A BEAM CAVITY INTERACTION COMPUTER CODE FOR LINACS

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Summary

A computer code BCI was used to calculate the energy loss of beam bunches in two types of cavities being studied in CRNL. The merit of the proposed on-axis coupled structure for CHEER is discussed in terms of beam energy loss. A calculation for a simple structure consisting of two accelerating cells and a coupling cell is performed. The amplitude modulation of the wake field is interpreted as the excitation of the zero and π coupled-resonator modes and coupling coefficients are derived. Beam loading of a cw linear accelerator is studied by calculating the bunch energy loss and cavity energy when a cavity is excited resonantly with a series of beam bunches. Calculations of non-resonant excitation are also presented.

Introduction

The computer code, BCI[1] (Beam-Cavity-Interaction), has been used extensively in CERN for calculating energy losses in traversing cavities and various beam line elements by a single beam bunch. These energy losses are important in electron storage rings. The code calculates the electromagnetic fields excited by arbitrarily shaped bunches of charged particles travelling in structures with cylindrical symmetry by numerically integrating Maxwell's equations in the time domain. Transient fields at any instant of time are obtainable. A new version of this code includes options of using an open boundary condition and finite conductivity[2].

The Electron Test Accelerator (ETA) group at Chalk River Nuclear Laboratories has been interested in studies of heavy beam loading of accelerator structures during high current cw operation. Experiments have been performed[3] where 80% of the rf power was transferred directly to the beam. In order to simulate this cw condition, the BCI code was modified to provide an option called LINAC which performs calculations with multiple beam bunches. For the Canadian High Energy Electron Ring (CHEER) study[4], a new on-axis coupling cell structure with iris coupling was proposed. The beam energy loss in this type of structure including the loss in the coupling cell can be investigated using the BCI code.

This paper will describe our experience with this code and calculations using multiple beam bunches.

Energy Loss Parameters

Two types of cavities have been examined. Cavity A is a typical accelerating cavity that has been used in the side-coupled structure[5] of ETA. Cavity B is an accelerating cavity of an on-axis coupled structure presently under study to be used in CHEER[4]. Both of the cavities are designed for a fundamental frequency of 804 MHz. Schematics of these cavities are shown in Fig. 1.

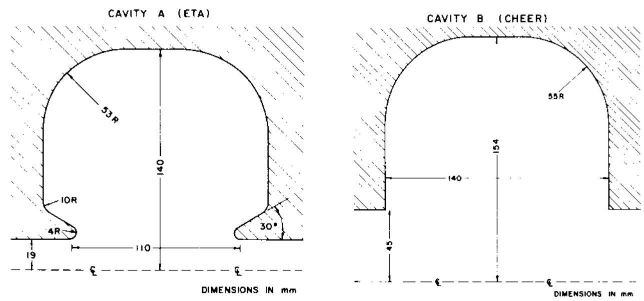


Fig 1: Profiles of cavity A and B.

The energy loss parameter k_T is defined as:

$$k_T = Q^2/\Delta E$$

where ΔE = total energy loss of the bunch and Q = total charge of the bunch. They were calculated assuming an open beam pipe at both ends of the cavities. A single beam bunch was used to excite the cavity. The shape of the beam bunch was taken as a gaussian with σ_{rms} width of 20 mm. The integrated charge for a single bunch was 2.5 x 10^{-11} C, equivalent to an electron beam of 20 mA. With the calculated energy loss of 3.70 x 10^{-10} J, the total energy loss parameter k_T of cavity A was found to be 0.592 V/pC. The k_T of cavity B was found to be 0.308 V/pC with the energy loss of the bunch equal to 1.9 x 10^{-10} J. This value of k_T can be compared to the value of 0.24 V/pC estimated by an analytic method[6]. The k_T of cavity B is relatively low compared to that of cavity A because of the missing nose cone and the increased pipe radius-to-gap length ratio. Further calculation also showed that a coupling cell used in an on-axis coupled structure with cavity B would increase the bunch energy loss by only 10%. Two unfavourable characteristics of the on-axis coupled structure are its lower shunt impedance and additional beam energy loss in the coupling cells. Our calculation for cavity B indicates that the latter is only a minor penalty and cavity B, having a relatively low energy loss parameter because of the missing nose cone, is a preferred design for CHEER in terms of beam energy loss.

