# AGS BOOSTER BEAM-BASED MAIN QUADRUPOLE TRANSFER FUNCTION MEASUREMENTS* 

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## Abstract

Physics models, particularly for online operations, such as MAD-X or Bmad models, depend on a good understanding of the magnet characteristics. Even thought detailed magnet measurements are used to verify that the magnets meet specifications before being installed, some magnet characteristics are often not fully understood during operations. Beam-based measurements can then be used to determine magnet properties. In this work, we present a new method for determining magnet properties using orbit response matrix (ORM) measurements. This new approach utilizes a neural network (NN) surrogate model to establish the mapping between ORM measurements and quadrupole strengths. The NN model is trained to identify quadrupole strength errors by observing the difference between measured ORM and model ORM. The evaluation of this NN during accelerator operation leads to a polynomial fit of the quadrupole strengths as a function of power supply settings. We will present results from preliminary beam studies in the AGS Booster.

## INTRODUCTION

The Alternating Gradient Synchrotron (AGS) Booster is used to increase beam intensity in the AGS by preaccelerating particles before they enter the AGS. It accepts heavy ions from EBIS or protons from the 200 MeV Linac. The Booster also serves as heavy ion source for the NASA Space Radiation Laboratory [1]. Accurate control of beam properties such as orbit and tune is indispensable to providing high quality beam to both the AGS, which serves as the injector for Relativistic Heavy Ion Collider (RHIC) and the future Electron Ion Collider (EIC), and the NSRL beamline for NASA.

In order to have better control of the beam, a good understanding of the magnet properties is necessary. One essential property is called the transfer function, which describes how the magnet field responds to change in power supply (PS) current. Traditionally, transfer functions are determined during magnet production measurements before installation, but how accurate they remain after the magnets are installed is unclear. In this work, we present a method which combines orbit response matrix (ORM) and machine learning (ML)

[^0]to determine magnet characteristics in the Booster. By establishing a surrogate model between change in ORM and quadrupole kick values, we can use polynomial fitting to obtain the relationship between quadrupole power supply current values, which are set by operators, and the actual kick values observed by the beam. The full algorithm is summarized in Fig. 1.


Figure 1: Conceptual representation of the magnet characteristic measurement method.

## QUADRUPOLE TRANSFER FUNCTION

There are 48 quadrupoles ( 24 horizontal, 24 vertical) in the Booster. The defocusing quadrupoles are slightly longer than focusing quadrupoles because quadrupoles and main bending dipoles are powered in series. Since dipoles have an intrinsic focusing component, making the defocusing quadrupoles slightly longer brought the vertical tune up closer to the horizontal tune. Most quadrupoles have round vacuum chamber, but two vertical quadrupoles (DQ5, FQ5) have special "eared" chambers.

The optical model of Booster is traditionally built with MAD-X [2]. The current quadrupole transfer functions used in the model are defined to match the two sets of tune measurement data taken in 1992 and 1993 [3].

The power supply current $I_{q}$ for a main quadrupole is a combination of main dipole current $I_{\text {dipole }}$, tune trim coils $I_{\text {trim }}$, and stop band corrector current $I_{\text {str }}$. Tune trim coils can shift vertical tune up to compensate for space charge tune shifts and avoid strong integer stop band at $v_{y}=4$. Stop band correctors are used to correct for all significant resonances between $v_{y}=4$ and $v_{y}=5$. An extra calibration term is also added in the model to compensate for observed $\dot{B}=\frac{\partial B}{\partial t}$ effects, such as eddy current in quadrupole vacuum chambers. Therefore, the Booster main quadrupole current
is given by:

$$
\begin{equation*}
I_{q}=I_{\text {dipole }}+0.2 \cdot\left(I_{\text {trim }}+\dot{B} \cdot C\right)+0.4 \cdot I_{s t r} \tag{1}
\end{equation*}
$$

where 0.2 and 0.4 are the turn ratios of trim and stop band coils to main dipole coils, and $C$ is the $\dot{B}$ calibration coefficient. According to measured data, the calibration coefficients are determined to be $C_{H}=3.4$ and $C_{V}=4.8 \mathrm{amp} / \mathrm{T} / \mathrm{sec}[3]$.

