AN ELECTRON MODEL OF VERTICAL FFA ACCELERATOR FOR HARMONYTRON

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Abstract

A new type of accelerator called Harmonytron has been proposed. The Harmonytron is based on a scheme of vertical Fixed-Field Alternating gradient (vFFA) focusing with harmonic number jump beam acceleration. An electron model of vFFA accelerator is under development at Kyushu University. The current status of the vFFA accelerator will be discussed.

INTRODUCTION

A new scheme of the high intensity hadron accelerator called Harmonytron [1] with harmonic number jump acceleration [2] and vertical scaling (zero chromatic) beam optics [3] has been proposed.

In the vFFA accelerator, the strength of the magnetic field changes exponentially in a vertical direction. Beam is accelerated and shifts in the vertical direction. As for the orbit radius, it is constant with all momentums. These characteristics represent that there is no transition energy.

The magnetic field for the vFFA includes a skew component, unlike an ordinary accelerator. Since the particle motion causes coupled horizontal and vertical motions, the particle motion in the vFFA becomes more complicated than in an ordinary accelerator and it makes design difficult. The vFFA has not been built so far.

At Kyushu University, a vFFA electron model is under construction aiming to determine an optics design scheme and magnetic field design procedure for the vFFA electron model. Here a new evaluation scheme in optics design using 3-D magnetic field analysis software code has been proposed. The vFFA electron model is under construction.

OPTICS DESIGN USING LINEAR APPROXIMATION

The optics design for the vFFA electron model is based on the methods used for a typical synchrotron. Transfer matrix of coupled motion is used to solved the equation of motion. In the vFFA, the vertical magnetic field distribution to satisfy the scaling law is expressed in the following equation [5].

$$B_y = B_0 \exp(my), \tag{1}$$

where B_0 is the magnetic flux density at $y = 0$ and m is a measure of the magnetic field gradient. The equations of motion for vFFA derived from Eq. (1) are shown in Eq. (2).

$$\begin{aligned} x'' + \frac{x}{\rho(s)^2} + \frac{1}{B\rho(s)}\left(\frac{\partial B_y}{\partial y}\right)y &= 0, \\ y'' - \frac{1}{B\rho(s)}\left(\frac{\partial B_x}{\partial x}\right)x &= 0. \end{aligned} \tag{2}$$

The particle motion in the vFFA is coupled horizontally and vertically. Therefore, the transfer matrix is approximately derived by solving two equations based on the normal dipole and the skew quadrupole components, respectively as shown in Eq. (3) and Eq. (4).

$$\begin{aligned} x'' + \frac{x}{\rho(x)} &= 0, \\ y'' &= 0. \end{aligned} \tag{3}$$

$$\begin{aligned} x'' + \frac{1}{B\rho(s)}\left(\frac{\partial B_y}{\partial y}\right)y &= 0, \\ y'' - \frac{1}{B\rho(s)}\left(\frac{\partial B_x}{\partial x}\right)x &= 0. \end{aligned} \tag{4}$$

Eq. (3) shows the equation of motion with only the normal dipole magnetic field and Eq. (4) the equation of motion with the skew quadrupole. The stability of the particle motion is verified by the eigenvalue of the transfer matrix.

VFFA ELECTRON MODEL

Optics

The focusing system of the vFFA electron model is based on a FD singlet sector. Sector type of magnet has some advantages. It has zero edge angle which can eliminate the vertical kick because the magnetic field component is only vertical on the design orbit and the design orbit can be closed.

The stability region of the vFFA electron model calculated in linear approxomation is shown in Fig. 1. In the Fig. 1, the horizontal axis shows the m value and the vertical axis shows a ratio of magnetic field strength between the F and D magnets.

Magnet

An electromagnet for the vFFA electron model is a multi-coil type, as shown in Fig. 2. The gray dashed line shows a superposition of the vertical fields generated by each coil. A multi-coil type magnet has two advantages. A simple pole shape makes easy adjust the field gradient of m value. The procedure has been verified in previous studies [6].

