GENERAL-PURPOSE RFQ DESIGN PROGRAM*

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Summary

We have written a general-purpose, radio-frequency quadrupole (RFQ) design program that allows maximum flexibility in picking design algorithms. This program optimizes the RFQ on any combination of design parameters while simultaneously satisfying mutually compatible, physically required constraint equations. It can be very useful for deriving various scaling laws for RFQs. This program has a "friendly" user interface in addition to checking the consistency of the user-defined requirements and is written to minimize the effort needed to incorporate additional constraint equations. We describe the program and present some examples.

Introduction

There are many different criteria that can be used to optimize a particular RFQ. We might maximize the brightness for a given current, or maximize the current for a given brightness while minimizing particle loss and vane-tip activation. We might constrain the overall length and power requirements for the design, or maintain a given betatron and synchrotron tune through the structure.

We do not wish to write a separate program to handle every new approach to RFQ design. Therefore, we have written a program that automates the design for RFQ accelerators, optimizes arbitrary given parameters while fixing others, and satisfies the constraint equations governing the physics. The program does a least-squares optimization of parameters coupled with a Lagrange multiplier method to handle the constraint equations.

In the next section, we define the problem for designing RFQs and present a solution. We then give an overview and several examples of the program "RFQDES." A friendly user interface and a table that checks the consistency of the users requirements is still being developed. When fully developed, the program will pick the appropriate set of constraint equations, given a set of requirements.

Method

RFQDES was written both to automate the design process for RFQ accelerators and to increase the flexibility of our traditional design process.[1] We design the accelerator by optimizing the design on certain parameters (desired current, emittance, emittance/acceptance ratio, energy gain per cell, longitudinal and transverse space-charge parameters) while simultaneously subjecting these parameters as well as other parameters (vane modulation, minimum vane radius, synchronous phase, betatron and synchrotron tunes, etc.) to design and physics constraints.

We solve this problem by defining a function I that is the squared sum of the (fitted minus desired) parameter values plus the sum of the constraint equations multiplied by Lagrange multipliers. The function I is then minimized by taking derivatives of the function with respect to all the nonfixed parameters of the system. The resulting equations, which are highly nonlinear, are solved with the Newton-Raphson technique.

All parameters can be placed in one of three categories: (1) fixed, (2) optimized about a desired value, or (3) free floating. Fixed parameters are ignored in the following derivation. They are treated as fixed numerical constants in the constraint equations. Optimized parameters are denoted by X_a where a is an index for the X variable array. Free-floating parameters are denoted by Y_i and i is the corresponding index of this array. We define the following parameters and functions:

X^d = the desired values for parameters X

X^f = the fitted values for the parameters X

σ_a = the weight parameter for X_a

Y = the RFQ floating variables (no desired or fixed value)

α_λ = the Lagrange multiplier for the λ^{th} constraint equation

F_λ = the λ^{th} constraint equation

$F_{\lambda,a} = \partial F_\lambda / \partial X_a$

$F_{\lambda,i} = \partial F_\lambda / \partial Y_i$

We define an effective chi-square function I where

$$I = 1/2\left[\sum_a \frac{\left(X_a^f - X_a^d\right)^2}{\sigma_a^2} + 2\sum_\lambda \alpha_\lambda F_\lambda(X^f,Y)\right] \quad . \tag{1}$$

We minimize I with respect to X, Y, and α, and obtain the following set of nonlinear equations:

$$J_a = \partial I/\partial X_a = (X_a^f - X_a^d)/\sigma_a^2 + \sum_\lambda \alpha_\lambda F_{\lambda,a} = 0 \quad , \tag{2}$$

$$J_i = \partial I/\partial Y_i = \sum_\lambda \alpha_\lambda F_{\lambda,i} = 0 \quad , \text{ and} \tag{3}$$

$$J_\lambda = \partial I/\partial \alpha_\lambda = F_\lambda = 0 \tag{4}$$

for all a, i, and λ indices. (If there are A optimized parameters, I free-floating parameters and Λ constraint equations, then $(1 \le a \le A)$, $(1 + A \le i \le A + I)$, and $(1 + A + I \le \lambda \le A + I + \Lambda)$. We solve the system of eqs. (2) to (4). We obtain the matrix equation

$$J^{n+1} = J^n + [\partial J/\partial(X,Y,\alpha)|_n] * [(X^{n+1},Y^{n+1},\alpha^{n+1}) - (X^n,Y^n,\alpha^n)] = 0 \quad , \tag{5}$$

which can be inverted to give

$$(X^{n+1},Y^{n+1},\alpha^{n+1}) = (X^n,Y^n,\alpha^n) - [\partial J/\partial(X,Y,\alpha)|_n]^{-1} * (J^n) \tag{6}$$

where n and n + 1 denote the iteration number.

