ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOPT036

DEPENDENCE OF BEAM SIZE GROWTH ON MACRO-PARTICLE'S INITIAL ACTIONS IN STRONG-STRONG BEAM-BEAM SIMULATION FOR THE ELECTRON-ION COLLIDER*

Y. Luo†, D. Xu, J. S. Berg, M. Blaskiewicz, W. Fischer, X. Gu, J. Kewisch, H. Lovelace III, C. Montag, S. Peggs, V. Ptitsyn, F. Willeke, Brookhaven National Laboratory, Upton, NY, USA
Y. Hao, Facility for Rare Isotope Beams, Michigan State University, East Lansing, MI, USA
J. Qiang, Lawrence Berkeley National Laboratory, Berkeley, CA, USA
G. Hoffstaetter, Cornell University, Ithaca, NY, USA
R. Gamage, H. Huang, V. Morozov, E. Nisson, T. Satogata,
Thomas Jefferson National Accelerator Facility, Newport News, VI, USA

Abstract

The Electron-Ion Collider (EIC) presently under construction at Brookhaven National Laboratory will collide polarized high energy electron beams with hadron beams with luminosities up to $1\times10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ in the center mass energy range of 20-140 GeV. We simulated the planned electron-proton collision of flat beams with Particle-In-Cell (PIC) based Poisson solver in strong-strong beam-beam simulation. We observed a much larger proton emittance growth rate than that from weak-strong simulation. To understand the numerical noises further, we calculate the beam size growth rate of macro-particles as function of their initial longitudinal and transverse actions. This method is applied to both strong-strong and weak-strong simulations. The purpose of this study is to identify which group of macro-particles contributes most of the artificial emittance growth in strong-strong beam-beam simulation.

INTRODUCTION

The Electron-Ion Collider (EIC) presently under construction at Brookhaven National Laboratory will collide polarized high energy electron beams with hadron beams with luminosities up to $1 \times 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ in center mass energy range of 20-140 GeV [1]. We focus on the collision between 275 GeV protons and 10 GeV electrons since both protons and electrons reach their highest beam-beam parameters for this collision mode in the EIC. Table 1 lists the beam-beam related design parameters for this study.

Both strong-strong and weak-strong models are used for EIC beam-beam interaction simulation studies [2, 3]. For weak-strong model, the electron bunch is assumed rigid and the proton bunch is represented with macro-particles. The beam-beam kick to proton macro-particles are analytically calculated. For strong-strong model, both bunches are represented with macro-particles. Particle-in-cell (PIC) method and Fast Fourier Transformation (FFT) are used to solve 2-d Poisson equation on rectangle grids. The charge of each macro-particle is deposited onto nearest 9 grids and the beam-beam kick to each macro-particle is interpolated from the potentials on those nearest 9 grids.

Table 1: Beam-Beam Related Machine and Beam Parameters for Collision between 275 GeV Protons and 10 GeV Electrons

quantity	unit	proton	electron
Beam energy	GeV	275	10
Bunch intensity	10^{11}	0.668	1.72
(β_x^*, β_y^*) at IP	cm	(80, 7.2)	(55, 5.6)
Beam sizes at IP	μm	(95, 8.5)	
Bunch length	cm	6	0.7
Energy spread	10^{-4}	6.8	5.8
Transverse tunes		(0.228, 0.210)	(0.08, 0.06)
Longitudinal tune		0.01	0.069

Strong-strong simulation is subject to numerical noises due to limited macro-particles, transverse grids, longitudinal slices, and the algorithm itself. For the EIC beam-beam simulation, we observed a much larger proton emittance growth rate than that from weak-strong simulation. Through converging studies, the emittance growth rate from strong-strong simulation can be be reduced with increased macro-particle number.

To better understand the sources of numerical noises in the PIC based strong-strong simulation, we carried out extensive studies with the EIC design parameters. In this article, we will study the dependence of beam size growth rate on the longitudinal and transverse amplitudes of macro-particles. The goal of this study is to identify which group of macro-particles producing most of the artificial emittance growth in strong-strong simulation.

SIMULATION SETUP

In the strong-strong beam-beam simulation for this study, each bunch is represented by 0.5 million macro-particles. There is only one interaction point. For beam-beam interaction simulation, each bunch is split into longitudinal slices, 15 for the proton bunch, 5 for the electron bunch. Crab cavities are placed on both sides of IP with an exact $\pi/2$ phase advance in the horizontal plane. The ring transfer map is simply represented by a 6-d uncoupled one-turn transfer

* Work supportedh by Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy.
† yluo@bnl.gov

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOPT036

matrix. In this model, beam-beam interaction is the only nonlinear force.

