ID 2497, MOOYSP1

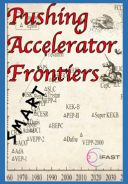
Impact of Longitudinal Gradient Dipoles on Storage Ring Performance

Frank Zimmermann, CERN

with Yannis Papaphilippou and Axel Poyet

13 June 2022, IPAC’22 Bangkok, Thailand

Work partially supported by the European Union's Horizon 2020 Research and Innovation programme under Grant Agreement No. 101004730(iFAST).



Equilibrium emittance due to synchrotron radiation in a storage ring

$$\varepsilon_x = C_q \gamma^2 \frac{I_5}{I_2}$$

$$I_5 = \oint \frac{\mathcal{H}_x}{|\rho|^3} ds \, , I_2 = \oint \frac{1}{\rho^2} ds$$

$$\mathcal{H}_x = \frac{1}{\beta_x}\{D_x^2 + (\beta_x D'_x + \alpha_x D_x)^2\}$$

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc}$$

M. Sands, "The Physics of Electron Storage Rings: An Introduction," SLAC-121, 1970.

R. H. Helm, M. J. Lee, P. L. Morton, and M. Sands, "Evalua tion of synchrotron radiation integrals," IEEE Trans. Nucl. Sci., vol. 20, pp. 900–901, 1973.

Reducing I_5 can minimise the emittance, or cost, of linear collider damping rings and storage-ring light sources, e.g. up to factor ~7 emittance reduction possible for the CLIC Damping Rings

Long history of proposals to tailor $\mathcal{H}_x / |\rho|^3$ to minimize ε_x

J. Guo and T. Raubenheimer, "Low emittance e- / e+ storage ring design using bending magnets with longitudinal gradient," **EPAC 2002**

R. Nagaoka and A. Wrulich, "Emittance minimisation with longitudinal dipole field variation," NIM A, vol. 575, pp. 292–304, **2007**

C.-x. Wang, "Minimum emittance in storage rings with uniform or nonuniform dipoles," PRST-AB, vol. 12, 061001 (**2009**)

A. Streun and A. Wrulich, "Compact low emittance light sources based on longitudinal gradient bending magnets," NIM A, vol. 770, pp. 98–112, **2015**

V. S. Kashikhin et al., "**Longitudinal Gradient Dipole Magnet Prototype for APS at ANL,**" IEEE Trans. on Appl. Superc., vol. 26, no. 4, pp. 1–5, **2016**

M. A. Dominguez Martinez, F. Toral, H. Ghasem, P. S. Papadopoulou, and Y. Papaphilippou, "Longitudinally variable field dipole design using permanent magnets for CLIC damping rings," IEEE Trans. Appl. Superc., vol. 28, no. 3, pp. 1–4, **2018**

P. Yang, W. Li, Z. Ren, Z. Bai, and L. Wang, "Design of a diffraction-limited storage ring lattice using longitudinal gradient bends and reverse bends," NIM A, vol. 990, 164968 (**2021**)

S. Papadopoulou, F. Antoniou, and Y. Papaphilippou, "Emittance reduction with variable bending magnet strengths: Analytical optics considerations and application to the compact linear collider damping ring design," PRAB 22, 091601 (**2019**)

A. Poyet et al., "Emittance Reduction with the **Variable Dipole for the ELETTRA 2.0 Ring**," this conference (**IPAC 2022**)

ID: 2169 - THPOPT013 Emittance Reduction With the Variable Dipole for the ELETTRA 2.0 Ring

