ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-WEPM049

FINITE ELEMENT SIMULATION OF FAST CORRECTOR MAGNETS FOR PETRA IV*

J. Christmann†, H. De Gersem, M. von Tresckow, TU Darmstadt, Darmstadt, Germany
A. Aloev, S. H. Mirza, S. Pfeiffer, H. Schlarb
Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

Abstract

The new synchrotron light source PETRA IV at DESY will use a fast orbit feedback system (FOFB) with hundreds of fast corrector magnets to meet stringent orbit stability requirements. These magnets are operated at high frequencies, creating strong eddy currents that result in Joule losses and a time delay between applied voltage and aperture field. User experiments impose challenging requirements on beam operation to preserve the point of the radiation source. To meet the demanding feedback requirements, finite element (FE) simulations are needed to understand the characteristics of the corrector magnet. However, due to the small skin depths at high frequencies and the laminated structure of the yoke, these simulations need a very fine mesh and are thus very costly. Therefore, we homogenize the laminated yoke which reduces the computational effort but captures the eddy current effects accurately. The reduction of simulation times from several hours to a few minutes allows us to conduct extensive studies of the eddy current losses and the field quality of the magnets.

INTRODUCTION

PETRA IV at DESY will be a fourth-generation synchrotron light source, relying on an ultra-low emittance ring to achieve an extremely high brightness [1]. Given the usual 10 percent beam size stability requirement [2, 3], the intended ultra-low emittance implies stringent orbit stability. To meet these requirements, PETRA IV needs a FOFB that counteracts distortions due to perturbations occurring on short time scales, such as ground vibrations, power supply noise, or changes in insertion device settings [4]. To react to these disturbances, the corrector magnets will be powered with currents containing frequencies in the kHz-range. Hence, strong eddy currents will be induced, resulting in attenuation and time delay of the aperture field as well as Joule losses. Investigating these effects via FE simulations over a broad frequency range is necessary to understand the characteristics of the magnets. However, the computational effort is prohibitive due to the small skin depths at high frequencies and the laminated structure of the magnet yoke. In this work, we homogenize the yoke which reduces the computational effort drastically, causing simulation times to drop from several hours to a few minutes. The homogenization technique allows us to conduct simulations over the full frequency range of interest up to 65 kHz. All simulations are carried out with the LF Frequency Domain Solver of CST Studio Suite® [5] assuming linear material properties, i.e., non-linearities and hysteresis are neglected.

First, we explain the homogenization technique. Next, we use a toy model for validation. Then, we apply the technique to a model of a fast corrector for PETRA IV. Herein, we investigate the Joule losses for different lamination thicknesses and the multipole coefficients along the axis. Finally, we give a conclusion.

HOMOGENIZATION TECHNIQUE

We conduct frequency domain FE simulations of a magnetoquasistatic problem. Let Ω be the computational domain and let $H_\mathrm{D}(\mathrm{curl};\Omega)$ be the Sobolev space consisting of all square-integrable vector fields $\underline{\vec{v}} : \Omega \to \mathbb{C}^3$ whose (weak) curl is square-integrable and whose tangential components vanish on the Dirichlet part of the boundary Γ_D, i.e.,

$$H_\mathrm{D}(\mathrm{curl};\Omega) := \{\underline{\vec{v}} \in L^2(\Omega;\mathbb{C}^3) : \nabla\times\underline{\vec{v}} \in L^2(\Omega;\mathbb{C}^3),\ \vec{n}\times\underline{\vec{v}}|_{\Gamma_\mathrm{D}} = 0\}.$$

Then, the weak formulation of our problem reads: Determine $\underline{\vec{A}} \in H_\mathrm{D}(\mathrm{curl};\Omega)$ such that

$$\int_\Omega \left(\nu\nabla\times\underline{\vec{A}}\right)\cdot\left(\nabla\times\underline{\vec{A}}'\right)\mathrm{d}V + j\omega\int_\Omega \sigma\underline{\vec{A}}\cdot\underline{\vec{A}}'\,\mathrm{d}V = \int_\Omega \underline{\vec{J}}_\mathrm{s}\cdot\underline{\vec{A}}'\,\mathrm{d}V \quad \forall\underline{\vec{A}}' \in H_\mathrm{D}(\mathrm{curl};\Omega),$$

where ω is the angular frequency, $\underline{\vec{A}}$ the magnetic vector potential, $\underline{\vec{J}}_\mathrm{s}$ the source current density, σ the electrical conductivity, and ν the reluctivity [6, 7].

