DYNAMICS OF OFF-AXIS INJECTION NEAR THE COUPLING RESONANCE AT PETRA IV

E. C. Cortés García*, I. V. Agapov, DESY, Hamburg, Germany

Abstract

The PETRA IV project will have a storage ring with an ultra-low natural emittance of 20 pm rad. For an off-axis injection scheme with working points at the difference resonance, it is critical to ensure that the vertical excursion caused by transversal coupling does not affect injection efficiency. In this contribution, we present simulation results of an off-axis injection near the coupling resonance, which provides equal equilibrium emittances. The advantages and disadvantages of such a scheme are discussed.

INTRODUCTION

PETRA IV is the project to upgrade DESY’s flagship storage ring, PETRA III, to the next generation. The PETRA IV storage ring is based on a hybrid six-bend achromat lattice and well described in Ref. [1]. It is anticipated that the particle loss brought on by Touschek scattering will have an impact on the beam lifetime and, consequently, the machine’s performance [2]. A new strategy for potentially mitigating this source of losses is to operate the machine with working points near or exactly at the transversal coupling resonance [3].

A problem that can be encountered when injecting off-axis near the coupling resonance is the vertical oscillation amplitude of the betatron motion when the initial horizontal oscillations couple into the vertical plane. Insertion devices (IDs), particularly in-vacuum devices, limit the physical aperture available for the beam to undergo betatron oscillations. At PETRA IV, the IDs are expected to restrict the physical aperture down to 2.5 mm (half aperture). Fortunately, the PETRA IV’s strong non-linear dynamics cause the change of transversal oscillation amplitudes to occur only after the betatron motion has already been substantially dampened by synchrotron radiation. In this contribution, we present simulation results of off-axis injection close to the coupling resonance. This will result in equal equilibrium emittances of the stored electron beam in the transversal planes.

EQUILIBRIUM EMITTANCE

The dynamics of the weakly coupled particle motion in storage rings is well understood and can be found in textbooks [4, 5]. Here we lay out the results that are collected in Ref. [6] and implied in Ref. [7] for the equilibrium emittances.

For weakly coupled lattices with equal partition numbers in horizontal and vertical planes the emittance ratio is [5]

\[ \frac{\epsilon_y}{\epsilon_x} = \frac{\kappa^2}{\Delta^2 + \kappa^2} \defeq \sin^2 \theta, \]  

(1)

where \( \Delta = q_y - q_x \) is the distance to the coupling resonance, \( \kappa \) is the resonance driving term. Equation (1) defines an angle \( \theta \) that describes the magnitude of the coupling in the system, \( \sin^2 \theta = 1 \) for full coupling and \( \sin^2 \theta = 0 \) for no coupling. The emittances satisfy \( \epsilon_x + \epsilon_y = \epsilon_{0,\text{nat}} \) where the natural emittance is closely related to the machine’s optical functions. It has been shown that Eq. (1) is only valid when the dynamics is away from the coupling resonance and the horizontal partition number \( j_x \) of the lattice is close to one [7]. In the general case the emittance ratio reads [6]

\[ \frac{\epsilon_y}{\epsilon_x} = \frac{(\tau_x + \tau_y) \sin^2 \theta}{4 \tau_x + (\tau_y - 3 \tau_x) \sin^2 \theta}. \]  

(2)

where \( \tau_{x,y} \) are the damping times of each transversal plane. Equation (2) implies that \( \epsilon_y + \epsilon_x > \epsilon_{0,\text{nat}} \) and for \( \sin^2 \theta = 1 \)

\[ \epsilon_x = \epsilon_y = \epsilon_{0,\text{nat}}/(1 + 1/j_x). \]  

(3)

The achievable horizontal emittance for PETRA IV at the coupling resonance is \( \epsilon_x > \epsilon_{0,\text{nat}}/2 (j_x = 1.25) \).