longitudinal gradient magnet prototype for APS Upgrade

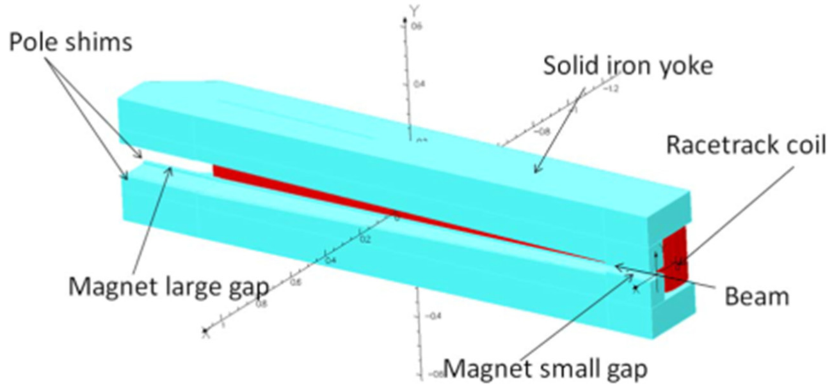
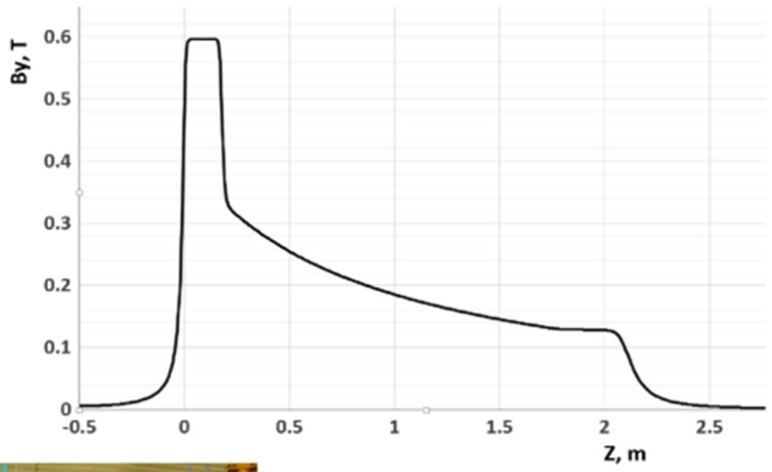
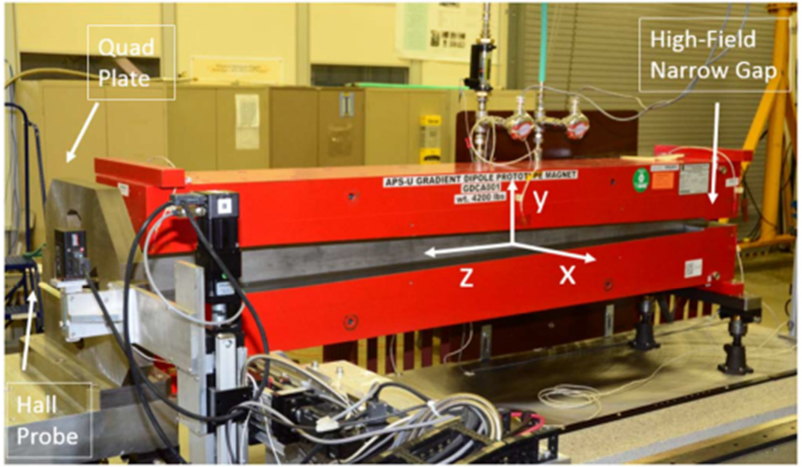


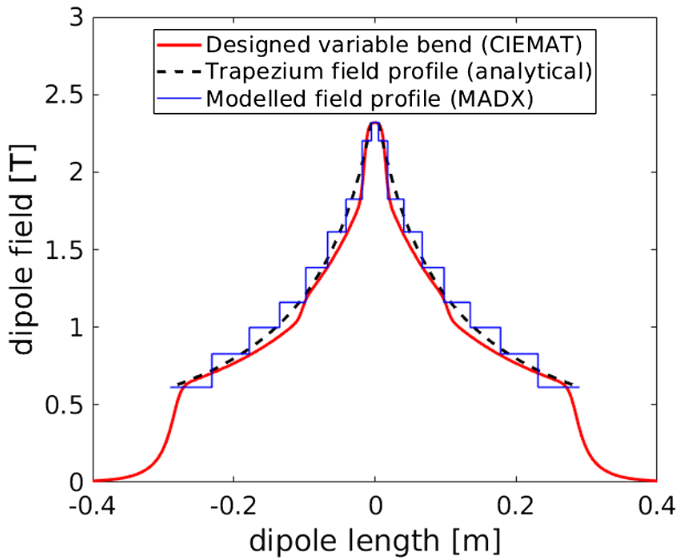
Fig. 2. L-bend dipole magnet geometry.

