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GENERALIZED GRADIENT MAP TRACKING IN THE SIBERIAN SNAKES OF THE AGS AND RHIC*

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Abstract

Accurate and efficient particle tracking through Siberian snakes is crucial to building comprehensive accelerator simulation models for the Alternating Gradient Synchrotron (AGS) and Relativistic Heavy Ion Collider (RHIC) at Brookhaven. In this work, we apply the Generalized Gradient (GG) formalism developed by Venturini and Dragt to model the complex fields of Siberian snakes. The GG formalism has recently been implemented in the Bmad and PTC toolkits for charged particle simulations. The implementation allows for generation of truncated power series maps. We present simulation results of the Siberian snakes in both the AGS and RHIC demonstrating that the GG formalism can provide fast and accurate particle tracking.

INTRODUCTION

High polarization of proton beams is important for operations at the Brookhaven National Laboratory complex, which includes the Relativistic Heavy Ion Collider (RHIC) and its injector, the Alternating Gradient Synchrotron (AGS). However, proton polarization is easily lost due to depolarizing spin resonances. During acceleration, depolarization happens when a particle encounters spin-perturbing magnet fields at the same frequency as its spin vector precession frequency. When such resonance occurs, the spin tune ν_{sp} satisfies [1]:

$$\nu_{\mathrm{sp}} = G\gamma = m + m_x Q_x + m_y Q_y + m_s Q_s \tag{1}$$

where m and $m_{x,y,s}$ are integers, $Q_{x,y,s}$ are orbital tunes, γ is the Lorentz factor, and $G = 1.7928$ is the anomalous magnetic moment of the proton.

One of the most efficient way to suppress depolarizing resonances is using Siberian snakes. A Siberian snake rotates the particle spin about an axis in the horizontal plane, without affecting orbital motion. Snakes installed in the AGS [2] and RHIC [3] are helically twisted dipole magnets, which minimize orbit distortion within the snakes. The spin tune of a ring with a Siberian snake of strength s is given by:

$$\cos \pi\nu_{\mathrm{sp}} = \cos\frac{s\pi}{2}\cos\pi G\gamma \tag{2}$$

where $s = 1$ indicates a full Siberian snake, which rotates the spin by 180°. When $s < 1$, the device is called a partial snake, and is referred to as a percentage of the full snake.

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For RHIC and the AGS, Siberian snakes are traditionally modeled in MAD [4] by matrices generated for specific current and energy configurations. This method falls short with simulations of energy ramping due to the nonphysical jumps between matrices. Another common method is to use grid field tables for the snake fields, but field table files are for snakes are typically very large and thus cumbersome to use and slow to track through. In this work, we present simulation results of tracking particles through the complex fields of Siberian snakes using the Generalized Gradient (GG) DC field description, developed by Venturini and Dragt [5]. This formalism has recently has been implemented [6] in the Bmad [7] and PTC [8] toolkits for charged particle and X-ray simulations in accelerators and storage rings. The GG formalism provides an analytic way to define the magnetic field for which Differential Algebra can be used to find accurate truncated power series maps that can be used for analysis and fast tracking. We show that the GG implementation in Bmad/PTC allows fast and accurate tracking through Siberian snakes in the AGS and RHIC. Interpolation between points z_i is accurately done using a polynomial of order $2N + 1$.

GENERALIZED GRADIENTS

The GG expansion starts with scalar potential ψ of the magnetic field satisfying $\vec{B} = -\vec{\nabla}\psi$. In cylindrical coordinates (ρ, ϕ, z) ψ can be written in the form

$$\psi = \sum_{m=0}^{\infty} \psi_{m,c}(\rho, z)\cos(m\phi) + \psi_{m,s}(\rho, z)\sin(m\phi) \tag{3}$$

The functions $\psi_{m,\alpha}$ ($\alpha = c$ or s) can be expressed as a Taylor series in ρ

$$\psi_{m,\alpha} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} m!}{4^n n!(n+m)!}\rho^{2n+m} C_{m,\alpha}^{[2n]}(z) \tag{4}$$

where $C_{m,\alpha}$ are the generalized gradients and the superscript $[2n]$ indicates the $2n^{th}$ derivative.

In a practical application, a finite set of $C_{m,\alpha}$ are chosen to represent the field and the Taylor series for each $C_{m,\alpha}$ used is truncated at some order N (which does not have to be the same for all functions). At a set of points z_i, the $C_{m,\alpha}$ are characterized by the function value and all derivatives up to order N. Sagan et al. [6] discuss the implementation of GGs into Bmad and PTC. The end result is a table of generalized gradients and derivatives which can be used to calculate the field at any point.

