# PROTON AND ELECTRON RLA OPTICS DESIGN* 

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## Abstract

We describe optics designs of the key components of proton and electron recirculating linear accelerators. They are presented in the context of a high-power hadron accelerator being considered at ORNL and a CEBAF electron energy doubling project, FFA@CEBAF, being developed at Jefferson Lab. Both concepts rely on the fixedfield alternating-gradient arc optics designs where multiple beam passes are transported by a single beam line.

## INTRODUCTION

Recirculating Linear Accelerators (RLAs) provide for efficient use of expensive normal and super-conducting RF cavities by letting the accelerating beam pass through them several times for additional energy gain. The maximum number of recirculating passes and, therefore, the maximum attainable energy gain is limited by several factors including (a) the maximum practical number of the beam return arcs, (b) the complexity of matching the different arcs to the same linear accelerator (linac), (c) the challenge of synchronizing the beam with RF timing, and (d) the cavity input power limit.

This paper focuses on aspects (a) and (b) above. They present similar challenges and are solved similarly in the context of the high-power hadron accelerator [1] being considered at ORNL and the CEBAF electron energy doubling project, FFA@CEBAF [2, 3], being developed at Jefferson Lab. Thus, the paper discusses the two accelerator concepts jointly while still highlighting some of their differences.
In both concepts, the problem of having to deal with multiple return arcs is mitigated by applying the FixedField Alternating-gradient (FFA) approach to the design of the arcs. This approach allows for transport of multiple different-energy beams by a single beam line. Multiple options are considered for matching the multiple beams coming in and out of the arcs to the linac. It is particularly challenging for the passes coming out of the same FFA arc, especially without separating them into individual channels, because all beams must be controlled simultaneously by common magnets. As described below, the matching options being considered include an adiabatic match [4] and a compact match based on a dogleg bending magnet configuration as well as an approach based on

[^0]parametric resonance control of the individual energies that is still in early stages of exploration.

## ORNL HADRON ACCELERATOR CONCEPT

The focus of the ORNL study is the feasibility of a hadron accelerator of a modest energy of about 1 GeV but high power of over 10 MW CW . One of the fundamental limiting factors that need to be addressed by the design is the maximum power that an SRF cavity can provide to the beam in CW mode. The typical approach to such a design is to use a straight linac [1] with near-crest beam acceleration in the cavities up to their voltage or power limit. However, acceleration at low relativistic $\beta$ is less efficient than at high $\beta$ in terms of energy gain per cavity. This creates potential for reusing some of the cavities for additional acceleration.


Figure 1: Comparison of straight linac and RLA parameters. The average beam current through a cavity is shown as a function of its relativistic $\beta$. The green line is the constant-power profile of 300 kW . It assumes the empirically obtained dependence of the energy gain per cavity on the beam $\beta$ [1].

Analysis of the cavity power available for acceleration as a function of the beam energy is illustrated in Fig. 1 [1]. For simplicity, it assumes the maximum power that can be delivered by a cavity of 300 kW CW. Combined with a conservative assumption that an FFA arc can transport a range of the beam momenta with the maximum ratio of 2, it suggests an accelerator scheme shown in Fig. 2 [1]. As shown in Fig. 1, the potential reduction in the required number of cavities is by a factor of about 4 . All the FFA arcs have the same average bending radii determined by the highest-energy field requirements, so that they can be housed in the same tunnel. All accelerating sections are concentrated in a relatively small area
simplifying delivery of cryogens and RF power. An estimated tunnel circumference is 400 m .


Figure 2: Schematic of the high-power hadron accelerator being explored at ORNL.

The main building block of an arc is a FODO cell composed of combined-function magnets. The periodic orbits and optics of the lower- and upper-bound energies of the highest-energy RLA, RLA4, are shown in Fig. 3. All energies within the range are bent by the same angle. The required extra or missing dipole field integral comes from the quadrupole field gradients when combined with the orbit offset. The quadrupole gradients also provide alternating focusing with strength inversely proportional to the beam momentum. This is the essence of the FFA approach [5]. The parameters of the combined-function magnets making up the cell in Fig. 3 are chosen not to exceed 1 T at 20 mm radius. Such magnets can potentially be built using the permanent magnet technology [6].


Figure 3: Orbits (top, left scale), momentum dispersions (top, right scale) and horizontal (bottom, left scale) and vertical (bottom, right scale) Twiss $\beta$ functions of a regular periodic arc cell of RLA4.

Placement of linac sections requires one or a series of sufficiently long straights with zero orbital offset and momentum dispersion of all passes. The adiabatic matching approach [4] allows one to adiabatically transform an arc cell into a straight linac cell. The magnet parameters are gradually adjusted over several cells from the values corresponding to a regular arc cell to those of a straight cell. This transition must be sufficiently smooth, i.e., it
must occur over a sufficiently large number of cells to ensure a good match.
Figure 4 shows the orbits and optics of an arc-to-linac matching section of RLA4. In Fig. 4, an arc FODO cell adiabatically transitions into a straight triplet cell. Matching in Fig. 4 is implemented over 22 cells using the following scaling law [7]:

$$
\begin{equation*}
\theta_{i}=f(i) \theta_{a}, l_{i}=l_{s}\left[\frac{l_{a}}{l_{s}}\right]^{f(i)}, k_{1}^{i}=k_{1}^{s}\left[\frac{k_{1}^{a}}{k_{1}^{s}}\right]^{f(i)} \tag{1}
\end{equation*}
$$