INJECTION AT PETRA IV

The operation of PETRA IV is anticipated to follow a conventional accumulation scheme, where particle losses are topped up by the off-axis injection of fresh bunches. The booster synchrotron DESY IV [8] will be able to deliver bunches with up to 1 nC per bunch to PETRA IV. The emittance of the injected beam is a crucial factor to take into account for the off-axis injection scheme because it could negatively impact injection efficiency when combined with PETRA IV’s aperture constraints (such as its dynamical aperture and collimators). Thus, numerous injection schemes have been considered to allow for flexibility if such restrictions are encountered [9]. For each injection scenario the theoretical injection efficiency was found to be 100%. Henceforth, one scenario is presented to evaluate the dynamics near the coupling resonance. In Table 1 the relevant parameters for the injected beam are listed, these are the horizontal \( \beta_{x,\text{inj}} \) of the transfer line at the injection point, the transversal emittances of the injected beam and the distance \( \Delta x \) to the stored beam. The simulations were performed with

Table 1: Injected Beam Parameters

<table>
<thead>
<tr>
<th>( \beta_{x,\text{inj}} ) [m]</th>
<th>( (\epsilon_x, \epsilon_y) ) [nm]</th>
<th>( \Delta x ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.405</td>
<td>(13.7, 13.7)</td>
<td>5.344</td>
</tr>
</tbody>
</table>

ELEGANT [10] with 6D tracking including synchrotron radiation and quantum excitation effects. No collective effects

---

* edgar.cristopher.cortes.garcia@desy.de
Table 2: General Injection Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of particles</td>
<td>(N_p)</td>
<td>10^4</td>
</tr>
<tr>
<td>No. of turns</td>
<td>(N_T)</td>
<td>(\geq 4680)</td>
</tr>
<tr>
<td>Tunes</td>
<td>((Q_x, Q_y))</td>
<td>(135.18, 86.27)</td>
</tr>
<tr>
<td>Frac. tunes</td>
<td>((q_x, q_y))</td>
<td>(0.18, 0.27)</td>
</tr>
<tr>
<td>Chromaticity (corr.)</td>
<td>((\xi_x, \xi_y))</td>
<td>(8.3, 7.1)</td>
</tr>
<tr>
<td>Long. beam size</td>
<td>(\sigma_s)</td>
<td>2.5 cm</td>
</tr>
<tr>
<td>Momentum dev.</td>
<td>(\delta)</td>
<td>(2.6 \cdot 10^{-3})</td>
</tr>
</tbody>
</table>

were included. Relevant parameters for the injection simulations are listed in Table 2. Since both the dynamic aperture and the momentum acceptance of PETRA IV are known to be sensitive to alignment errors, all simulations were done with perturbed optics. We used a reduced set of alignment errors that cause approximately 5 percent beta beating (RMS), since it has been demonstrated that it is equivalent to carrying out a full start-up and tuning procedure with the full set of expected errors [11].

**SIMULATION RESULTS**

**Injection Efficiency**

First, a vertical tune scan was performed in steps of 0.01 from the nominal value (see Table 2) \(q_y = 0.27\) to \(q_x = q_y = 0.18\). This was performed with a single realization of the machine, including misalignment errors to induce beta beating and a leakage of \(\kappa\) of the order of 1%. The simulated injection efficiency is depicted in Fig. 1. The efficiency does not appear to be compromised even when the working point lies exactly on the coupling resonance. This is similar to the experimental results reported for SPEAR3 [3]. The injection efficiency can be better understood with the results illustrated in Figs. 2 and 3. In Fig. 2 particles were tracked with initial coordinates \(\vec{z}_0 = (x_0, 0, y_0, 0, 0, 0)^T\) to determine the edge, where stable bounded motion is found. The solid lines indicate the average over 20 machine realizations and the shaded area represents the RMS. In Fig. 3 the same calculation was performed but by changing stepwise the longitudinal coordinate \(\delta = \Delta p/p\) (zero else) until particle loss is observed. For both simulations if a particle with initial coordinates \(\vec{z}_0\) survived 1024 turns, the motion was considered stable. Note that the horizontal damping time due to synchrotron radiation is \(\tau_{SR} = 2339\) turns. The available transversal aperture is marginally deteriorated compared to the nominal working point. This working point however has a noticeably larger local momentum acceptance (LMA) (see Fig. 3). Note that shifting the working point is the usual way to find the best balance between LMA and dynamic aperture.