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SIBERIAN SNAKES IN THE AGS

There are two partial Siberian snakes 120 degrees apart in the AGS ring, as shown in Fig. 1. For a medium energy accelerator like the AGS (with energy range of 5 to 25 GeV), partial snakes are more practical than full snakes due to their smaller orbit disturbances and shorter straight section requirement. The 5.9% (rotation angle of 10.6°) normal conducting (warm) snake was installed in 2004, and the super-conducting (cold) snake, capable of a strength of up to 22%, was installed in 2006. With the two snakes, 65% polarization was achieved in the AGS for acceleration of 1.5×10^{11} protons/bunch to 24 GeV in 2007 [2].

Traditionally, the AGS partial snakes are modeled in MAD [9] with matrices generated at a prefixed energy configuration that is closest to the operation energy. This method is fast at constant energies but it produces nonphysical jumps during ramping because there is no interpolation between prefixed matrices. It also does not simulate the orbit distortion and spin rotation within the snakes at all. Another traditional method to simulate snakes is to use grid field tables that specify the field strengths at grid locations within the snakes and track particles in Zgoubi [10]. But such field map files are very large (at least 40 megabyte per file) and tracking through them is generally very slow. Additionally, interpolation errors with grid fields will cause tracking errors.

Using the generalized gradient fitting algorithm in Bmad and PTC [6], we generate generalized gradient maps for the two partial snakes from their grid field tables. The GG expansion is calculated with Eq. 3 and 4. The m values we picked for both cos and sin terms in Eq. 3 are 1, 3, 5, 7. The Taylor series for each $C_{m,\alpha}$ is truncated at order $N = 5$.

Figure 2 shows the comparison of the warm snake's magnetic field values reconstructed from GG fitting algorithm (left) and the original grid field tables (right). Figure 3 shows the comparison of particle tracking results within the warm snake using GG field map (left) and grid field tables (right). We can see that GG field map is able to give accurate tracking results. Similarly satisfactory results are also obtained for the cold snake.

To check whether spin rotation angles are also accurate for the two snakes using GG maps, we track three particles

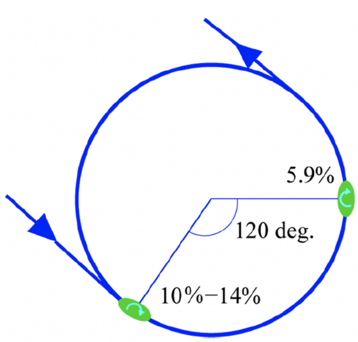

Figure 1: Locations of two partial snakes in the AGS ring.

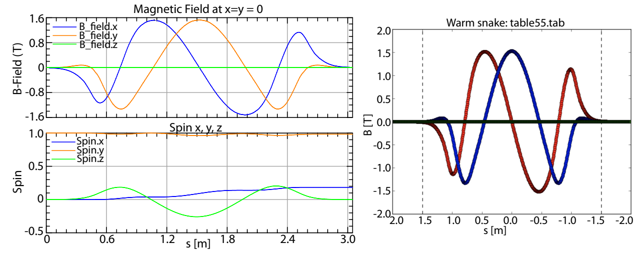

Figure 2: Magnetic field within AGS warm snake with GG map (left) and with grid table (right). Left plot also shows spin tracking result with GG map with initial spin (0,1,0).

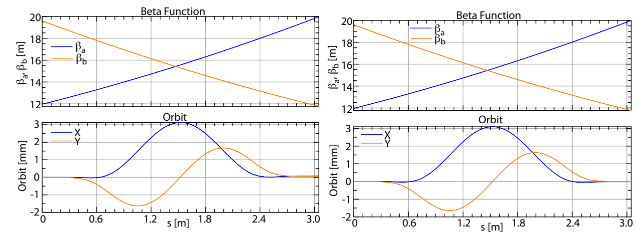

Figure 3: Beta function and orbit within AGS warm snake with GG map (left) and with grid table (right).

with three initial spin configurations: (1,0,0), (0,1,0), and (0,0,1). The final spin after the snake is given by:

$$\vec{s}_1 = \underline{R}\,\vec{s}_0 \tag{5}$$

where $\underline{R}$ is the 3-D rotation matrix :

$$\underline{R} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

So by observing the three final spin vectors, we can solve for rotation angle θ using Eq. 5. The snake strength is then calculated as the ratio $\theta/180°$ in percentage form. Table 1 summarizes the calculation results for GG maps and grid tables, and we can see that GG maps can reconstruct the two AGS partial snakes with correct strengths.

Table 1: AGS Snake Strength Calculation Results

Snake	GG map	grid table
warm	5.9%	5.86%
cold	11.4%	11.4%

In terms of the time to track a particle, three different tracking methods were tested: Tracking using a 3^{rd} order Taylor map derived from the GG field description, Tracking using the grid table and a 4^{th} order Runge-Kutta integrator, and finally tracking using a matrix. The results are summarized in Table 2. Matrix tracking is the fastest but, as mentioned before, the matrix does not include spin rotations, so it is not useful for our purpose. Tracking using the Taylor map derived from the GGs is around 1000 times faster than Runge-Kutta tracking with the grid table. This shows that using GGs is both accurate and faster.