Coupled Resonator Modes

A simple structure consisting of two cavities of cavity B resonantly coupled by a simple cylindrical cavity has been excited using a single bunch with σ_{rms} of 0.14 m. The long bunch was used to predominantly excite modes around the fundamental frequency f_0. The wake field of this bunch showed, in addition to the oscillations at the fundamental frequency observed in a single

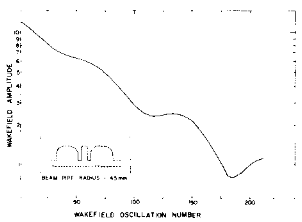


Fig.2: Wakefield modulation.

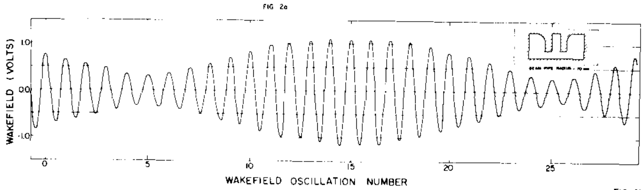


FIG 2b

cavity, modulation on the amplitude of these oscillations. This modulation is shown in Fig. 2a as the scalloping on a roughly exponential curve. The general decreasing trend of the amplitude is a result of field energy leaving the structure through the two open ends. The modulation is more clearly defined with the geometry shown in Fig. 2b where reflective boundary conditions were placed at the two ends to eliminate the decreasing trend of the amplitude. The amplitude modulation of the wake field oscillation for this geometry is very prominent. Since this modulation does not appear with a single cavity, its appearance may be inferred as the excitation and interference of zero and π modes of the simple structure having frequencies $\pm\Delta$ displaced from the fundamental frequency. The values of Δ is then equal to f_0 divided by the number of wake field ocsillation in each modulation period. Furthermore, the values of Δ has been found to increase with the radius of the bore hole that couples the cavities together. This is consistent with the proposed interpretation of coupled resonator modes because the frequency splitting of zero and π modes increases with an increase in coupling. Table 1 shows the modulation period, confirmed by Fast Fourier Transform Analysis, and the derived coupling coefficients for three sizes of beam pipe. The coupling coefficients are calculated as:

$$k = \frac{\text{frequency difference of zero and } \pi \text{ modes}}{f_0} = \frac{2\Delta}{f_0}$$

They are compared to coupling coefficients calculated using SUPERFISH[7] as reported in reference 4. Though good agreement is found for a pipe radius of 0.045 m, there are discrepancies at large apertures and further studies will be required to resolve these discrepancies.

TABLE 1 Coupling Coefficients Determined from BCI and SUPERFISH

Beam Aperture Radius (m)	Modulation Period (bunches)	k BCI (%)	k SUPERFISH (%)
0.07	19	10.5	6.8
0.06	29	6.9	4.5
0.045	102	1.96	2.12

Multiple Bunches Excitation

Since an understanding of cw beam loading is necessary for optimum structure design, an option called LINAC has been added to BCI at CRNL. This option will repeat the beam bunches at specified regular time intervals, simulating the cw condition in an acccelerator. Using this option, a cavity without external power source was beam excited. The bunch energy loss and total cavity energy were examined as the cavity approached equilibrium. Both resonant and non-resonant conditions were studied.

In these calculations, cavity A was used. The individual bunch had a gaussian shape with a σ_{rms} of 0.02 m. The fundamental frequency of the cavity was determined as 797.65 MHz (λ_0 = 0.37585 m) by studying the wake field oscillation after a bunch, whose length was large compared to λ_0, had traversed the cavity. Calculations with different conductivities Σ of the cavity wall were performed. This is advantageous because the time required for a cavity to achieve equilibirium is inversely proportional to the wall losses and one can effectively compress the time scale for achieving equilibrium to save computer time by decreasing the conductivity.