In the magnets' yoke, σ and ν are functions of the spatial coordinate, since they are different for the conducting laminates and isolation sheets. The homogenization technique, as proposed in [8], consists in replacing σ and ν in the yoke with spatially constant material tensors

$$\overline{\overline{\sigma}} = \gamma\sigma_\mathrm{c}\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},$$

$$\overline{\overline{\underline{\nu}}} = \frac{\sigma_\mathrm{c} d\delta\omega\,(1+j)}{8}\frac{\sinh\left((1+j)\,\frac{d}{\delta}\right)}{\sinh^2\left((1+j)\,\frac{d}{2\delta}\right)}\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix} + \nu_\mathrm{c}\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix},$$

* Work funded by Deutsches Elektronen Synchrotron DESY and by the Deutsche Forschungsgemeinschaft (DFG) – Project-ID 264883531 – GRK 2128 ”AccelencE”

† jan-magnus.christmann@tu-darmstadt.de

ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-WEPM049

where σ_c denotes the conductivity of the laminates, ν_c their reluctivity, d their thickness, and $\delta = \sqrt{\frac{2\nu_c}{\sigma_c \omega}}$ is the skin depth. The parameter γ is the percentage of the yoke's volume consisting of conducting material, called "stacking factor". An explanation for the choice of the conductivity tensor can be found in [9]. The derivation of the reluctivity tensor is detailed in [8].

VALIDATION STUDY

To validate the homogenization technique for usage in the context of fast corrector magnets, we use a toy model of such a magnet, see Fig. 1. The toy model is a dipole magnet with a C-shaped iron yoke and a copper beam pipe. The geometrical dimensions are given in Table 1. The relative permeability of iron is set to $\mu_{r,\mathrm{Fe}} = 1000$ and its conductivity is $\sigma_{\mathrm{Fe}} = 10.4\,\mathrm{MS\,m^{-1}}$. For copper, we have $\mu_{r,\mathrm{Cu}} = 1$ and $\sigma_{\mathrm{Cu}} = 58\,\mathrm{MS\,m^{-1}}$. The coils have 250 turns and a peak current of 10 A. To reduce the computational

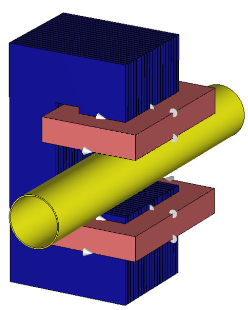


Figure 1: Toy model in CST Studio Suite® [5].

Table 1: Geometrical Dimensions of the Toy Model

Yoke	Beam Pipe
Height: 95 mm	Outer Radius: 10.5 mm
Width: 50 mm	Thickness: 0.5 mm
Length: 40 mm	Length: 140 mm

effort, we use both symmetry planes that are present in the model. We investigate the eddy current losses and the multipole coefficients in the magnet's center at test frequencies $f = 10\,\mathrm{Hz}, 100\,\mathrm{Hz}, 500\,\mathrm{Hz}, 1\,\mathrm{kHz}$.

To capture the eddy current effects accurately also at higher frequencies beyond 1 kHz, a fine mesh is necessary. This is illustrated in Fig. 2 which shows the eddy current losses at the test frequencies over the number of tetrahedra in the mesh. Using a computer with an Intel(R) Xeon(R) W-2995 CPU @ 3 GHz with 18 Cores and 256 GB RAM, the total simulation time for the finest mesh we have been able to use with the available memory is 3 h 37 min.

Figure 3a shows a comparison of the eddy current losses obtained for the full model with the finest mesh featuring $6.4 \cdot 10^6$ tetrahedra to the homogenized model with a mesh featuring $1.8 \cdot 10^5$ tetrahedra. We observe a good agreement between the homogenized and the full model while the total simulation time for the homogenized model is reduced to only 4 min 37 s. The maximum deviation of the losses in the yoke of the homogenized model from those in the full model is 4.5% at $f = 10\,\mathrm{Hz}$. We omit the plots of the beam pipe losses but note that they are also well approximated with a maximum relative deviation of 0.8% at $f = 1\,\mathrm{kHz}$.

