# INVESTIGATION OF POLARIZED PROTON SPIN COHERENCE TIME AT STORAGE RINGS 

A. Melnikov*, A. Aksentyev ${ }^{1}$, Y. Senichev,<br>Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia<br>E. Syresin, Joint Institute for Nuclear Research (JINR), Dubna, Russia<br>${ }^{1}$ also at National Research Nuclear University "MEPhI," Moscow, Russia

## Abstract

The idea of the Electric Dipole Moment (EDM) search using the storage ring with polarized beam demands long Spin Coherence Time (SCT). It is the time during which the RMS spread of the orientation of spins of all particles in the bunch reaches one radian. Long SCT is needed to observe a coherent effect on polarization induced by the EDM. The possibility of getting a 1000 s SCT for deuterons has been shown experimentally at COoler SYnchrotron (COSY), accelerator at FZJ Jülich, Germany. Reaching high values of SCT for protons is more challenging due to a higher anomalous magnetic moment. Obtaining sufficient proton SCT is obligatory for planned EDM search experiments at COSY and the ProtoType EDM Ring (PTR). It has been shown that the second order momentum compaction factor $\left(\alpha_{1}\right)$ has to be optimized along with chromaticities $\left(\xi_{x, y}\right)$ to get high SCT. Three families of sextupoles have to be used. The optimal values of chromaticities and $\alpha_{1}$ are discussed. The racetrack option of PTR is investigated.

## PRINCIPLES OF SCT OPTIMIZATION

To study the effects of spin decoherence one has to observe spin precession component orthogonal to the invariant axis. The in-plane spin precession is governed by the TBMT equation and in a storage ring with magnetic bending the spin-tune is $v_{s}=G \gamma$, where $G$ is anomalous magnetic moment. For non-reference particles the spin-tune is determined by the oscillation amplitudes in the phase space and lattice parameters such as chromaticities.

The first step in optimization of SCT, referring to [1], is turning on an RF cavity to suppress the first order spindecoherence. It helps to increase SCT about three orders of magnitude.

In the presence of an RF cavity longitudinal motion is nonlinear in general case. The solution of nonlinear equations for the principle of synchronous acceleration gives the rise of equilibrium (average) energy level $\Delta \delta_{e q}[1]$ :

$$
\begin{align*}
\Delta \delta_{e q}=\frac{\gamma_{s}^{2}}{\gamma_{s}^{2} \alpha_{0}-1} & {\left[\frac{\delta_{m}^{2}}{2}\left(\alpha_{1}+\frac{3}{2} \frac{\beta_{s}^{2}}{\gamma_{s}^{2}}-\frac{\alpha_{0}}{\gamma_{s}^{2}}+\frac{1}{\gamma_{s}^{4}}\right)\right.} \\
& \left.+\left(\frac{\Delta L}{L}\right)_{\beta}\right] \tag{1}
\end{align*}
$$

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where betatron orbit lengthening term is:

$$
\begin{equation*}
\left(\frac{\Delta L}{L}\right)_{\beta}=-\frac{\pi}{L_{0}}\left[\epsilon_{x} \xi_{x}+\epsilon_{y} \xi_{y}\right] \tag{2}
\end{equation*}
$$

Here $\alpha_{0}$ and $\alpha_{1}$ is a first and second order momentum compaction factor. $\gamma_{s}$ is a Lorentz factor and $\beta_{s}$ is a relativistic beta factor for the synchronous particle. $\delta_{m}$ is an amplitude of synchrotron oscillations in $\frac{\Delta p}{p} . \xi_{x, y}$ are beam chromaticities and $\epsilon_{x, y}$ are the Courant-Snyder invariants.

The shift of equilibrium energy level for different values of $\alpha_{1}$ is presented in Fig. 1. The calculations were done for the case of synchrotron motion without betatron oscillations.


Figure 1: Phase trajectories in longitudinal plane for different values of $\alpha_{1} ; \xi_{x, y}=0 . \frac{\Delta K}{K}$ is a relative kinetic energy deviation.

From (1) and (2) it can be seen that $\xi_{x}, \xi_{y}, \alpha_{1}$ have to be optimized to influence nonlinear longitudinal and spin motion. Three sextupole families need to be placed at points with different optical functions and dispersion to optimize these three parameters. Denoting the ansatz in Eq. (1) at $\delta_{m}^{2}$ as $\kappa$, one can say that the sextupoles have to be tuned to zero $\xi_{x}, \xi_{y}, \kappa$ to achieve long SCT. This comes from the fact that spin-tune deviation is determined by equilibrium energy level shift:

$$
\begin{equation*}
\Delta v_{s}=G \Delta \gamma_{e q} . \tag{3}
\end{equation*}
$$

The investigation of this concept is presented below.

## SCT OPTIMIZATION AT COSY

COSY is a racetrack synchrotron providing unpolarized and polarized protons and deuterons in the momentum range from $300 \mathrm{MeV} / \mathrm{c}$ up to $3.7 \mathrm{GeV} / \mathrm{c}$. The typical optical setting with zero dispersion for the experiment is depicted in Fig. 2. The polarization of the injected beam is flipped from vertical into the ring plane with an rf-solenoid. And SCT is determined as the decay time of the in-plane polarization signal of particle ensemble. Three sextupole families: MXS, MXL and MXG are used for SCT optimization [2].


Figure 2: Layout of COSY Twiss functions. Dashed lines indicate the locations of three sextupole families in the arcs.