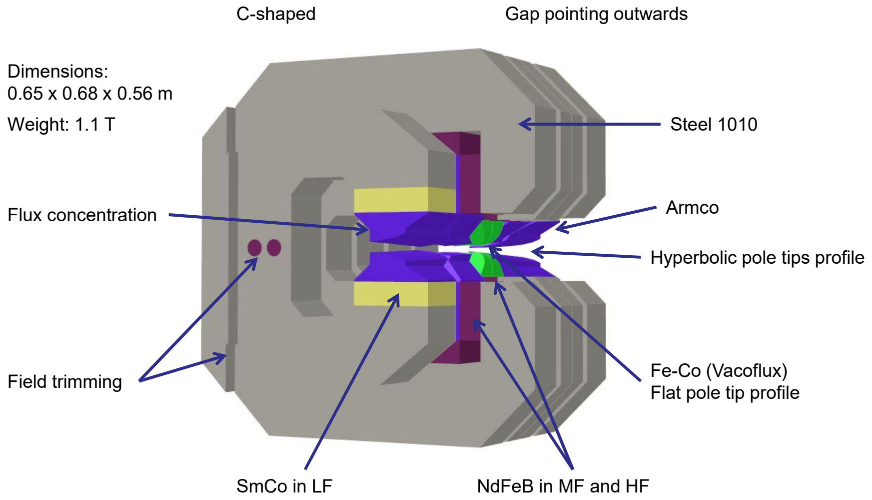




V. S. Kashikhin et al., "Longitudinal Gradient Dipole Magnet Prototype for APS at ANL," IEEE Trans. on Appl. Superc., vol. 26, no. 4, pp. 1–5, 2016

longitudinal gradient magnet for ELETTRA 2.0





demonstrator magnet built at CIEMAT

M. Dominguez Martinez, F. Toral

classical quantum excitation (e.g., M. Sands SLAC-R-121)

radiation integral I_5 in Eq. (1) stems from classical quantum excitation due to photon emission in *constant magnetic field*

$$\Delta\varepsilon_x = \int \dot{N}_{\mathrm{ph}} \langle u^2 \rangle \mathcal{H}_x ds$$

"long-magnet" photon spectrum for single electron passing through magnet with constant field B_0 and length $2L$:

$$\frac{dN_{\mathrm{ph}}}{d\nu} = \frac{4r_e e}{9\hbar}\frac{B_0 2L}{\nu} S\left(\frac{\nu}{\nu_c}\right) \qquad S\left(\frac{\nu}{\nu_c}\right) = \frac{9\sqrt{3}}{8\pi}\frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(x')dx'$$

$$\langle u^2 \rangle = \frac{11}{27}\epsilon_{\mathrm{crit}}^2 \qquad \dot{N}_{\mathrm{ph}} = \left(\frac{15\sqrt{3}}{8}\right)\frac{P_\gamma}{\epsilon_{\mathrm{crit}}} \qquad P_\gamma = \frac{cC_\gamma}{2\pi}\frac{E_b^4}{\rho^2} \qquad C_\gamma = \left(\frac{4\pi}{3}\right)\frac{r_e}{(m_e c^2)^3} \qquad \frac{1}{\rho} = \frac{B_0 e}{p}$$

Note the normalizations: $\int_0^\infty S(u)du = 1$ $\qquad \int_0^\infty \frac{S(u)}{u}\, du = \frac{45}{8\sqrt{3}}$

longitudinal gradient dipoles according to classical formulae

for a magnet with varying dipole field $B_y(s)$, using previous formulae, the magnetic field variation can naively be taken into account as

$$\frac{dN_{\mathrm{ph}}}{d\nu\, ds}(s) = \frac{4r_e e}{9\hbar} \frac{B_y(s)}{\nu} S\left(\frac{\nu}{\nu_c(s)}\right) \qquad \nu_c(s) = \frac{3}{2} c\gamma^3 \frac{B_y(s)e}{p}$$

integrating over the magnet length $2L$ we obtain the classical photon spectrum for this dipole

$$\frac{dN_{\mathrm{ph}}}{d\nu} = \int_{-L}^{+L} \frac{dN_{\mathrm{ph}}}{d\nu\, ds}(s)\, ds$$