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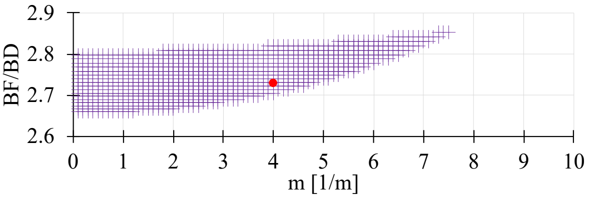


Figure 1: Stability region in the FD singlet lattice.

The each coil current is optimized by an iteration procedure based on the following equation.

$$I_n = I_0 \exp(m y_n). \tag{5}$$

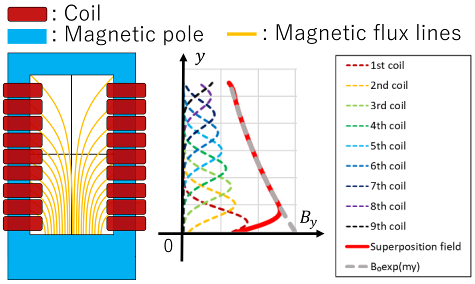


Figure 2: Schematic diagram of a multi-coil type magnet (9 coils).

The optics parameters chosen in the calculation for the vFFA electron model are shown in Table 1. The multi-coil type magnet has 18 coils and the vertical aperture is 0.24 m.

Table 1: Ring Parameters of the vFFA Electron Model

Lattice	FD singlet
Magnet type	Sector
Ring radius	1.0 m
Energy	20.0 to 40.0 keV
m value	$-4.0\,\mathrm{m}^{-1}$
Excursion	0.1 m
B_0(F)	60.8 Gauss
B_0(D)	22.2 Gauss

MAGNETIC FIELD DESIGN

In the vFFA, the particle motion is complicated by the presence of skewed magnetic field components, nonlinear magnetic fields, and leakage fields. These must be taken into account in the transverse beam optics design method. Therefore, a new design index "effective magnetic field gradient (the effective *m*)" is defined.

The effective *m* is obtained by using the BL integral, which is the integral of the vertical magnetic field on a closed orbit for each energy. The BL integral distinguishes when the sign of the magnetic field *By* is positive as defocused and when it is negative as focused. The effective *m* takes into account the complicated particle motion due to skew and nonlinear magnetic field components. It can also take into account changes in the closed orbit due to changes in optical conditions.

In the vFFA electron model, the effective *m* is optimized by adjusting the current value of the multi-coil type magnets. The following procedure was used to iterate the current value. The current values are optimized using the least-squares method.

1. Determine the initial current value from Eq. (5).
2. Create a magnetic field for tracking simulation.
3. Derive a closed orbit. (Using Runge-Kutta 4th).
4. Calculation of BL integral on the closed orbit.
5. Calculation of the effective *m*.
6. Update current values using the least-squares method.

The magnetic field obtained in "2." is calculated using OPERA-3D [7] for each coil, the current value is the initial value from Eq. (5) .The magnetic fields of each coil with adjusted current values are superimposed to reproduce the magnetic field if all coils were adjusted. This method has the advantage of not requiring the additional OPERA-3D calculations within the iteration loop.

The closed orbit displacement and the effective *m* before and after the current value adjustment are shown in Fig. 3. Before adjustment, the effective *m* was not constant with respect to the ring parameter, and the closed orbit displacement was different from the ideal displacement. When the effective *m* was adjusted to be constant with respect to the ring parameter, the closed orbit displacement almost matched the ideal one. Fig. 4 shows the closed orbit at each energy. The orbit position remains almost constant in the radial (*r*) direction and changes only in the vertical (*y*) direction during acceleration. The vertical height of the closed orbit decreases with increasing energy. This is due to the negative sign of the *m* value.