## Investigation of Spin Dynamics

In this section spin-tracking simulation results are presented to verify the theoretical predictions for the optimal values of $\xi_{x}, \xi_{y}$ and $\kappa$ coming from Eq. (3). Calculations are made with COSY Infinity code [3].

First the dependence of spin-tune on $\xi_{x}, \kappa$ for nonreference particles was investigated. Here the amplitude dependent spin-tune was computed with the normal form algorithm. The results from Figures 3 and 4 confirm that optimal $\xi_{x}, \kappa=0$.


Figure 3: Spin-tune deviation for non-reference particles with different horizontal betatron amplitudes.

However, vertical betatron motion is subject to the influence of spin resonances, hence optimal $\xi_{y} \neq 0$ [4]. For protons there are several intrinsic resonances in the entire momentum range of COSY with the strongest one: $\gamma G=8-Q_{y}$ (Fig. 5). The working point for longest SCT corresponds


Figure 4: Spin-tune deviation for non-reference particles with different energies.
to the point where $\Delta v_{s}=0$ for different betatron amplitudes. It can be adjusted by varying $\xi_{y}$ so that the location of zero crossing corresponds to the energy or $\gamma G$ of the experimental setup. The results from Figure 6 show that positive vertical chromaticity shifts the working point to lower energies.

It has to be pointed out that normal form algorithm diverges near spin resonances due to small denominators. And the spin-tune was calculated by tracking particles and averaging one turn spin phase advance during several RF periods The influence of imperfection resonances was not taken into account in this study.


Figure 5: Spin-tune deviation for non-reference particles with different vertical betatron amplitudes; $\xi_{y}=0$.


Figure 6: Spin-tune deviation for non-reference particles with different vertical betatron amplitudes; $\xi_{y}=6.5$.

## SCT OPTIMIZATION AT PROTOTYPE EDM STORAGE RING

The PTR is a 45 MeV proton storage ring with combined E+B cylindrical bending elements [5]. Combined elements were chosen to allow for frozen spin operation. In this mode EDM signal is observed as a buildup of vertical polarization for a coherent ensemble of longitudinally polarized particles.

The PTR has a superperiodicity $P=4$ and "weak" focusing in the vertical plane, $Q_{y} \sim 0.1 \div 1.6$ (Fig. 7), to allow for a substantial spatial separation of the two beams by residual radial magnetic fields. Compensating this separation means mitigating systematic effects, which is crucial for the EDM experiment. For this purpose the PTR supports clockwise and counterclockwise injection of proton beams after the procedure of magnetic field reversal.


Figure 7: Layout of one superperiod of PTR with $P=4$ with optical functions. Dashed lines indicate the locations of sextupole families.

## Racetrack Option of PTR

At the PTR there are three sextupole families placed on top of the quadrupoles. As it can be seen from Fig. 7, the phase advance between the first two sextupoles is quite small.


Figure 8: Layout of one superperiod of a racetrack option of PTR with $P=2$ with optical functions. Dashed lines indicate the locations of sextupole families.

To allow for efficient manipulation of three parameters: $\xi_{x}, \xi_{y}, \kappa$ and to achieve long SCT a racetrack option of PTR is proposed (Fig. 8). In this lattice the superperiodicity is reduced to $P=2$ to insert three families of sextupoles at points with different ratios of optical functions and dispersion. Dispersion-free straight sections are also favorable in terms of beam dynamics.

Spin-tracking simulation results for the PTR lattice with $P=4$ show that with two families of sextupoles it is possible to achieve proton SCT $\sim 100 \mathrm{~s}$. While for a racetrack option with three families of sextupoles it comes to $\sim 1000 \mathrm{~s}$. That is sufficient for the purpose of the EDM experiment.

## CONCLUSION

The numerical calculations prove that SCT is influenced by three parameters: $\xi_{x}, \xi_{y}, \kappa$. The optimal values for $\xi_{x}, \kappa$ are zero, that matches the theoretical predictions. However, for protons optimal $\xi_{y}$ is not zero due to the influence of intrinsic resonances.

To optimize $\xi_{x}, \xi_{y}, \kappa$ one needs three families of sextupoles with substantial phase advance between them. This concept is realized in the proposed racetrack option of PTR. While the initial design of the PTR is based on the idea of mitigating systematic effects observing vertical separation of two beams, "weak" vertical focusing and less ability to increase SCT come as a drawback. The racetrack option of PTR is more suitable in achieving high SCT. Detailed study on how both lattices can handle systematic effects is foreseen.

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## REFERENCES

[1] Y. Senichev, R. Maier, D. Zyuzin, and N. V. Kulabukhova, "Spin Tune Decoherence Effects in Electro- and Magnetostatic Structures", in Proc. IPAC'13, Shanghai, China, May 2013, paper WEPEA036, pp. 2579-2581.
[2] G. Guidoboni et al., "How to Reach a Thousand-Second inPlane Polarization Lifetime with $0.97-\mathrm{GeV} / \mathrm{c}$ Deuterons in a Storage Ring," Phys. Rev. Lett., vol. 117, no.5, 2016, 054801.
[3] COSY INFINITY, cosyinfinity.org
[4] M. Rosenthal, "Experimental Benchmarking of Spin Tracking Algorithms for Electric Dipole Moment Searches at the Cooler Synchrotron COSY", Ph.D. thesis, Phys. Dept., RWTH Aachen University, Aachen, Germany, 2016.
[5] F. Abusaif et al., "Storage Ring to Search for Electric Dipole Moments of Charged Particles - Feasibility Study", CERN, Geneva, Switzerland, Rep. CERN-2021-003, Vol. 3, 2021.


[^0]:    * alexei.a.melnikov@gmail.com