$\longrightarrow$ so far the basis for gradient dipole designs

correct if effective magnet length much longer than photon emission length

another, "short" magnet regime

effective "local" magnet length

$$l_{\text{eff}}(s) \approx \left. \frac{B_y(s)}{dB_y(s')/ds'} \right|_{s'=s}$$

emission "source length" of synchrotron radiation

$$l_{\text{source}}(s) \approx \pm \frac{\rho(s)}{\gamma}$$

if $l_{\text{eff}}(s) \leq l_{\text{source}}(s)$ or $\frac{1}{B_y^2(s)} \left| \frac{dB_y(s')}{ds'} \right|_{s'=s} > \frac{e}{m_e c}$

radiation spectrum extends to higher frequencies

for $l_{\text{eff}} \ll l_{\text{source}}$ previous formulae are replaced by

independent of beam energy, but dependent on particle mass !

$$\frac{dN_{\text{ph}}}{d\nu} \approx \frac{r_e e^2 c}{2\pi m_e \hbar \nu} \int_1^{\infty} \frac{y^2 - 2y + 2}{y^4} \left| \tilde{B}\left(\frac{\nu}{2\gamma^2} y\right) \right|^2 dy$$

with

$$\tilde{B}(\nu) = \int_{-\infty}^{\infty} B(t) e^{-i2\pi\nu t} dt$$

a pioneer of the new regime: R. Coïsson, 1979

PHYSICAL REVIEW A VOLUME 20, NUMBER 2 AUGUST 1979

Angular-spectral distribution and polarization of synchrotron radiation from a "short" magnet

R. Coïsson

Istituto di Fisica, Università di Parma, Italy
and Gruppo Nazionale de Struttura della Materia (Consiglio Nazionale della Ricerche), Parma, Italy

(Received 18 January 1979)

Power per unit solid angle, spectrum and polarization as a function of angle, and integrated spectrum are calculated for the radiation from a beam of ultrarelativistic ($\gamma \gg 1$) charged particles in a magnet causing a deflection much smaller than $1/\gamma$, with an arbitrary form of the magnetic field $B(z)$. Some examples are given, and the connection with the "edge effect" is shown.

NUCLEAR INSTRUMENTS AND METHODS 164 (1979) 375-380; © NORTH-HOLLAND PUBLISHING CO.

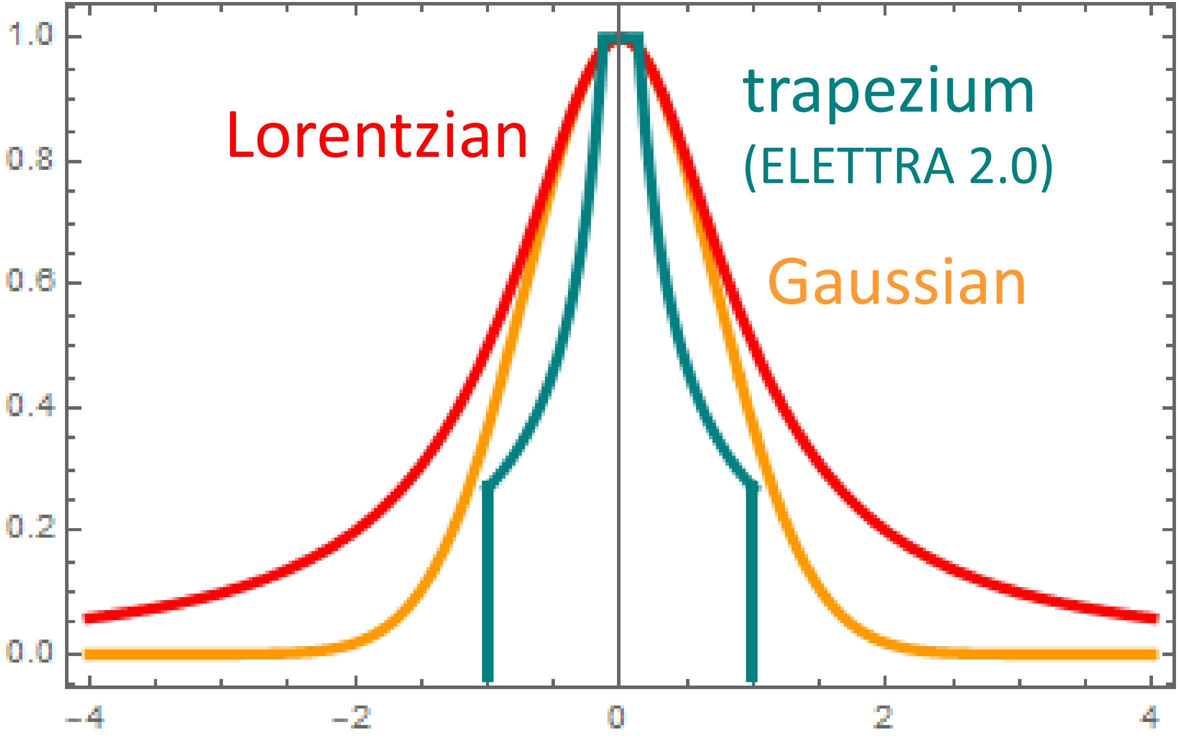
OBSERVATION OF VISIBLE SYNCHROTRON RADIATION EMITTED BY A HIGH-ENERGY PROTON BEAM AT THE EDGE OF A MAGNETIC FIELD

