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BEAM-BASED ALIGNMENT OF INDIVIDUAL MEMBERS OF SEXTUPOLE FAMILIES

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Abstract

In order to steer beams through the center of focusing elements, the field center with respect to adjacent Beam Position Monitors needs to be known precisely. Often individual qudrupoles are varied to find their center, where the orbit does not change, but this requires costly field control for each quadrupole. Here we analyze a beam-based Alignment (BBA) technique that utilizes sextupoles. BBA can easily be performed on sextupoles if each can be changed individually; one simply finds the orbit where a change of a sextupole's strength does not change the tune. However, often also sextupoles are powered in families. We therefore analyze a BBA method for sextupoles for which the sextupole strength does not have to be changed, but where the orbit can be changed in one sextupole alone by means of a closed-orbit 3-bump. A sextupole deflects the beam in the same direction for positive and for negative beam offsets, due to its quadratic field. Minimal beam deflection therefore occurs in the center of the sextupole. By changing the position at which the beam enters the sextupole while measuring the corrector strengths that close the 3-bump, one can therefore find the sextupole center. Here, we explore the precision to which this method can reconstruct the sextupole alignment.

INTRODUCTION

In the Electron Ion Collider (EIC), and in most storage rings, family-powered sextupoles are set up in a configuration where they are separated by drift regions, quadrupoles, and other sextupoles powered in separate families. Figure 1 depicts the 3-bumps simulated in this procedure, where a single sextupole in the "SX1_3" family of the Electron Storage Ring (ESR) of the EIC is placed nearby three quadrupoles and in between the first two. Because corrector coils are not yet placed into the lattice, we model them as thin correctors after each quadrupole. The three correctors around a sextupole create a 3-bump that contains no other sextupole.

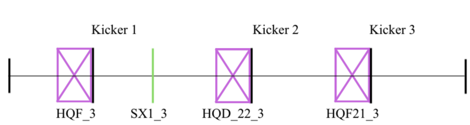

Figure 1: Diagram of a three-corrector bump system with a sextupole between the first and second corrector.

Without the sextupole, the corrector strengths that close the 3-bump can be computed to first order using the Twiss parameters, or they can be measured with beam while the sextupole families with members in the 3-bump are turned off. A sextupole deflects the beam proportional to the square of its distance from the sextupole center, by

$$\theta_s = \frac{1}{2} k l_2 (x_s - x_o)^2, \tag{1}$$

where kl_2 is the field coefficient of the sextupole times it's length, x_0 is the horizontal location of the sextupole center and x_s is the closed-orbit position in the sextupole. By comparing the corrector strength to close the 3-bump with and without sextupole, we can reconstruct the positioning of the center of the sextupole based on Eq. (1).

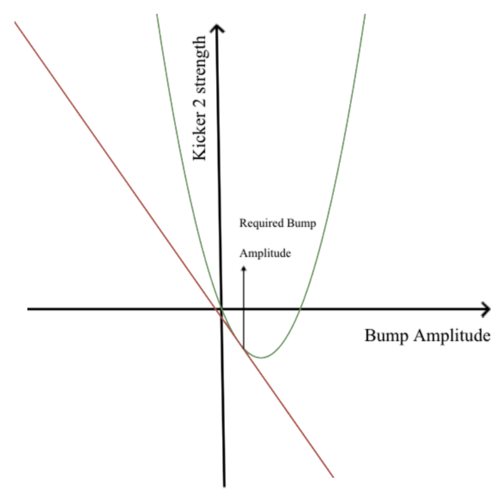

Figure 2: Strength of corrector-2 that closes the 3-bump with (green parabola) and without (red line) a sextupole as a function of the bump amplitude, which is created by corrector-1. The two curves meet when the orbit is in the center of the sextupole.

For this method, several sextupoles can exist in the same bump, as long as they are powered by seperate families and only one of them is powered at a time.