Accelerating cavities are usually operated at the resonant condition, when the bunch frequency f_B is equal to the cavity fundamental frequency f_0 or its sub-harmonics. Figures 3 and 4 shows respectively the cavity energy and bunch energy loss as a function of bunch number under this condition. Initially, the bunch energy loss increases linearly with the number of bunches and the cavity energy increases as the square of the number of bunches. This shows the increases of bunch energy loss are mainly because of the increase of the beam induced wake field, which has an amplitude proportional to the square root of the cavity energy acting to decrease the beam energy.

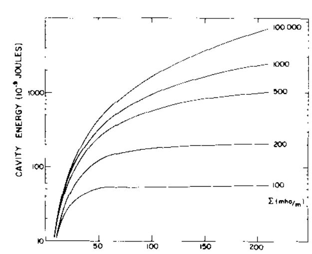
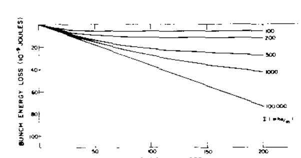


Fig.3,4: Resonant condition.



Both the bunch energy loss and cavity energy approach constant levels, when the bunch energy loss is equal to the wall loss in the cavity. According to simple electromagnetic theory this energy loss is proportional to the square of the field strength at the wall surface and the skin depth of the conductor. In terms of cavity energy E_c and wall conductivity Σ, the energy loss is proportional to $E_c/\Sigma^{1/2}$. This proportionality is approximately followed by the equilibrium energy losses found in Figs. 3 and 4 for conductivity values of 100, 200 and 500 mho/meter.

The non-resonant condition, where the bunch frequency is not equal to the cavity mode frequency or its sub-harmonics, can be realized in any recirculating accelerator when modes other than the accelerating mode of the accelerator are considered. This general case is of great importance when beam line elements are considered. Figure 5 shows the bunch energy losses when these two frequencies differ by various amounts. Because of the difference in frequency between the wake field and beam bunches, the bunches will either lose or gain energy depending on their relative phase with the wake field. With the relative phase between the bunches and wake field continuously shifting, beat patterns in the bunch energy loss (Fig. 5) and cavity energy (Fig. 6) are observed. The bunch energy loss is maximum when the bunch is exactly out of phase with the wake field, and minimum when exactly in phase. The number of bunches, N, in a beat cycle is equal to the ratio of f_0 and the difference of the two frequencies f_B and f_0, i.e.,

$$N = f_0/|f_B - f_0|.$$

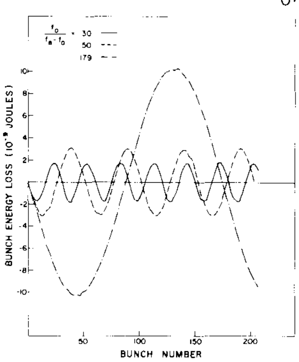


FIG 5

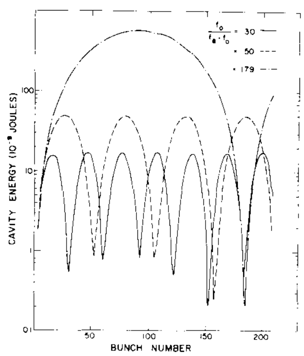


FIG 6

Fig.5,6: Beat pattern, wall conductivity= 10^5 mho/m.

Figures 7 and 8 show the bunch energy loss and cavity energy when N is equal to 30 for different conductivities of cavity wall. One can make four observations. Firstly, the bunch energy loss has a beat pattern which repeats every 30 bunches as expected. Secondly, the amplitude of the beat pattern decreases with the bunch number to a constant value when equilibrium is achieved. As expected in forced oscillation, the field in the cavity is oscillating with the same frequency as the bunch frequency at this time. Thirdly, the equilibrium bunch energy loss and cavity energy are smaller than those in the resonant excitation calculation because the wake field in the cavity has not been able to build up substantially with some bunches taking energy out of the cavity at regular intervals. Fourthly, with the equilibrium cavity energies almost equal for all conductivities, the bunch energy loss at equilibrium is mainly a function of conductivity.