In the simulation, we group proton macro-particles according to their initial transverse or longitudinal actions in units of σ,

$$N_{x,y,z} = \sqrt{J_{x,y,z}/J_{x,y,z,rms}}, \quad (1)$$

Here $J_{x,y,z}$ are the actions for the horizontal, vertical, and longitudinal motions, $J_{x,y,z,rms}$ are their RMS values. The emittances are given by $2J_{x,y,z,rms}$. For the transverse plane, we define a normalized transverse action in unit of σ_r,

$$N_t = \sqrt{N_x^2 + N_y^2}. \quad (2)$$

During the regular strong-strong beam-beam simulation, we calculate and record the geometric transverse RMS sizes for each group of macro-particles, which is given by, for example in the horizontal plane,

$$\sigma_x = \sqrt{\sum (x_i - < x >)^2 / M_p}. \quad (3)$$

Here $< x >$ is the mean of all particles' position and M_p is number of macro-particles in one group.

We simply calculate each group's beam size growth rate with a linear fitting between 20k to 50k turns. The electron beam sizes reach their equilibrium state after 20k turns. We normally extrapolate the relative growth rate $d\sigma_{x,y}/\sigma_{x,y}dt$ to %/hour. In the following, we use the relative growth rate of beam size as an observable to measure the stability of macro-particles in each group.

For a Gaussian distribution bunch, there are much less macro-particles at longitudinal or transverse amplitudes larger than 4σ. The error bar in the growth rate calculation for these amplitudes will be big and can not be trusted.

WITH STRONG-STRONG SIMULATION

First we calculate the beam size growth rate as function of longitudinal amplitude of macro-particles in strong-strong simulation. For the EIC, the full crossing angle is 25 mrad. Crab cavities are used to compensate the geometric luminosity loss and to restore head-on collision. However, due to a relatively short wavelength of crab cavities and a relatively long proton bunch length, protons in the bunch head and bunch tail are not perfectly crabbed. They will feel offset beam-beam interaction and therefore will have faster emittance growth rates than particles in the bunch center.

Figure 1 shows the horizontal beam size growth rate as function of longitudinal amplitude of macro-particles. The horizontal axis is the longitudinal action in unit of σ. The vertical axis is the growth rate in units of %/hour, which we use for qualitative measure of stability of macro-particles. To reduce the statistical error in this study, for each condition, we tracked several seeds of initial macro-particle distributions. The results from different seeds converge well as shown in the plot.

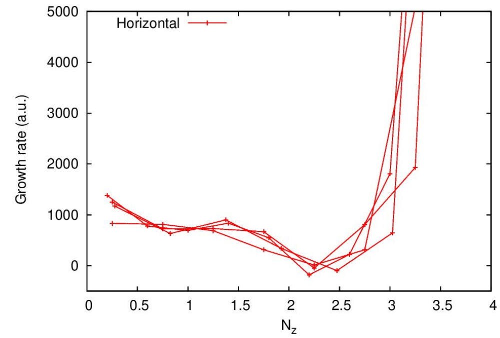

Figure 1: Horizontal beam size growth rate as function of longitudinal action of macro-particles in strong-strong simulation

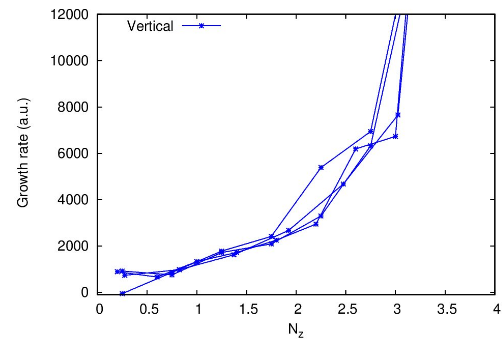

Figure 2: Vertical beam size growth rate as function of longitudinal action of macro-particles in strong-strong simulation

From Fig. 1, the horizontal beam size growth rate decreases with increase in longitudinal amplitude until the longitudinal amplitude reaches $N_z = 2.25$. Then the growth rate of macro-particles begin to increase and quickly go to a huge growth rate beyond $N_z = 3.0$. As discussed above, the growth rate beyond $N_z = 3.0$ can not be trusted due to a large statistical error there.

Figure 2 shows the vertical beam size growth rate as function of longitudinal amplitude of macro-particles. Different from the horizontal beam size, the vertical beam size growth rate increases with the longitudinal amplitude from a small longitudinal action $N_z = 0.5$.