METHODS

In order to simulate and numerically close these bumps, we used the program Tao of the Bmad toolkit. We exited a bump amplitude with corrector-1 and then ran a least square optimization on $M = 83$ beam position monitors to find the stengths of correctors 2 and 3 that changed the BPMs outside the 3-bump of Fig. 2 as little as possible. To each BPM we added the same rms of Gaussian noise, results for 0 µm, 1 µm, 10 µm will be shown. The following list summarizes our BBA procedure:

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- Turn all sextupole families off that have a sextupole within the 3-bump and store the closed orbit around the ring as a reference. Give corrector-1 several strengths that lead to bump amplitues up to several mm and close the bump with correctors 2 and 3. This determines the linear relation $a = \theta_2/\theta_1$ and $b = \theta_3/\theta_1$. The relationship is linear, because the bump contains no sextupole strength.

- Turn the three correctors off again and activate the family of the studied sextupole as strongly as reasonable whilst keeping all families turned off that also have a sextupole in the bump. Make sure the studied family has only one sextupole within the bump. Store the closed orbit around the ring as a reference.

- Excite various bump amplitudes up to several mm. For each, close the 3-bump so the orbit remains unchanged outside the bump region. Record the quadratic relations $\theta_2(\theta_1)$ and $\theta_3(\theta_1)$.

- For each bump excitation θ_1, repeat the process of bump closing and record the resulting bump strength to increase the accuracy of θ_2 and θ_1 by averaging. Here we averaged over $B = 10$ bump closings.

- Observe the quadratic curves $\theta_2(\theta_1)$ and $\theta_3(\theta_1)$ and find their intersection with the linear relations obtained without sextupoles: $\theta_2 = a\theta_1$ and $\theta_3 = b\theta_1$. At the resulting bump amplitude, the orbit goes through the center of the sextupole.

To better observe the quadratic relationship, we chose to remove the linear term from the raw data by subtracting $a\theta_1$ and $b\theta_1$ from the strength of the second and third corrrector, leaving only the quadratic term that is due to the sextupole, as shown in Fig. 3. It's minimum determines the bump amplitude that leads through the sextupole center. Without system errors, the extremum for the second and third corrector occur at the same bump amplitude. With BPM errors, there is a discrepancy between the two extremum locations, which gives an experimental measure of the result's uncertainty.

RESULTS

For any bump, excited by an angle θ_1 in the first magnet, we record which angle θ_2 in the second magnet closes the bump. The difference in this θ_2 setting when the sextupole is on, vs. when it is off is referred to as $\Delta\theta_2$ and it is a parabola with respect to the beam's position in the sextupole. We use the minimum of this parabola to determine the center of the sextupole. In the following, we always subtract this linear term, and θ_2 therefore will refer to the difference $\Delta\theta_2$ for simplicity, it describes the angle in the second corrector which compensates the deflection θ_s in the sextuple at the end of the 3-bump, at the locaiton of the third corrector, $\theta_2\sqrt{\beta_2}\sin(\psi_3 - \psi_2) = -\theta_s\sqrt{\beta_s}\sin(\psi_3 - \psi_s)$ with the common notation for Twiss parameters at the positions of the

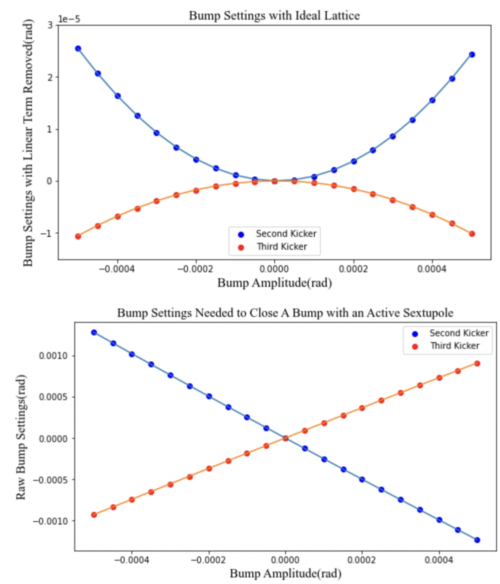

Figure 3: second and third corrector data with linear term removed (Top) and without linear term removed (Bottom) in an ideal lattice without BPM errors. Not that the extrema of the parabola are at the orgigin, because the sextuple was not misaligned in this example.

sextupole and the 2nd and 3rd corrector, indicated by indices s, 2, and 3.

