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GLOBAL BETATRON COUPLING COMPENSATION FOR THE HADRON STORAGE RING OF THE ELECTRON-ION COLLIDER*

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Abstract

The Electron Ion Collider (EIC), to be constructed at Brookhaven National Laboratory, will collide polarized high-energy electron beams with hadron beams, achieving luminosities up to $1 \times 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ in the center-mass energy range of 20-140 GeV. The Hadron Storage Ring (HSR) of the EIC will utilize the arcs of the Relativistic Heavy Ion Collider (RHIC) and construct new straight sections connecting the arcs. In this article, we will examine all available skew quadrupoles currently in the latest HSR lattice design and explore possible schemes for future global betatron coupling correction with RHIC-like decoupling feedback system. The effects of detector solenoids and quadrupole rolls are estimated at injection and stored energies. We performed a preliminary simulation of global coupling correction with the 12 skew quadrupole families.

INTRODUCTION

The Electron Ion Collider (EIC), to be constructed at Brookhaven National Laboratory, will collide polarized high-energy electron beams with hadron beams, achieving luminosities up to $1 \times 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ in the center-mass energy range of 20-140 GeV [1, 2]. The EIC includes two storage rings: the hadron storage ring (HSR) and the electron storage ring (ESR). Both ring will be housed in the tunnel of the Relativistic Heavy Ion Collider (RHIC) which is scheduled to shut down in 2025.

The Hadron Storage Ring (HSR) of the EIC will utilize the arcs of the RHIC rings with new straight sections connecting the arcs. Figure 1 shows the schematic plot of skew quadrupole grouping and power supply connection for the Blue ring of RHIC. For each ring of RHIC, there are totally 12 global skew quadrupole power supplies. Each skew quadrupole power supply feed the same current to 8 global skew quadrupoles in a same sector.

For the RHIC operation, we further group these 12 sub-skew quadrupole families or power supplies into 3 skew quadrupole families. Since the global coupling coefficient is a complex number, in principle, only two orthogonal global skew quadrupole families are sufficient for its correction. For example, for the RHIC global decoupling feedback, we further paired the skew quadrupole Family 1 and Family 3 into a new family, Family 13, whose contribution to the global coupling coefficient is about orthogonal to the contribution of Family 2 [3]. The polarity definition is based the assumption of 1 unit transverse integer tune split. Table 1 lists all the skew quadrupoles used for the RHIC global decoupling.

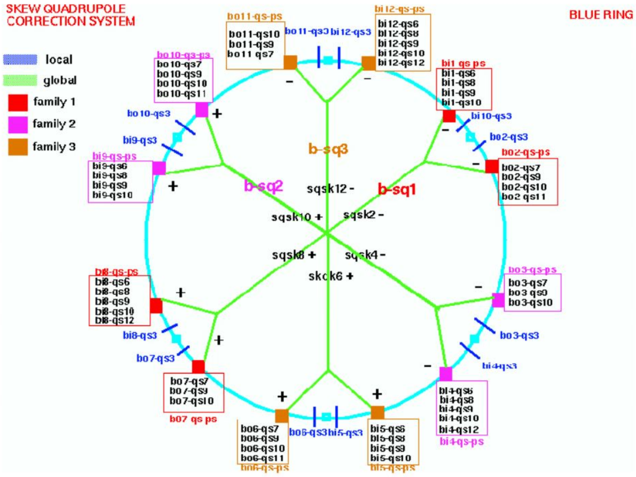

Figure 1: The schematic plot of skew quadrupoles in the Blue ring of the RHIC.

In the following, we will examine available skew quadrupoles in the latest HSR lattice design and its possibility of implementing similar RHIC-like skew quadruopole grouping for efficient global coupling correction. We estimate the coupling contributions from detector solenoid and random quadrupole roll errors. With the current HSR skew quadrupoles, we performed a quick simulation test for HSR global coupling correction.

HSR GLOBAL COUPLING ESTIMATE

The HSR Lattice

In the following, we will adopt the latest lattice design for the HSR with 1 interaction point at IP6. The lattice is designed for the collision mode between 275 GeV proton and 10 GeV electron beams. Due to the crossing angle collision with crab cavities and spin rotators in the IRs, the HSR lattice design does not maintain the regular 6-folded symmetry of RHIC lattice design. Furthermore, the design integer tune split is 2 unit, while for the RHIC it is 1 unit.