The current model uses a fifth order polynomial to model the gradient of a quadrupole based on its current:

$$
\begin{equation*}
\frac{\partial B}{\partial r}=a_{0}+a_{1} \cdot I_{q}+a_{2} \cdot I_{q}^{2}+a_{3} \cdot I_{q}^{3}+a_{4} \cdot I_{q}^{4}+a_{5} \cdot I_{q}^{5} \tag{2}
\end{equation*}
$$

The normalized gradient equations for the quadrupoles (short, long, eared vacuum chamber) are:

$$
\begin{align*}
K_{1, \text { short }} & =\left(1-0.00004179 \cdot \frac{\dot{B}}{B}\right) \cdot \frac{1}{B \rho L}\left\langle\frac{\partial B}{\partial r}\right\rangle  \tag{3}\\
K_{1, \text { long }} & =\left(1-0.000041942 \cdot \frac{\dot{B}}{B}\right) \cdot \frac{1.003}{B \rho L}\left\langle\frac{\partial B}{\partial r}\right\rangle  \tag{4}\\
K_{1, \text { ear }} & =\left(1-0.000062913 \cdot \frac{\dot{B}}{B}\right) \cdot \frac{1.003}{B \rho L}\left\langle\frac{\partial B}{\partial r}\right\rangle . \tag{5}
\end{align*}
$$

Assuming we can take measurements at a flat porch during the Booster cycle, where $B$ stays constant, the normalized gradient equations we need to consider become:

$$
\begin{align*}
& K_{1, H}=\frac{1}{B \rho L}\left\langle\frac{\partial B}{\partial x}\right\rangle  \tag{6}\\
& K_{1, V}=\frac{1.003}{B \rho L}\left\langle\frac{\partial B}{\partial y}\right\rangle \tag{7}
\end{align*}
$$

where the main Booster quadrupole strengths are determined by two fifth-order polynomials $\frac{\partial B}{\partial x}$ and $\frac{\partial B}{\partial y}$ in the format of Eq. 2. The two sets of polynomial coefficients used in the current Booster model, summarized in Table 1, are derived from a least square linear regression fitting to the measured magnetic data.

Table 1: Booster Quadrupole Transfer Function Coefficients

| Coefficient | H Quad K1 | V Quad K1 |
| :---: | :---: | :---: |
| $a_{0}$ | 0.001818 | 0.002099 |
| $a_{1}$ | $9.080 \times 10^{-4}$ | $9.257 \times 10^{-4}$ |
| $a_{2}$ | $6.657 \times 10^{-9}$ | $1.164 \times 10^{-8}$ |
| $a_{3}$ | $7.225 \times 10^{-12}$ | $1.046 \times 10^{-11}$ |
| $a_{4}$ | $3.239 \times 10^{-15}$ | $4.057 \times 10^{-15}$ |
| $a_{5}$ | $5.07 \times 10^{-19}$ | $5.75 \times 10^{-19}$ |

## ORM MEASUREMENT ROUTINE

The orbit response matrix (ORM) quantifies the mapping between orbit measurements $(\vec{x}, \vec{y})$ and corrector settings
$\left(\vec{\theta}_{x}, \vec{\theta}_{y}\right)$ via [4]:

$$
\begin{equation*}
\binom{\Delta \vec{x}}{\Delta \vec{y}}=\underline{R}\binom{\Delta \vec{\theta}_{x}}{\Delta \vec{\theta}_{y}} . \tag{8}
\end{equation*}
$$

Since the Booster orbit responds linearly to change in corrector settings, each element of $\underline{R}$ can be calculated using:

$$
\begin{equation*}
R_{i j}=\frac{\Delta\left(x_{i} \text { or } y_{i}\right)}{\Delta \theta_{j}} \tag{9}
\end{equation*}
$$

## Simulation with Bmad and Pytao

The Booster ring has a total of 48 BPMs and 48 correctors, so a double-plane $\underline{R}$ has a dimension of $(48,48)$. We rebuilt the Booster optical model using Bmad [5] in order to develop a streamlined process to get simulated ORMs. In MAD-X, getting multiple ORMs requires running multiple simulations and dealing with separate output files. Bmad's simulation program Tao has a python interface PyTao, so we can run accelerator simulations in combination with Python functions. Therefore, we developed a Python routine to get simulated ORMs with different quadrupole settings. ORMs are obtained by changing each corrector successively, combining all the output orbits into an array, and calculating each matrix element using Eq. (9).