where $\theta, l$, and $k_{1}$ are the magnet reference bending angle, length, and normalized quadrupole gradient, respectively. Indices $i, a$, and $s$ refer to the parameter values of the $i$ th, arc and straight cells, respectively. The scaling function $f(i)$ is given by

$$
\begin{equation*}
f(i)=1-3\left[\frac{i}{n_{T}+1}\right]^{2}+2\left[\frac{i}{n_{T}+1}\right]^{3} \tag{2}
\end{equation*}
$$

where $i$ is the matching cell number from 1 to $n_{T}$ and $n_{T}$ is the total number of matching cells. $f(i)$ is a third-order polynomial going from 1 to 0 as $i$ changes from 0 to $n_{T}+$ 1. In addition, the coefficients of $f(i)$ are chosen so that $f^{\prime}(0)=f^{\prime}\left(n_{T}+1\right)=0$ to ensure gradual ramp up and ramp down of the parameter change rate.


Figure 4: Orbits (top, left scale), momentum dispersions (top, right scale) and horizontal (bottom, left scale) and vertical (bottom, right scale) Twiss $\beta$ functions of an adiabatic matching section of RLA4.

The quantities in Eq. (1) that do not go to zero, namely, $l$ and $k_{1}$, scale in such a way that the $\log$ of their ratio varies from 1 to 0 according to $f(i)$ [7]. The scaling law of Eq. (1) maintains the betatron phase advance per cell nearly constant. We find this criterion to be important for attaining a good match.
An important feature of the adiabatic approach demonstrated in Fig. 4 is that it provides a match of both orbital and optical functions continuously over the entire energy range. Other matching approaches are also possible as discussed in the subsequent sections.

## FFA@CEBAF

It is of great interest for nuclear physics experiments at CEBAF to increase the electron beam energy up to $22 \mathrm{GeV}[2,3]$. Such an increase can be attained by doubling the number of beam passes through CEBAF linacs.

It is not practical to realize this by simply doubling the number of conventional recirculation arcs due to the space constraints of the existing tunnel. Use of compact FFA arcs transporting multiple beam passes through the same sting of magnets offers a solution to this problem [2,3]. The arcs are built of permanent magnets providing high fields of the order of 1 T while having small profiles [6].

The design of period arc FODO cells follows the FFA concept briefly mentioned above. In the envision design, CEBAF linacs accelerate the beam by a factor of about 33. Given that the linac optics must provide stable transport of the lowest energies, the reduction in their focusing strengths with energy makes them effectively appear as drifts to the highest-energy passes recirculated by the FFA arcs. The linac straight are about 250 m long. For the Twiss $\beta$ functions to stay under control, they must be on the order of 150 m and have nearly zero slopes at the middle points of the linac straights as illustrated in Fig. 5. Therefore, the relatively small beams coming out of the FFA arcs must be expanded to match the linac optics.


Figure 5: Example CBAF linac horizontal optics starting from the center of one of the linac straights with Twiss $\beta_{x}=150 \mathrm{~m}$ and $\alpha_{x}=0$ for all energies.

Applying adiabatic matching in FFA@CEBAF presents a challenge due to the tunnel space constraints and the fact that it is not possible to keep the betatron phase advance per cell constant during match. Thus, we explore a two-step matching strategy. The different-energy orbits and dispersions are suppressed in the first step with their associated constraints thus eliminated in the second step when only Twiss $\beta$ and $\alpha$ must be matched.
It has been suggested $[8,9]$ to realize the first step nonadiabatically using a two-dipole dogleg structure because it generates a pattern of different-energy orbits and dispersions resembling that of an FFA FODO cell. In fact, one subtlety in this picture is that the dispersion energy ordering is reversed in the two cases, i.e., contrary to a dogleg, the dispersion magnitude in an FFA cell becomes higher at higher energies. Nevertheless, with additional optical knobs involved, this approach works as illustrated in Fig. 6.
The remaining second step involving match of Twiss $\beta$ and $\alpha$ is still challenging due to the necessity to control the optics of 6 beams simultaneously using common magnets. It is desirable to develop orthogonal knobs for
independent control of the individual energies. We take advantage of the fact that different energies have different betatron phase advances in a periodic FFA cell [10]. Applying periodic quadrupole kicks over several cells at twice the rate of the betatron oscillations at a certain energy selectively excites a parametric resonance in Twiss $\beta$ of that particular energy.


Figure 6: Orbits (top, left scale), momentum dispersions (top, right scale) and horizontal (bottom, left scale) and vertical (bottom, right scale) Twiss $\beta$ functions of a nonadiabatic orbit and dispersion suppression section.

We use a linear model to illustrate parametric resonance excitation process in a channel consisting of a series of periodic cells. A single plane is considered for simplicity. Each cell is represented by a periodic linear matrix with $\beta=10 \mathrm{~m}, \alpha=0$, and $v_{r}=v+\Delta v_{r}$. A thin quadrupole kick is applied at the end of each cell with the quadrupole strength modulated as $k_{1} \propto \cos \left(2 \pi v \times 2 n+\varphi_{r}\right)$. As shown in Fig. 7, when betatron motion is in resonance with the quadrupole kicks, i.e., $\Delta v_{r}=0$, it gets amplified. The effect of the kicks gets averaged out for other tunes including those as close as 0.01 to the resonance tune. The resonance phase $\varphi_{r}$ determines positions of maxima and minima along the channel. We are currently exploring this mechanism in application to optics control and matching.


Figure 7: Illustration of resonant Twiss $\beta$ excitation at a particular tune in a channel of periodic cells.

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