**Transient Transversal Tunes**

Strong amplitude detuning leads to the initial transverse oscillations of \(\Delta x = 5.344\) mm to have the tunes of \((q_x, q_y) = (0.43, 0.02)\). Therefore, the difference resonance is not excited in the early stages of the injection process. The emittance exchange mechanism is only triggered when the transverse motion is sufficiently damped. This is illustrated in Fig. 4, where the evolution of Eq. (1) is plotted together.
with the tune difference. The transient tune here is the average one turn phase advance of the \( N_p = 10^4 \) particles.

![Figure 4: Transient tune difference and coupling (see Eq. (1)) as function of turn number. The maximal coupling value (dashed blue line) is 48% for the first 5000 turns.](image1)

**Equilibrium Emittance**

The dynamics of the particles was investigated with a long tracking simulation to corroborate that the transversal emittances converge in the fully coupled case to \( \epsilon_x = \epsilon_y = 10.9 \text{ pm rad} \). The damping of the transversal beam size due to synchrotron radiation is illustrated in Fig. 5. The emittance values for 10 different machine realizations read

\[
\epsilon_x = (12.5 \pm 0.9) \text{ pm rad}, \\
\epsilon_y = (11.9 \pm 0.7) \text{ pm rad}.
\]

(4)

The error is given by the RMS value of the machine realizations probed. The introduction of misalignments perturbs the machine’s linear optics such that the calculated equilibrium emittances lie slightly above the expected value. This is also true for the uncoupled case.

![Figure 5: Normalized beam size (RMS) evolution versus turn number.](image2)

**Stable Resonance Islands**

The formation of stable resonance islands is an intriguing aspect of the dynamics brought about by shifting the vertical working point close to the coupling resonance. Figure 6 shows this feature in detail. Over ten realizations, approximately 90% of the injected particles are damped to the beam core, while approximately 10% are damped towards resonance islands.

![Figure 6: Visualization of the stable resonance islands. The right dashed gray ellipse represents the injected 3 RMS beam distribution.](image3)

**SUMMARY AND OUTLOOK**

In this contribution, we presented the results of injection simulations at the coupling resonance. The simulations show that shifting the working point to the fully coupled case is not detrimental to the injection efficiency. The amplitude detuning induced by the non-linear dynamics shifts the tune noticeably away from the coupling resonance. The resonance is only driven when the injected particles have damped their motion considerably. Then the linear dynamics determine the transversal motion. The introduction of misalignment errors in the elements, which represent a more realistic setting, shows that the equilibrium emittance is slightly higher than the achievable emittance in a perfect machine. This was expected, since the errors perturb the linear optics of the ring. Finally, stable resonance islands were observed in the simulation, which is an intriguing finding. The percentage of particles trapped is ca. 10%. Mechanisms of avoiding, cleaning, or exploiting the resonant islands should be further explored for this operation scenario.

Off-axis injection at the coupling resonance appears to be a viable option for PETRA IV. Particle losses due to Touschek scattering can be mitigated and emittance growth caused by intra-beam scattering (IBS) could likely be reduced. Quantitative studies on the Touschek lifetime and IBS growth rates are beyond the scope of this contribution. In conclusion, this operation mode could potentially improve the machine’s performance.

**ACKNOWLEDGMENTS**

We would like to thank the PETRA IV beam physics group and especially J. Keil, Y.-C. Chae, C. Li and S. Antipov for fruitful discussions.
REFERENCES