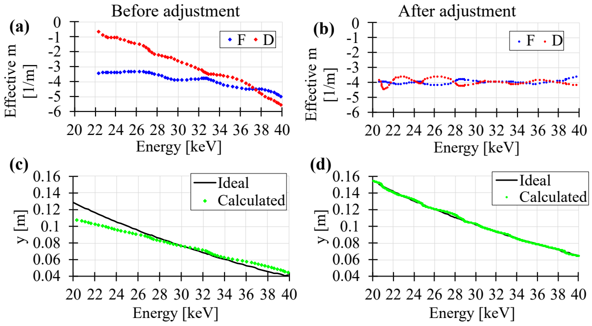


Figure 3: The closed orbit displacement and the effective *m*. (a) and (b) are the effective *m*, (c) and (d) are the closed orbit displacements before and after adjustment.

ACCELERATION

Longitudinal tracking simulation has been performed. Two RF cavities are used for COD suppression. The two RF cavities are 180 degrees apart. The acceleration method is

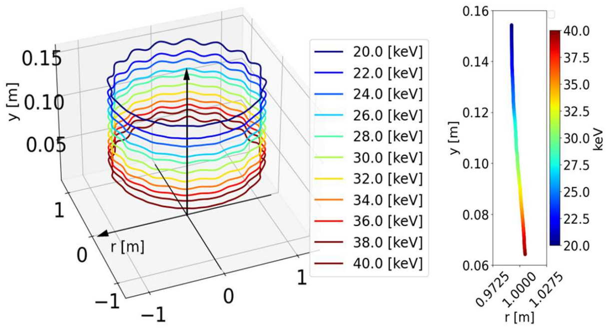
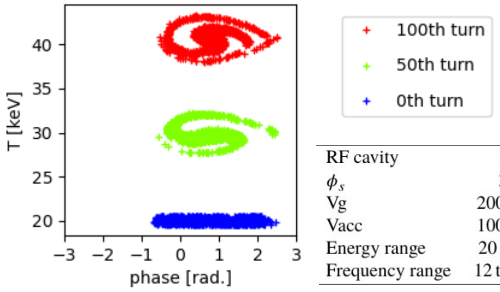


Figure 4: Closed orbits of the vFFA electron model at various energies.

the same as that of a normal synchrotron. The results are shown in Fig. 5.

In the vFFA electron model, beams can be accelerated stably using an ordinary rf acceleration.



RF cavity	2 units
ϕ_s	30 deg
Vg	200 V/cavity
Vacc	100 V/cavity
Energy range	20 to 40 keV
Frequency range	12 to 17 MHz

Figure 5: The beam tracking simulation results for the longitudinal beam motion from 0 to 100 turns (20 to 40 keV for synchronous particle).

EXPERIMENT PREPARATION

Photograph of the experimental setup under construction is shown in Fig. 6. Six magnets (3-cell) has been installed. A magnetic field measurement device was also installed next to it. Plans are to measure the magnetic field between the central cell (F and D magnets). One more cell will be added and a 4-cell beam transport line will be constructed to perform beam transport experiments in near future.

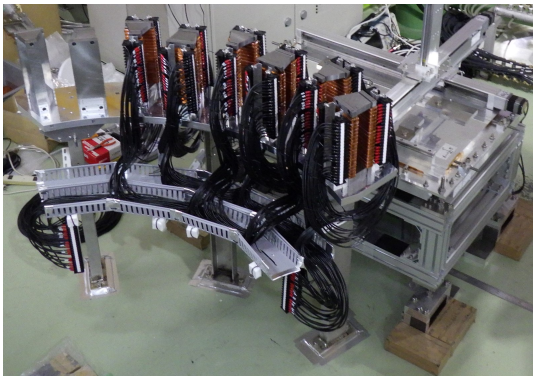


Figure 6: Experimental setup (Under construction).

CONCLUSION

Aiming at the proof-of-principle experiment of the vFFA accelerator for the realization of the Harmonytron, optics design and investigation of beam dynamics for the vFFA electron model were carried out.

In optics design, a scheme has been proposed to derive the approximated transfer matrix by separating the equation of motion including couplings. Stability regions of optical parameters were obtained for the vFFA electron model.

An optimizing procedure using the effective magnetic field gradient was proposed for the magnetic field design of the multi-coil type magnet. The closed orbit having an ideal displacement ranging from 20 to 40 keV was obtained. In addition, it is shown to accelerate the electrons stably using an ordinary rf acceleration.

ACKNOWLEDGMENT

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