## Real Machine

In the real Booster machine, the Collider Accelerator Department (CAD) Controls Group uses various software tools to control and monitor accelerator elements. The correctors are managed by FunctionEditor, which allows users to upload a time dependent current function to the power supply of the magnets. In order to set correctors to a constant kick, we define a trapezoid-shaped function whose flat top value is the desired current value in FunctionEditor, and send it to the machine (make live). We developed a script [6] that sets three current functions for each corrector: zero kick (baseline value), positive kick, and negative kick. After setting the corrector, live BPM data is saved and the ORM values is calculated by finding the slope of orbit data. The script work flow is outlined in Fig. 2.

When processing data obtained from the ORM scipt, we discover there are 1 missing corrector and 11 bad BPMs (gets NaN values) in the real machine. As a result, the actual ORM has a dimension of $(37,47)$. The raw matrix values has unit of $\mathrm{mm} /$ Amps, and we convert them to $\mathrm{m} /$ radian to match the values taken from simulation. We take data at 1.82 GeV , which means 1 Amp in a corrector is $1.877 \times 10^{-5}$ radian kick. Figure 3 shows an example of an ORM taken from the real machine after unit conversion.

## QUADRUPOLE STRENGTH MODEL

We started with building a NN model for only the vertical quadrupoles. Since all quadrupoles in the same plane are wired in series, this means the NN model only has one output, which is the k 1 value for 24 vertical quadrupoles. The reference ORM $\underline{R}^{\text {mod }}$ is taken in simulation with all


Figure 2: Work flow of ORM measurement script.


Figure 3: Real ORM obtained using measurement script.
quadrupoles set to zero. Only vertical ORM is used for training this model because there is no coupling between the horizontal and vertical plane.

When the quadrupoles have a non-zero kick, the measured ORM differs from the reference ORM by:

$$
\begin{equation*}
\Delta \underline{R}=\underline{R}^{\text {meas }}-\underline{R}^{\text {mod }} \tag{10}
\end{equation*}
$$

A single-plane simulated ORM of the Booster has a dimension of $(24,24)$, so the flattened $\Delta \underline{R}$ has a dimension of $(24 \times 24,1)=(576,1)$. The NN model we train takes the flattened $\Delta \underline{R}$ as input, and predicts the vertical quadrupole strength that caused the difference in ORM values.

In order to better represent ORM data from real machine, we added a Gaussian noise with unit width and an amplitude of 80 micron to the simulated orbit data during ORM calculation. The amplitude is inferred from the amplitude of noise in the off-diagonal blocks of the real ORM. Since there is no coupling, all the fluctuations in the off-diagonal blocks come from noise.

The NN model we built is a fully connected feed-forward neural networks (FFNN) [7] with three layers and Exponential Linear Unit activation function [8]. The model is trained on 800 sets of simulated data, and reaches $99.5 \%$
accuracy on 200 sets of simulated test data. In order to get the quadrupole transfer function, we then increase the PS current of vertical quadrupoles incrementally from 0 to 200 Amps, and use the trained NN model to predict the actual quadrupole strength k1. Figure 4 shows the comparison between k 1 values calculated from the traditional formula Eq. (7) and k 1 values predicted by NN model given ORM measurements.


Figure 4: NN prediction of the mapping between vertical quadrupole strength and power supply current.

Once we have predictions for k1 values, we can calculated the gradient by reversing Eq. (7):

$$
\begin{equation*}
B_{1, V}=\left\langle\frac{\partial B}{\partial y}\right\rangle=\frac{B \rho L}{1.003} \cdot K_{1, V} \tag{11}
\end{equation*}
$$

To match the traditional formula (Eq. (2)), we do a 5th degree polynomial fit on gradients and PS currents. There are several polynomial fit packages readily available in Python, we found the best results come from scipy.optimize.curve_fit, which allows the user to define the value ranges on all the fit coefficients. numpy.polyfit works fine within the range of given data, but performs poorly on data outside the given fitting range. Figure 5 shows the fitting results for the vertical quadrupole transfer function.


Figure 5: Polynomial fit results for vertical quadrupole transfer function.

## CONCLUSION

In this work, we developed and tested a beam-based method to determine magnet properties using ORM measurements and NN models. Operational script is developed to take ORM measurements in the AGS Booster. Preliminary studies show that the proposed algorithm is able to reproduce a reasonable transfer function for vertical quadrupoles in the Booster.

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