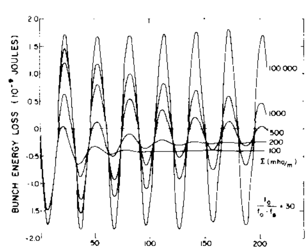


FIG 7

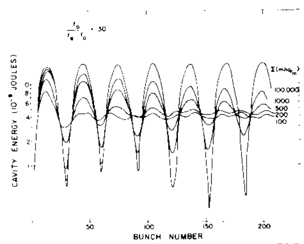


FIG 8

Fig.7,8: Non-resonant condition.

In addition to what are shown in the figures, it is also found that: in the resonant condition, the most negative wake field is calculated for the peak of the beam bunch, showing zero phase shift; in the non-resonant condition, the most negative wake field occurs at a time later than the peak of the beam bunch, corresponding to a finite phase shift. This is again as expected for simple forced oscillation as in observations one to three above.

The higher energy loss in the resonant condition shows that one must be careful to recognize the various excitation modes of the accelerating structure in a recirculating accelerator. One must avoid having the bunch frequency equal to the frequency of any of the higher order modes, or the coupled resonator modes of the accelerating structure, or the mode frequency of any beam line elements.

Conclusions

In order to understand beam energy losses in cw linear accelerator with heavy beam loading, calculations using the computer code BCI (Beam-Cavity Interaction) were performed. The total energy loss parameters with a single bunch calculated for the accelerating cells used in ETA (CRNL) and in CHEER were found to be 0.592 and 0.308 V/pC respectively. The additional energy loss in the coupling cells used in the CHEER on-axis coupled structure was calculated to be only 10% of that for the complete structure. The modulation of the wake field of a simple structure consisting of a coupling cell and two accelerating cells was interpreted as the excitation of zero and π coupled-resonator modes. The derived coupling coefficients agree with those calculated by SUPERFISH for small beam pipes but differ for large beam pipes. A new option in the code for simulating cw operation was used to study the resonant and non-resonant excitation of a cavity. The calculations show that the bunch energy loss and cavity energy follow relations of generalized forced oscillations. The comparison of the resonant and non-resonant results reaffirmed that, in a recirculating accelerator and for beam energy loss considerations, one should avoid situations in which the bunch frequency is equal to any mode frequency (or its subharmonics) of any beam line element except that of the accelerating mode.

References

1. T. Weiland, "On the Computation of Electromagnetic Fields Excited by Relativistic Bunches of Charged Particles in Accelerating Structures", CERN report, CERN/ISR-TH/80-07 (January 1980).
2. T. Weiland, "Transient Electromagnetic Fields of Bunched Beam in Accelerating Structures with Infinitely Long Tubes", CERN report, CERN/ISR-TH/80-46 (November 1980).
3. G.E. McMichael, J. McKeown and J.S. Fraser, "Beam Loading Experiments with a Side-Coupled Structure", Proceedings of 1979 Linear Accelerator Conference, Brookhaven National Laboratory Report, BNL-51134 (1979) pp. 180.
4. J. McKeown and S.O. Schriber, "Accelerating Structure with On-Axis Couplers for Electron Storage Rings", Proceedings of 1981 Particle Accelerator Conference, IEEE Trans. Nucl. Sci., NS-28 (3) (1981) pp. 2755.
5. G.E. McMichael and J. McKeown, "Operational Experience with Coupled-Cavity Structures in a High Duty Factor Accelerator", Proceedings of 1979 Particle Accelerator Conference, IEEE Trans. Nucl. Sci., NS-26 (3) (1979) pp. 4108.
6. K. Bane, A. Chao, and T. Weiland, "A Simple Model for the Energy Loss of a Bunched Beam Traversing a Cavity", CERN report, CERN/ISR-TH/81-02 (February 1981).
7. K. Halbach, R.F. Holsinger, W.E. Jule and D.A. Swenson, "Properties of the Cylindrical Rf Cavity Evaluation Code SUPERFISH", Proceedings of the 1976 Proton Linear Accelerator Conference, Atomic Energy of Canada Limited, Report AECL-5677 (November 1976) pp. 122.