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Program RFQDES Overview

We adhered to the FORTRAN 77 standard in writing RFQDES. The structure is modularized to facilitate implementing future changes. All derivatives are calculated analytically to improve the precision and rate of convergence of the Newton-Raphson technique. We used the chain rule for taking derivatives so that modifications to a formula for calculating a constraint equation will have a limited or negligible impact on other formulations in the program.

The nonlinear least-squares optimizer routine, though written for this program, was designed so that it can be lifted easily from this package and used elsewhere. Each variable, except in the optimizing package where array variables are used, is given a name that indicates its function. One subroutine is used to convert back and forth between named variables and the array variables needed by the optimizer. This procedure introduces a slight run-time inefficiency but greatly facilitates coding and debugging. The resulting program is more readable and modifiable.

All parameters internal to the code are dimensionless [except for beam current, which is scaled by $1/(m_0c^2/e)$.] All lengths are scaled by the rf wavelength, potentials by $1/(m_0c^2/e)$, and electric fields by rf wavelength/(m_0c^2/e). The result is that the user's input data define the system of units.

We assumed a uniformly filled, ellipsoidal charge distribution for calculating the space-charge defocusing force. We also approximated the form factor[1,2] as

$$f(\text{length/width}) = \text{width}/(3 \times \text{length}) \quad . \tag{7}$$

Other space-charge models can be substituted in the program in place of this model.

Examples

We illustrate the code by designing (using different criteria) a single cell for an RFQ. A complete design is an extension to the design of a single cell, and this procedure is automated in the program RFQDES. All cases presented are designed for a 1.0-MeV proton beam in a 425-MHz RFQ. We define the maximum electric field on the RFQ vanes in terms of the Kilpatrick criterion[3] (19.9×10^6 V/M at 425 MHz).

The first two runs given in table I show the effect on the beam-current limit obtained in raising the maximum electric field from 2 to 3 times Kilpatrick. The normalized acceptance for both cases was $2 \times 10^{-6}\pi$ m•rad, with the beam contained in a total normalized emittance of $1.0 \times 10^{-6}\pi$ m•rad. The longitudinal and transverse space-charge μ's were fixed at 0.84 in both cases. For Case 1, the current was maximized, while the accelerating gradient was only weakly optimized. The accelerating gradient for Case 2 was fixed to that obtained for Case 1 (30 kV/cell). The current limit obtained for Case 1 was 0.060 A and for Case 2 was 0.208 A.

There are two reasons for the large increase in current for the same emittance and acceptance. First, the synchronous phase went from -21.8 to -41.3°, which effectively doubles the beam length. Secondly, the increased external electric field made the beam smaller for the fixed emittance. Since the ratio of acceptance/emittance was fixed at 2, the RFQ vanes were brought closer together. This reduced vane radius and increased electric field increased the ponderomotive focusing term (B in Ref. 1), which scales as electric-field/minimum-vane-radius.

Cases 3 and 4 were the same as Case 1 except that the maximum electric field was increased from 2 to 3 times the Kilpatrick field; all other physical parameters were held fixed (that is, the same physical RFQ). The Case 3 acceptance/emittance ratio was 2 (same as Cases 1 and 2), whereas the Case 4 acceptance/emittance ratio was 2.55, which gave the same emittance as Cases 1 and 2 ($1.0 \times 10^{-6}\pi$ m•rad). We see that the current limit for Case 3 compared to Case 1 increased to 0.157 A, but the emittance grew to $1.28 \times 10^{-6}\pi$ m•rad. The current in an emittance of $1.0 \times 10^{-6}\pi$ m•rad is 0.120 A (see Case 4). Case 4 should be contrasted with Case 2.

Conclusion

The above examples were meant to show a little of the freedom we have to design RFQs using RFQDES. We can conveniently try many different RFQ design approaches and look at various scaling laws. The code is modular, internally well documented, and has a simple data-flow structure (COMMON is not used). We can, with minimum effort, modify the program as needs change.

References

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TABLE I

THE 425-MHz RFQ CELL FOR A 1-MeV PROTON BEAM

Kilpatrick Field	Current (A)	Emittance π m•rad	Sync-Phase (°)	Modulation	Sym-Radius (m)	Vane Voltage (V)	Acceptance/ Emittance	Zero-Current Phase Advance (°) Transverse	Longitudinal
2.00E+00	6.00E-02	1.00E-06	-2.18E+01	1.53E+00	4.92E-03	1.24E+05	2.00E+00	1.35E+01	1.57E+01
3.00E+00	2.08E-01	1.00E-06	-4.13E+01	1.66E+00	2.75E-03	1.15E+05	2.00E+00	4.93E+01	2.33E+01
3.00E+00	1.57E-01	1.28E-06	-5.18E+01	1.53E+00	4.92E-03	1.85E+00	2.00E+00	1.72E+01	2.79E+01
3.00E+00	1.20E-01	1.00E-06	-5.18E+01	1.53E+00	4.92E-03	1.85E+00	2.55E+00	1.72E+01	2.79E+01