A at 2.0 MeV and above 400 A at 3.0 MeV (table based on a 70-cm drift). Additional radial and emittance growth would be expected at most currents in the lower energy range and in high currents at all energies if all drift regions in the linac are included.

The manner in which emittance growth affects the radius of a beam with space charge is indicated in the table. Taking the ratio of the trajectory radius to that produced by the universal curve, at each current and energy, we see that an rms emittance of the order of 10 cm-mrad causes a 50% growth in the radius of the beam envelope.

TABLE I

EMITTANCE AND BEAM RADIUS AS A FUNCTION OF BEAM ENERGY AND BEAM CURRENT (FOR A 70-cm DRIFT OF AN INITIALLY CONVERGENT BEAM WITH 0.9054-cm INITIAL RADIUS AND 10.01 cm-mrad INITIAL EMITTANCE)

Energy (MeV)	Current (A)				
	100	200	300	400	500
2.0	9.96* 0.95** 0.576† 0.599††		12.13* 1.45** 0.975† 0.599††	13.69* 1.70** 1.12† 0.599††	14.49* 1.80** 1.24† 0.599††
3.0		9.98* 0.85** 0.552†	10.06* 0.98** 0.576†		10.33* 1.10** 0.687†
4.0	9.99* 0.65** 0.426†		10.01* 0.75** 0.504†	9.98* 0.80** 0.537†	9.94* 0.90** 0.549†
5.0	10.01* 0.60** 0.408†			9.99* 0.72** 4.445†	9.96* 0.75** 0.506†

*Beam rms emittance (cm-mrad)
**Maximum beam radius (cm) from trajectory
†Maximum beam radius (cm) from universal beam spread curve
††Beam radius (cm) from TRANSPORT

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