R. BOSSART, J. BOSSER, L. BURNOD, R. COISSON*, E. D'AMICO, A. HOFMANN and J. MANN

CERN, Geneva, Switzerland

Received 22 March 1979

example field profiles



$$B_y(s) = \frac{B_0}{1 + s^2/L^2}$$

$$B_y(s) = B_0 e^{-z^2/L^2}$$

$$B_y(s) = \frac{p}{e\rho(s)}$$ with

$$\rho(s) = \rho_1 + \frac{(L_1 - |s|)(\rho_1 - \rho_2)}{L_2}$$

general short magnet spectrum (Coïsson)

$$\frac{dN_{\mathrm{ph}}}{d\nu} \approx \frac{r_e e^2 c}{2\pi m_e \hbar \nu} \int_1^\infty \frac{y^2 - 2y + 2}{y^4} \left| \tilde{B}\left(\frac{\nu}{2\gamma^2} y\right) \right|^2 dy$$

Lorentzian profile (Coïsson)

$$\frac{dN_{\mathrm{ph}}}{d\nu} = \frac{\pi r_e e^2 c^2 L^2 B_0^2}{2 m_e c^2 \hbar c \nu} \left[\frac{2}{3} e^{-x} \left(1 + x + \frac{x^2}{2}\right) + x \left(1 + x - \frac{x^2}{3}\right) \mathrm{Ei}(-x) \right]$$

$$x = 4\,\nu/\nu_1$$

$$\nu_1 = 2\gamma^2\, c/(\pi L)$$

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{e^{x'}}{x'} dx'$$

Gaussian profile (Coïsson)

$$\frac{dN_{\mathrm{ph}}}{d\nu} = \frac{r_e e^2 c^2 L^2 B_0^2}{2 m_e c^2 \hbar c \nu} \left[\frac{1}{3} e^{-x^2} (1 + 4x^2) + x\sqrt{\pi} \left(1 + \frac{4x^2}{3}\right) \mathrm{erfc}(x) - x^2 \mathrm{Ei}(-x^2) \right]$$