To estimate the global betatron coupling, we will utilize the Hamiltonian perturbation theory of betatron coupling in

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Table 1: Global Skew Quadrupoles Used in the RHIC

Blue:			
Family	Power Supply	Skew Quadrupoles	Polarity
F1	bo7-qs-ps	bo7-qs 7,9,10	+1
F1	bi8-qs-ps	bi8-qs 6,8,9,10,12	+1
F1	bo2-qs-ps	bo2-qs 7,9,10,11	-1
F1	bi1-qs-ps	bi1-qs 6,8,9,10	-1
F2	bo10-qs-ps	bo10-qs 7,9,10,11	+1
F2	bi9-qs-ps	bi9-qs 6,8,9,10	+1
F2	bo3-qs-ps	bo3-qs 7,9,10	-1
F2	bi4-qs-ps	bi4-qs 6,8,9,10,12	-1
F3	bo6-qs-ps	bo6-qs 7,9,10,11	+1
F3	bi5-qs-ps	bi5-qs 6,8,9,10	+1
F3	bo11-qs-ps	bo11-qs 7,9,10	-1
F3	bi12-qs-ps	bi12-qs 6,8,9,10,12	-1
Yellow:			
Family	Power Supply	Skew Quadrupoles	Polarity
F1	yi2-qs-ps	yi2-qs 6,8,9,10	-1
F1	yo1-qs-ps	yo1-qs 7,9,10,11	-1
F1	yi7-qs-ps	yi7-qs 6,8,9,10,12	+1
F1	yo8-qs-ps	yo8-qs 7,9,10	+1
F2	yi3-qs-ps	yi3-qs 6,8,9,10,12	-1
F2	yo4-qs-ps	yo4-qs 7,9,10	-1
F2	yo9-qs-ps	yo9-qs 7,9,10,11	+1
F2	yi10-qs-ps	yi10-qs 6,8,9,10	+1
F3	yi11-qs-ps	yi11-qs 6,8,9,10,12	-1
F3	yo12-qs-ps	yo12-qs 7,9,10	-1
F3	yo5-qs-ps	yo5-qs 7,9,10,11	+1
F3	yi6-qs-ps	yi6-qs 6,8,9,10	+1

circular accelerators [4]. The two eigentunes Q_1, Q_2 with betatron coupling are given by

$$Q_1 = Q_{x,0} - \frac{\Delta}{2} + \frac{1}{2}\sqrt{\Delta^2 + (C^-)^2}\,, \tag{1}$$

$$Q_2 = Q_{y,0} + \frac{\Delta}{2} - \frac{1}{2}\sqrt{\Delta^2 + (C^-)^2}\,. \tag{2}$$

Here $|\Delta|$ is the integer tune split $\Delta = Q_{x,0} - Q_{y,0} - p$, p is the integer tune split. C^- is the coupling coefficient which is defined as

$$\begin{aligned} C^- = \frac{1}{2\pi} \oint \sqrt{\beta_x \beta_y} \Big[& k_{1s} + k_s \Big(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\Big) \\ & - i k_s \Big(\frac{1}{\beta_x} + \frac{1}{\beta_y}\Big) \Big] e^{i(\Psi_x - \Psi_y)} dl. \end{aligned} \tag{3}$$

Detector Solenoid Effect

The full length of the EIC detector solenoid in IR6 is 4 meters long. The design longitudinal magnetic field is about 2 Tesla. Figure 2 shows the amplitude of the coupling coefficient $|C^-|$ as a function of the longitudinal solenoid strength. The coupling coefficient amplitude from the detector solenoid at injection with proton energy 24 GeV is about 10 times larger than at storage with proton energy 275 GeV with the same magnetic field.