The goal of this study was to test whether or not this Sextupole BBA Method could realistically produce results for the sextupole alignment to a level of precision that is useful for accelerator applications (within at least 100 µm of uncertainty). As expected, when there are no Gaussian errors implemented in the BPMs, this sextupole BBA method works to machine precision. When 1 µm of Gaussian error was implemented into the BPMs, the method was precise to the order of 10 µm. When 10 µm of Gaussian error was implemented into the BPMs, the method was precise to the order of 50 µm, following approximately a square law of $\sigma_{x0} \approx 7\sigma_{\mathrm{BPM}}$.

Assuming Gaussian BPM-noise errors at the order of 10 µm is not unreasonable.

EQUATION ESTIMATING THE UNCERTAINTY

The following equation was derived to estimate the expected precision with which the Sextupole BBA Method can determine the center of a sextupole.

We create a 3-bump through the sextupole and scan its amplitude in the sextupole from a range of θ_1 values centered symmetrically around 0, scanning this region with N data points. The rms uncertainty with which we can determine θ_2 is σ_{θ_2}; the parabolic relationship between these correctors is $\theta_2 = c + d\theta_1 + e\theta_1^2$, where the three coefficients are determined

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by a least-square fit for the N data points of the parabola. The least-square-fit results are $e = (\overline{\theta_2}\,\overline{\theta_1^2} - \overline{\theta_2\theta_1^2})/(\overline{\theta_1^2}^2 - \overline{\theta_1^4})$ and $d = \overline{\theta_2\theta_1}/\overline{\theta_1^2}$, where overlines indicate averages over the N data points. Note that these are symmetrically chosen so that $\overline{\theta_1^n} = 0$ for odd n.

The minimum of the parabola is $\theta_0 = -\frac{1}{2}d/e$. To estimate the uncertainty of this minimum, one finds that for every series of N measurements $\Delta\theta_0 = -\frac{1}{2e^2}(e\Delta d - d\Delta e)$. Squaring and averaging over many of such series leads to $\sigma_{\theta_0}^2$. Note that $\Delta d\Delta e$ averages to 0 because even orders of θ_1 average to 0, leading to

$$\sigma_{\theta_0}^2 = \frac{\sigma_{\theta_2}^2}{Ne^2}\left(\frac{1}{4\overline{\theta_1^2}} + \frac{d^2}{4e^2(\overline{\theta_1^4} - \overline{\theta_1^2}^2)}\right) . \tag{2}$$

Because the corrector angle needed to close the bump without sextupole is subtracted, θ_2 only compensates the orbit deflection θ_s from the sextupole, i.e. $\theta_2\sqrt{\beta_2}\sin(\psi_3 - \psi_2) = -\theta_s\sqrt{\beta_s}\sin(\psi_3 - \psi_s)$ and $\theta_s = \frac{1}{2}kl_2\beta_1\beta_s\sin^2(\theta_s - \theta_1)(\theta_1 - \theta_0)^2$. Therefore $\theta_2 = e(\theta_1 - \theta_0)^2$ where the proportionality constant e only depends on Twiss parameters, and then

$$\sigma_{\theta_0}^2 = \frac{\sigma_{\theta_2}^2}{Ne^2}\left(\frac{1}{4\overline{\theta_1^2}} + \frac{\theta_0^2}{(\overline{\theta_1^4} - \overline{\theta_1^2}^2)}\right) . \tag{3}$$

Choosing θ_1 to consist of a large number N, uniformly and symmetrically spaced in the interval of width $2\Delta\theta_1$, simplifies to

$$\sigma_{\theta_0}^2 = \frac{3\sigma_{\theta_2}^2}{4Ne^2\Delta\theta_1^2}\left(1 + \frac{15\theta_0^2}{\Delta\theta_1^2}\right) . \tag{4}$$