$$x = \sqrt{2}\,\nu/\nu_1$$

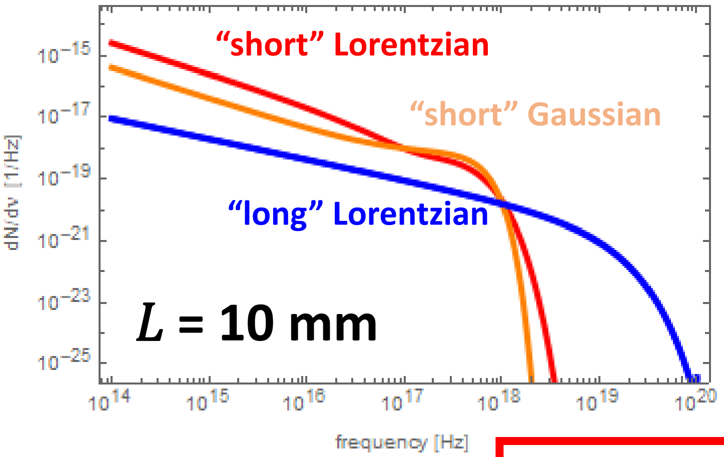
$$\nu_1 = 2\gamma^2\, c/(\pi L)$$

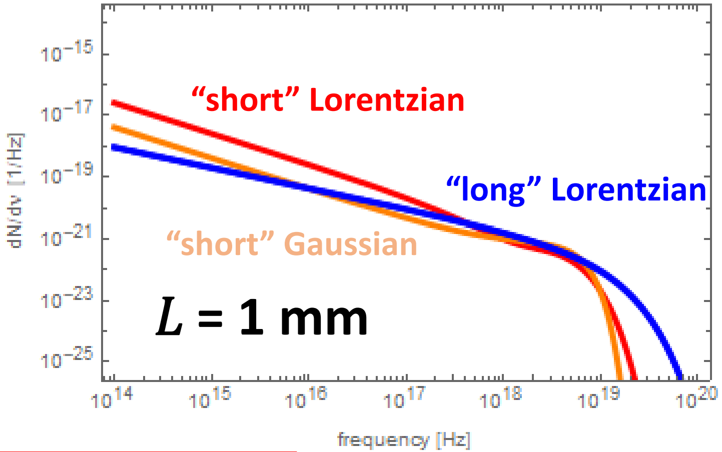
$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-x'^2} dx'$$

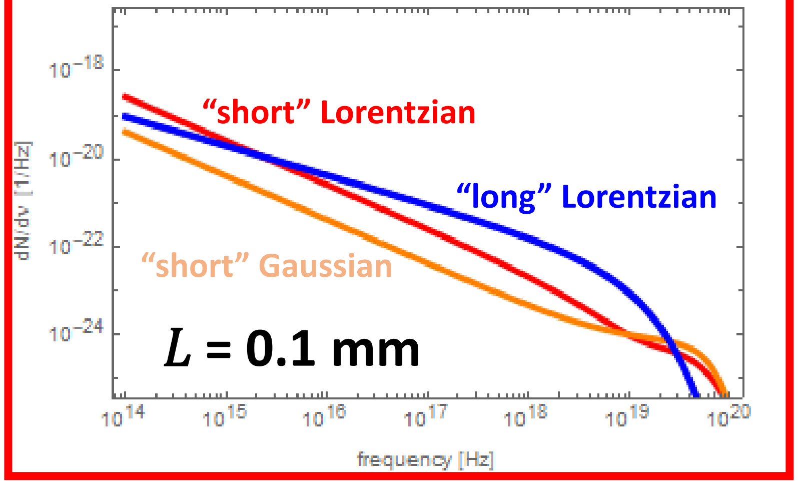
trapezium profile (ELETTRA magnet)