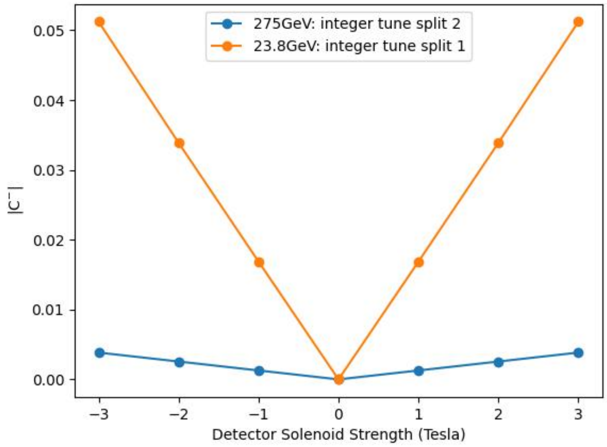

Figure 2: $|C^-|$ as a function of solenoid strength for the HSR.

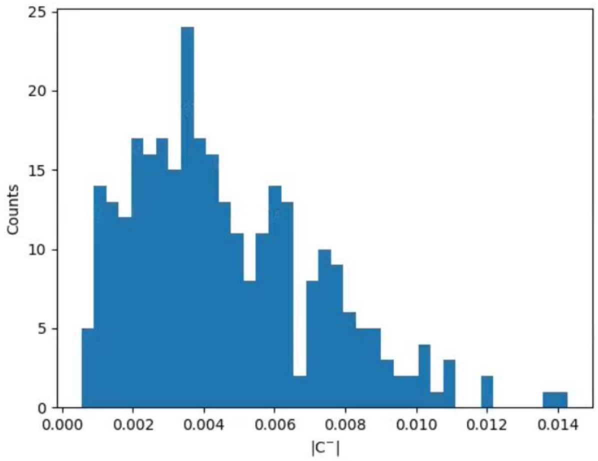

Figure 3: Histogram of $|C^-|$ with RMS 100 µrad quadrupole roll errors for the 275 GeV HSR lattice.

Effect of Quadrupole Roll Errors

For a random distribution of quadrupole roll angles along the ring, we can estimate the amplitude of the coupling coefficient with

$$|C^-|^2 = \frac{1}{\pi^2}\left(\sum \beta_x \beta_y (k_1 l)^2\right) < \theta^2_{\mathrm{roll}} > . \tag{4}$$

Here $< \theta_{\mathrm{roll}} >$ is the RMS of quadrupole roll errors. With the latest 1-IR HSR store lattice, assuming $< \theta_{\mathrm{roll}} >= 100$ µrad, $|C^-|$ is 0.0054.

Here we carry out a numerical calculation of $|C^-|$ with 300 seeds of random quadrupole roll errors. The RMS value of quadrupole roll error is assumed to be 200 µrad. Figure 3 shows its histogram. The mean of $|C^-|$ is 0.0046, which is close to the analytical estimation. However, for some worst cases, the amplitude of the coupling coefficient can go up to 0.014.

Table 2: Skew Quadrupoles in the HSR Lattice Design

Sector	Power Supply	Skew Quadrupoles
5	yo5-qs-ps	yo5-qs 7,9,10,11
6	yi6-qs-ps	yi6-qs 6,8,9,10
7	yi7-qs-ps	yi7-qs 6,8,9,10,12
8	yo8-qs-ps	yo8-qs 7,9,10
9	yo9-qs-ps	yo9-qs 7,9,10,11
10	bo10-qs-ps	bo10-qs 7,9,10,11
11	bo11-qs-ps	bo3-qs 7,9,10
12	bi12-qs-ps	bi4-qs 6,8,9,10,12
1	bi1-qs-ps	bi8-qs 6,8,9,10
2	yi2-qs-ps	yi2-qs 6,8,9,10
3	yi3-qs-ps	yi3-qs 6,8,9,10,12
4	yo4-qs-ps	yo4-qs 7,9,10

Requirement of Global Coupling Compensation

To achieve high luminosity in the EIC, we adopt flat beams at the interaction point. For the highest luminosity collision mode between 275 GeV protons and 10 GeV electrons, the vertical beam size is about 11 times smaller than the horizontal beam size at the IP. To achieve that, we aim for ratios of the transverse β^* functions and transverse emittances to be 11:1. To generate and maintain this large emittance ratio, we need very good control of the global coupling in the HSR. Early numerical simulation studies and beam experiments performed in RHIC show that the amplitude of the coupling coefficient needs to be 10 times smaller than the transverse tune split. For the HSR design tunes (0.228, 0.210), we need the coupling coefficient $|C^-|$ to be less than 0.0018.