Expression e by the Twiss parameters leads to

$$e = -\frac{1}{2}kl_2\beta_1\beta_s\sin^2(\psi_s - \psi_1)\frac{\sqrt{\beta_s}\sin(\psi_3 - \psi_s)}{\sqrt{\beta_2}\sin(\psi_3 - \psi_2)} . \tag{5}$$

The 3-bump can only be closed to a precision σ_{θ_0} that is proportional to the precision of the BPM that closes the bump. This uncertainty will lead to an uncertainty of the sextupole offset x_0, given by

$$\sigma_{x_0}^2 = \beta_s\beta_1\sin^2(\psi_s - \psi_1)\sigma_{\theta_0}^2 . \tag{6}$$

The uncertainty in θ_2 is due to the error in BPM readings σ_{BPM}. With the M BPMs at phases ψ_m all having the same uncertainty, a least square minimization of these BPM readings by means of the second corrector strength leads to

$$(\text{corrector strength})^2 = \frac{\sigma_{\text{BPM}}^2}{B\sum_{m=1}^{M}\beta_2\beta_m\sin^2(\psi_m - \psi_2)} . \tag{7}$$

The error reduction from averaging over B BPM minimizations is included in the denominator. This formula holds for the corrector strength with sextupoles and also for the one without sextupoles. The difference θ_2 therefore has a $\sigma_{\theta_2}^2$ that is twice as large.

Some more approximations: If one scans far across the center of the sextupole, i.e. $\Delta\theta_1 \ll \theta_0$, the term $15(\theta_0/\Delta\theta_1)^2$ can be neglected against 1. If we set all sin factors to an average value of $1/\sqrt{2}$ and use an average $\overline{\beta}$ for each beta function, we obtain

$$\sigma_{\theta_0}^2 \approx \frac{12\sigma_{\theta_2}^2}{Kl_2^2\overline{\beta}^4Ne^2\Delta\theta_1^2} , \tag{8}$$

$$\sigma_{x_0}^2 \approx \frac{\overline{\beta}^2}{2}\sigma_{\theta_0}^2 , \tag{9}$$

$$\sigma_{\theta_2}^2 \approx \frac{4\sigma_{\text{BPM}}^2}{BM\overline{\beta}^2} \text{ and finally} \tag{10}$$

$$\sigma_{x_0}^2 \approx \frac{24\sigma_{\text{BPM}}^2}{BMNkl_2^2\overline{\beta}^4\Delta\theta_1^2} . \tag{11}$$

In the ESR of the EIC, $\overline{\beta} \approx 20\,\text{m}$, and $Kl_2 \approx 1\,\text{m}^{-2}$. As noted above, we used $B = 10$, $M = 83$, and measured $N = 21$ setting of the first corrector while scanning over $2\Delta\theta_1$ of 1 mrad, leading to

$$\sigma_{x_0} \approx \frac{\sigma_{\text{BPM}}}{5} . \tag{12}$$

This formula would approximate a precision of the method significantly superior to the simulation result, including averaging over all BPMs, over several measurements, and for many data points along the parabola. It is therefore concluded that a careful analysis of these averaging procedures may increase the precision significantly beyond current simulation results.

Even though the simulation results already indicate useful applicability of this method with sub 100 µm precision, significantly better precision may yet be possible.

CONCLUSION

The here presented Sextupole BBA technique involving the study of an orbital bump through one member of a sextupole family was simulated with the goal of achieving an accuracy of at least 100 µm, to be useful in accelerator applications. When tested upon an ideal lattice without random errors as a control group, the Sextupole BBA method was able to return the center of the sextupole to machine precision. In this study we analyze BPM errors as high as 10 µm, a realistic order of error for modern instrumentation. The Sextupole alignment can be determined with uncertainty to the order of 50 µm, demonstrating potential for this method to be used in storage rings that power sextupoles in families. And yet better precision may be possible, because an Eq. (11) was derived that estimates the order of precision to which this BBA technique should be able to locate the offset of the sextupoles. This estimation for the uncertainty is significantly smaller than the precision found in simulated results. Therefore, studies to expand upon this should be done that involve testing how to simulated precision improves with the use of more BPMs or more averaging.

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