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Table 2: AGS Snakes Tracking Times (sec)

Snake	GG map	grid table	Matrix
warm	1.7×10^{-5}	1.63×10^{-2}	2.7×10^{-6}
cold	1.98×10^{-5}	2.42×10^{-2}	3.2×10^{-6}

SIBERIAN SNAKES IN RHIC

In the range of beam energies accessible by RHIC, 25 - 255 GeV , the largest spin-orbit resonances are almost 100 times stronger than the AGS. To compensate this, in each RHIC ring there are two full Siberian snakes on opposite sides, making a total of four in RHIC [3], as shown in Fig. 4. RHIC itself is 4km in length. This large ring size slows down any tracking simulation. Each snake field map ranges from 150 MB to 600+ MB depending on sparsity, making lattice initialization and/or tracking computationally expensive, so it is extremely beneficial to have a compact analytic representation of a magnetic field. A set of generalized gradient coefficients representing one snake is as small as 2 MB, and initializes Twiss parameter computation of the lattice in less than 5 seconds, compared with 4-5 minutes with the field map.

In the absence of snakes, ramping beam energy across linear spin-orbit resonances conditions generates coherent depolarizing precession of beam spins away from their stable orientation, the invariant spin field, which defines their equilibrium polarization at each fixed energy. In the single-resonance approximation, tracking at a fixed energy nearby a resonance condition indicates the presence of the nearby resonance with an oscillating vertical component of any spin around some average value (the invariant spin field at that orbital position). When a particle's energy satisfies the linear resonance condition exactly without energy oscillations, at the center of the RF bucket, the linear resonance is seen as a rotation of spin around a horizontal axis.

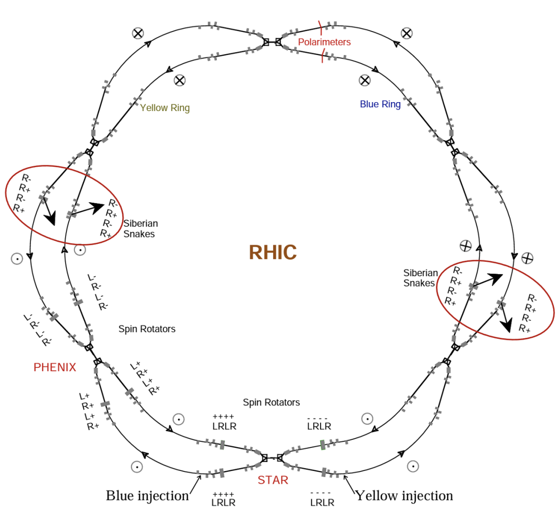

Figure 4: Locations of four full snakes in RHIC.

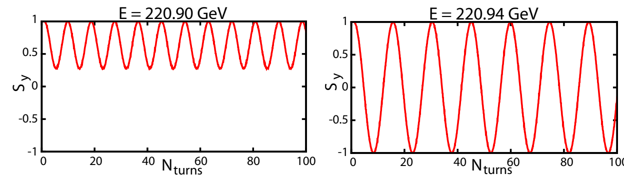

Figure 5: Spin tracking without snakes near resonance (left) and on resonance (right). Vertical betatron emittance is 0.5nm.

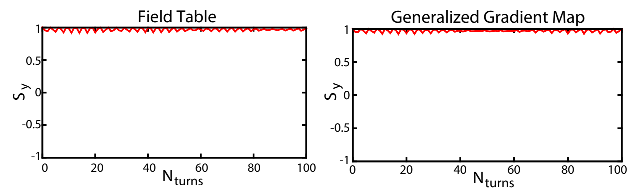

Figure 6: Spin tracking on resonance with snakes using grid field (left) and generalized gradient field (right). Vertical betatron emittance is 0.5nm.

Siberian snakes can coherently cancel linear depolarizing perturbations with each turn, allowing the invariant spin field to remain close to vertical at imperfection and linear intrinsic resonances. While this makes apparent the presence of higher-order resonances, it successfully avoids all linear resonances in the RHIC energy range.

In Fig. 5, we track a single particle of low emittance nearby and on the strongest resonance in RHIC in the absence of snakes. It is seen that the vertical spin projection oscillates with a fixed frequency in each case, corresponding to the resonance strength when on-resonance.

We utilize generalized gradient snake tracking at resonance energy (220.94 GeV) to present their success at canceling resonant oscillations. This is compared with the same tracking performed with the original grid field map, showing resounding similarity in Fig. 6 and successfully performing its intended effects.

CONCLUSION

In this work, we use the generalized gradient formalism to simulate Siberian snakes of the AGS and RHIC in Bmad. We show tracking results that demonstrate GG maps are accurate and faster than traditional grid tables.

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