spectrum computed numerically

example spectra: Lorentzian and Gaussian

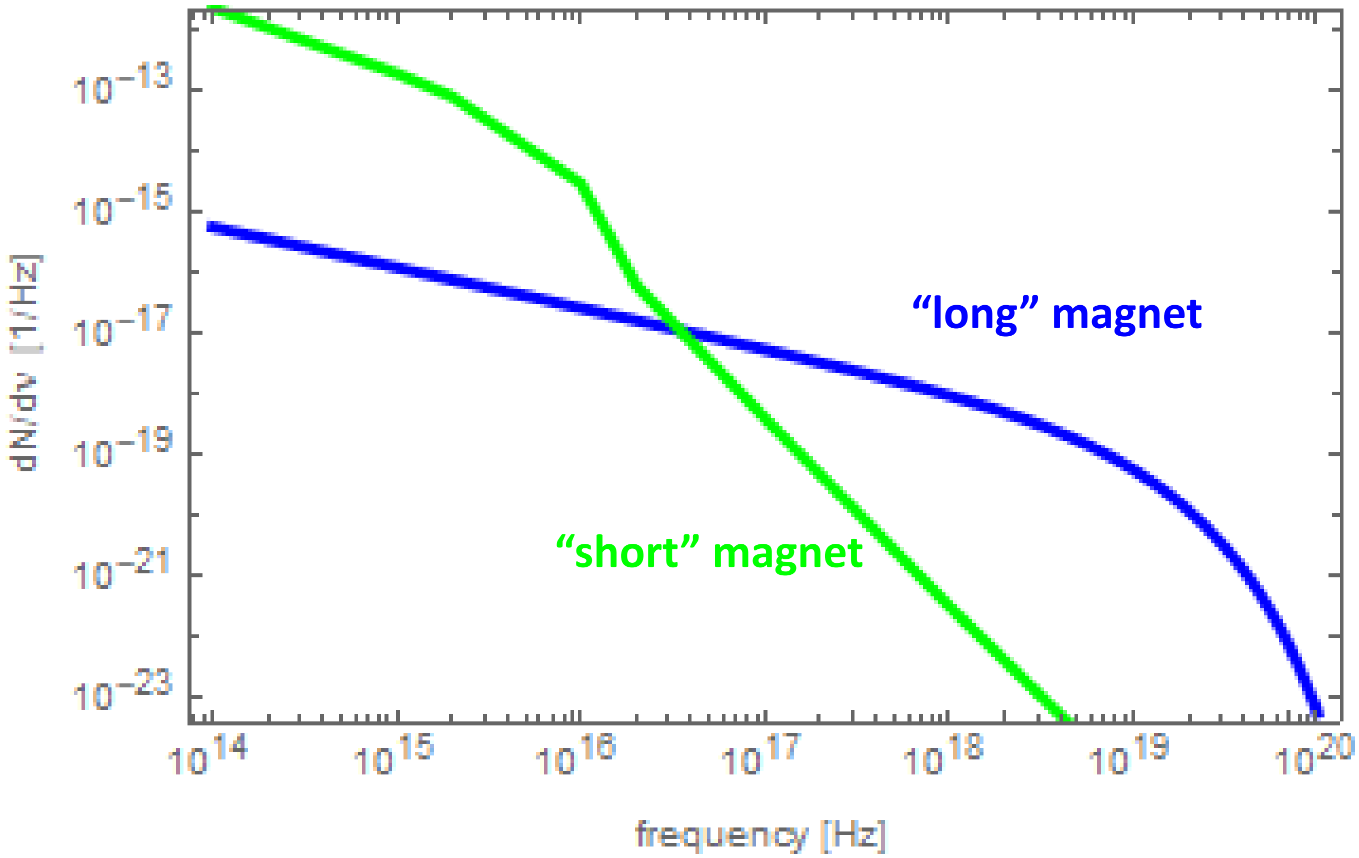






beam energy 3 GeV,
peak field 1.0 T

example spectra: trapezium for ELETTRA 2.0



0.8 m long dipole,
beam energy 2.4 GeV,
peak field 2.32 T

quantum excitation

$$\langle u^2 \rangle N_{\mathrm{ph}} = h^2 \int_0^\infty \nu^2 \frac{dN_{\mathrm{ph}}}{d\nu} d\nu$$

quantum excitation term $\langle u^2 \rangle N_{\mathrm{ph}}$, in units of (eV)^2 for a dipole with peak field B_0 = 1 T and of varying length L, and a 3 GeV e$^-$ beam

L	"long" Lorentzian*	"short" Lorentzian	"short" Gaussian
0.1 mm	$4\text{x}10^3$	$6\text{x}10^5$	$9\text{x}10^5$
1 mm	$4\text{x}10^4$	$6\text{x}10^5$	$9\text{x}10^5$
10 mm	$4\text{x}10^5$	$6\text{x}10^5$	$9\text{x}10^5$
100 mm	$4\text{x}10^6$	$6\text{x}10^5$	$9\text{x}10^5$

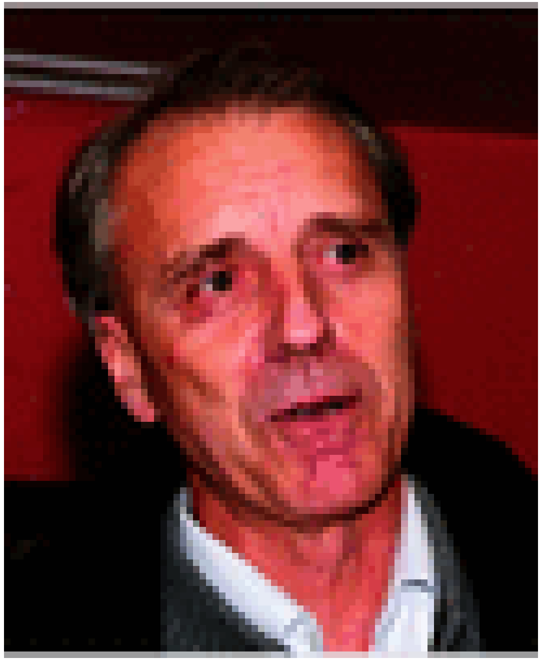
For the short magnet spectra, the quantum excitation is independent of L, whereas for the long-magnet formula it is proportional to the length of the magnet $2L$. This can also be directly seen by inspecting the spectral formula. **The quantum excitation terms are approximately equal for L ≈14 mm.**