Next, we come to the approximation of the multipole coefficients by the homogenized model. Figure 3b shows the dipole coefficients for both models at the test frequencies. The maximum relative deviation is 1% at $f = 1\,\mathrm{kHz}$. We have also investigated the approximation of quadrupoles and sextupoles. Here, the maximum relative deviations are 0.6% and 0.4%, respectively.

Moreover, we have tested two other homogenization techniques described in [10,11], but they give significantly worse approximations.

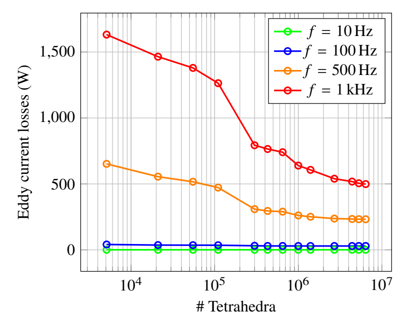
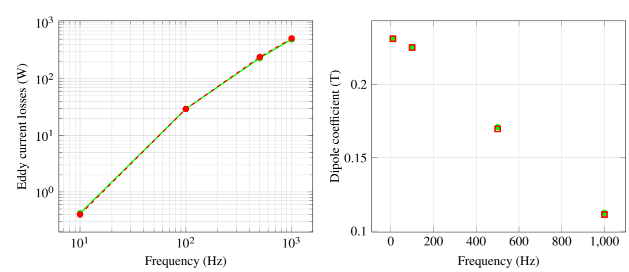


Figure 2: Eddy current losses in the yoke over the number of tetrahedra in the mesh.



(a) Eddy current losses. (b) Dipole coefficients.

Figure 3: Comparison homogenized model (red) vs full model (green).

APPLICATION TO CORRECTOR MAGNET

Next, we apply the homogenization to a model of a corrector magnet for PETRA IV, see Fig. 4. The design is reminiscent of an octupole magnet but produces a dipole field. On each of the eight posts pointing to the center, there is one thick main coil with 53 turns and one thinner auxiliary coil with 22 turns, only one of which carries current. The current-carrying coils are shown in red and the inactive coils in gray. Both the main and auxiliary coils get a current of 27.4 A. The iron yoke's diameter is 580 mm and its length is 90 mm. The model does not include a beam pipe.

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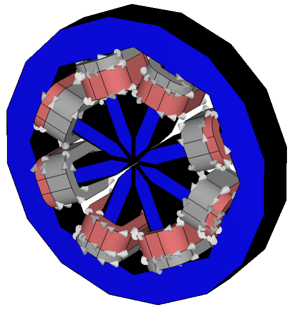

Figure 4: Corrector magnet model in CST Studio Suite® [5].

Eddy Current Losses for Different Lamination Thicknesses

The lamination thickness is an important design parameter. Generally, thinner laminations lead to less eddy current losses and thus less generated heat that must be cooled away. On the other hand, magnets with very thin laminations are difficult to manufacture as the individual laminations are very brittle. Hence, an investigation of the dependence of the eddy current losses on the lamination thickness is necessary to make a reasonable design choice.

We compute the eddy current losses in the yoke over the frequency range of interest up to 65 kHz for lamination thicknesses between 0.2 mm and 0.5 mm. The results are shown in Table 2 for some selected frequencies. We note that the simulation assumes a constant current at all frequencies, while in reality, the power supply will not be able to keep the current constant as the frequency is raised.

We observe that at low frequencies, the eddy current losses depend much more on the lamination thickness than at high frequencies. According to [12], this is due to the fact that at low frequencies, the eddy currents are restricted by a lack of space or high resistivity, while at high frequencies, they are mostly limited by the effect of their own field, i.e., they are mainly flowing in a thin layer close to the surface, such that the lamination thickness is less important.