HSR GLOBAL DECOUPLING

Global Skew Quadrupoles

Table 2 lists the skew quadrupoles in the current HSR design lattice. The HSR skew quadrupoles include skew quadrupoles from both the Blue and the Yellow rings of RHIC, depending on which RHIC arcs are to be used for the HSR.

Figure 4 shows the contributions to the coupling coefficient from the 12 HSR skew quadrupole families. The RHIC skew quadrupole families are almost uniformly distributed along 0 to 360 degrees, while the coupling contributions from HSR skew quadrupole families are not. The reason for this difference between HSR and RHIC is that the HSR lattice does not maintain the 6-folded symmetry of the RHIC lattice design. This will make global coupling correction less effective and less efficient.

Numerical Simulation for Global Decoupling

For a quick simulation test of global coupling compensation, with 200 µrad random quadrupole roll errors along the HSR, we optimize the strengths of 12 skew quadrupole families to minimize the off-diagonal coupling elements of the one-turn matrix and the vertical dispersion and its prime at IP6. Figure 5 shows the calculated determinant of the local coupling matrix **C** along the ring before and after correction. It can be seen that the betatron coupling along the ring is well corrected.

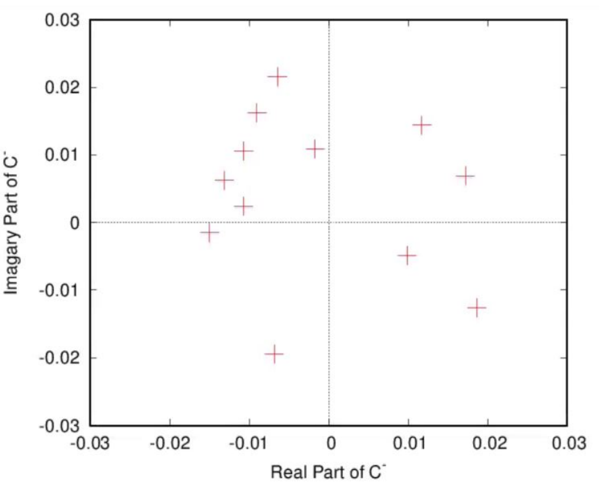

Figure 4: Contributions to coupling coefficient from 12 skew quadrupole families in the HSR.

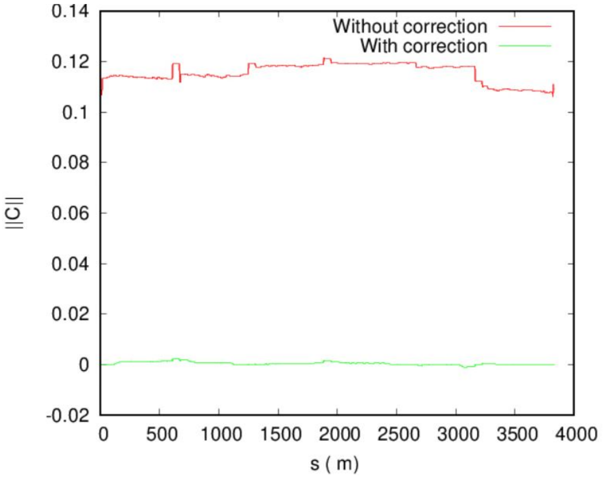

Figure 5: Determinant of coupling matrix before and after coupling correction.

SUMMARY

In this article, we examined the available skew quadrupoles currently in the HSR lattice design. The contributions from the detector solenoid and the random quadrupole roll errors are estimated. We noticed that the contributions to the coupling coefficient from the 12 HSR skew quadrupole families are not uniformly distributed in the coupling coefficient space as for RHIC. We performed a quick simulation test of coupling correction with these 12 HSR skew quadrupole families. Next, we will explore the possibility of having a 1-unit tune split for the HSR design lattice and figure out how to group the 12 skew quadrupoles for effective and efficient global coupling correction, as done in RHIC.

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