*The long-magnet values for the Gaussian are lower than for the Lorentzian by a factor ~3.7/4.0.

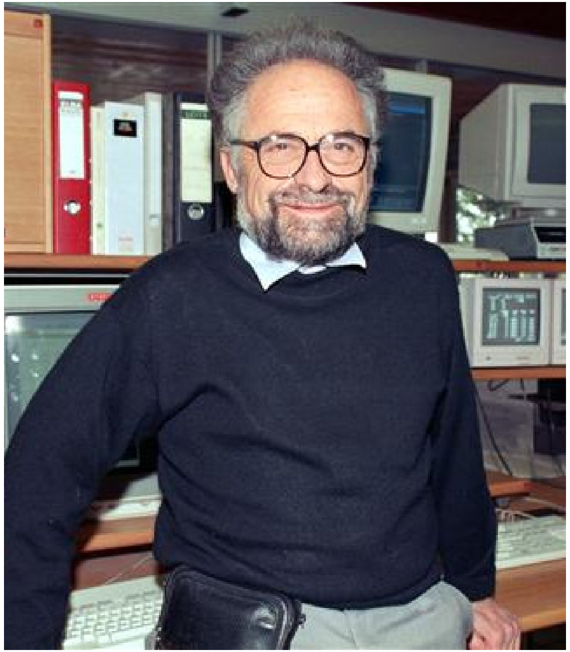
Conclusions and Outlook

- for e^-, if magnetic field changes over a length of < ~1 cm , quantum excitation can be much larger than naively expected !
- this effect may need to be considered when optimizing magnet field profiles for future extreme electron storage rings
- for protons, replacing m_e by m_p, the quantum excitation already important for field changes over a few metres → equilibrium emittance for future highest-energy hadron storage rings, such as the 100 TeV collider FCC-hh
- here only the two limiting cases of “long” and “short” magnets, respectively
- more accurate total emission spectra for arbitrary magnetic field shape, valid also in the transition region between “short” and “long” magnets, could be obtained numerically by starting from the Liénard-Wiechert retarded potentials and/or from the theory of Schwinger or others

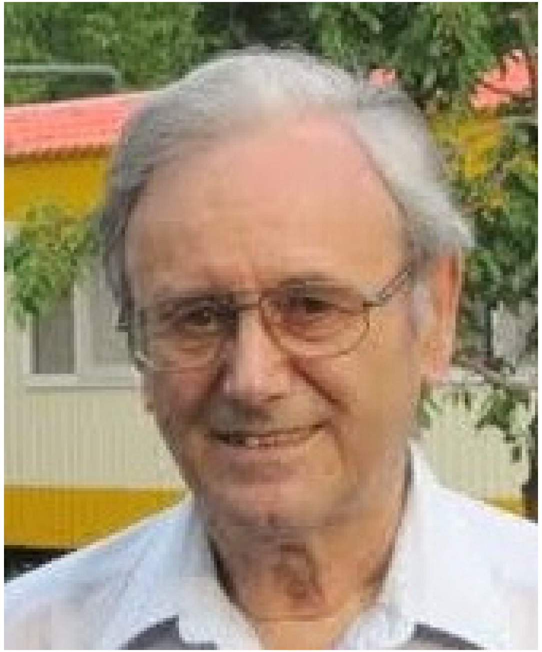
in the past I would have consulted with one of these three synchrotron radiation experts, who sadly left us much too early



Pascale Elleaume
(1955–2011+)
+French Alpes



Albert Hofmann
(1933–2018+)
+Geneva, Switzerland



Helmut Wiedemann
(1938–2020+)
+Chiang Mai, Thailand

PBP CMU Electron Linac Laboratory at Chiang Mai University