Table 2: Losses for Different Lamination Thicknesses

f	**Eddy Current Losses (W)**			
	$d = 0.2$ mm	$d = 0.3$ mm	$d = 0.4$ mm	$d = 0.5$ mm
10 Hz	$5.8 \cdot 10^{-1}$	$6.5 \cdot 10^{-1}$	$7.6 \cdot 10^{-1}$	$9.0 \cdot 10^{-1}$
1 kHz	$1.4 \cdot 10^{3}$	$2.1 \cdot 10^{3}$	$3.1 \cdot 10^{3}$	$4.0 \cdot 10^{3}$
10 kHz	$4.4 \cdot 10^{4}$	$4.9 \cdot 10^{4}$	$5.5 \cdot 10^{4}$	$5.8 \cdot 10^{4}$
65 kHz	$3.5 \cdot 10^{5}$	$3.6 \cdot 10^{5}$	$3.6 \cdot 10^{5}$	$3.5 \cdot 10^{5}$

Multipole Coefficients Along the Axis

To understand the magnet's influence on the beam dynamics, it is important to analyze the multipole coefficients along the longitudinal axis and their integrated values. Figure 5 shows the dipole coefficients along the axis for some frequencies and Table 3 shows the integrated dipoles, sextupoles, decapoles, 14-poles, and 18-poles. The $4n$-poles (quadrupoles, octupoles, etc.) are zero due to symmetry. On the one hand, we observe the dipoles, 14-poles, and 18-poles decreasing by 50-60% over the frequency range up to 65 kHz. This can be attributed to the attenuation of the main field due to the eddy currents in the yoke according to Lenz's law. On the other hand, the sextupoles are increasing by a factor of 6 and the decapoles by a factor of 4.7. This can mostly be attributed to strongly increased values of these coefficients in the magnet's end regions.

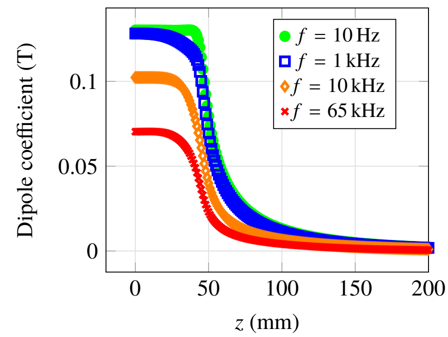

Figure 5: Dipole coefficients along the longitudinal axis.

Table 3: Integrated Multipole Coefficients

f	**Integrated Multipole Coefficients** (T m)				
	Dipole	Sextupole	Decapole	14-pole	18-pole
10 Hz	$1.72 \cdot 10^{-2}$	$-9.02 \cdot 10^{-6}$	$-3.87 \cdot 10^{-6}$	$4.71 \cdot 10^{-4}$	$-4.52 \cdot 10^{-5}$
1 kHz	$1.59 \cdot 10^{-2}$	$-1.99 \cdot 10^{-5}$	$-7.15 \cdot 10^{-6}$	$4.48 \cdot 10^{-4}$	$-4.26 \cdot 10^{-5}$
10 kHz	$1.14 \cdot 10^{-2}$	$-4.52 \cdot 10^{-5}$	$-1.52 \cdot 10^{-5}$	$3.41 \cdot 10^{-4}$	$-3.20 \cdot 10^{-5}$
65 kHz	$7.39 \cdot 10^{-3}$	$-5.38 \cdot 10^{-5}$	$-1.80 \cdot 10^{-5}$	$2.24 \cdot 10^{-4}$	$-2.08 \cdot 10^{-5}$

CONCLUSION

In this paper, we have addressed the issue of simulating fast corrector magnets with laminated yokes over a broad frequency range. We have selected a homogenization technique and, using a toy model, we have shown that it can be used to approximate the eddy current losses and multipole coefficients without resolving the individual laminates in the FE mesh. Then, we have applied this technique to a model of a corrector magnet for PETRA IV which has enabled us to study the eddy current losses for different lamination thicknesses and the multipole coefficients along the axis. With respect to the eddy current losses, we have found a strong dependence on the lamination thickness at lower frequencies. For higher frequencies, the lamination thickness does not make a difference. Regarding the multipoles, we have found that with rising frequency, the dipole-, 14-pole, and 18-pole coefficients decrease, while sextupoles and decapoles increase strongly.

ACKNOWLEDGEMENTS

We acknowledge the support from Deutsches Elektronen-Synchrotron DESY and the Deutsche Forschungsgemeinschaft (DFG) - GRK 2128 "AccelencE". Furthermore, we thank Dassault Systèmes for providing the CST Studio Suite® license.

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