Comparing Figs. 1 and 2, we notice that the vertical beam size growth rates are much greater than the horizontal beam size growth rates. For example, at $3N_z$, the vertical beam size growth rate is about 3-4 times faster than the horizontal beam size growth rate.

Figure 3 shows the horizontal and vertical beam size growth rates as function of transverse amplitude of macro-particles in strong-strong simulation. From the plot, macro-particles with amplitudes less than $1.5N_t$ have much faster growth rates in transverse beam sizes than macro-particles outside the bunch core. This hints that macro-particles in the bunch core contribute most of the emittance growth in strong-strong beam-beam simulation. With the nominal design parameters, there is no beam-beam introduced coherent instability. From beam physics, particles in the bunch core

ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOPT036

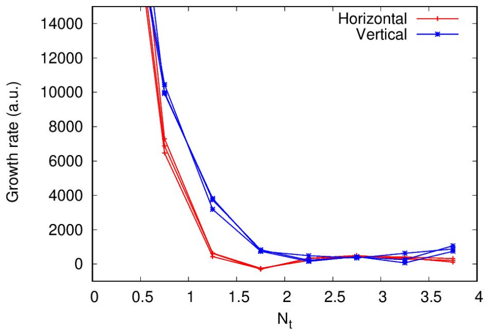

Figure 3: Horizontal and vertical beam size growth rates versus initial transverse action from strong-strong simulation

normally tend to be more stable than particles with larger transverse amplitudes. Also from the plot, the vertical beam size growth rates of macro-particles in the bunch core are at least 2-3 times higher than the horizontal growth rates.

COMPARED TO WEAK-STRONG MODEL

For comparison, we go through the same procedure in weak-strong simulation to calculate the dependence of beam size growth rate as function of initial longitudinal or transverse amplitude of macro-particles. We developed another beam-beam simulation model. In this model, we first perform a regular strong-strong simulation. After the electron bunch reaches its equilibrium state, we freeze its charge distribution and potentials on grids. Then we continue an extended weak-strong tracking with the frozen charge distribution and potentials. We call this 'frozen model'.

Figure 4 compares the vertical beam size growth rate as function of longitudinal action of macro-particles from three models. For both weak-strong and frozen models, the vertical beam size growth rate stays at a very low level until the longitudinal amplitude reaches $2.25N_z$. Then the growth rate begins to increase before it quickly takes off at 3.25 N_z due to statistical error. From the plot, the growth rates from the PIC based strong-strong simulation is much higher than both weak-strong model and frozen model.

Figure 5 compares the vertical beam size growth rate as function of transverse action of macro-particles with above three models. For weak-strong and frozen models, the vertical beam size growth rate stays at a very low level up to $4N_t$. However, for strong-strong model, macro-particles in the bunch core have a huge growth rate than macro-particles outside the bunch core. The macro-particles in the bunch core contribute most of the artificial emittance growth observed in strong-strong simulation. We found that PIC based Poisson solver generates a larger numerical variation in beam-beam force calculation for macro-particles in the bunch core, which will be discussed in our another article in these proceedings [4].

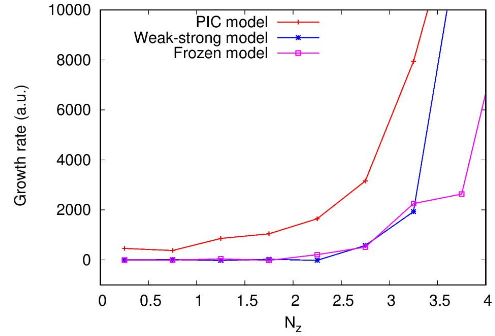

Figure 4: Comparison of beam size growth rates of macro-particles as function of longitudinal action from three models

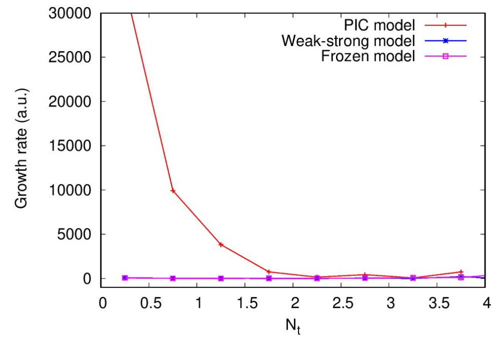

Figure 5: Comparison of beam size growth rates of macro-particles as function of transverse action from three models

SUMMARY

In the EIC beam-beam simulation, we observed a much larger emittance growth rate with PIC-based strong-strong beam-beam simulation than that from weak-strong simulation. To understand the numerical noises in strong-strong simulation, we calculated and compared the beam size growth rate as function of initial longitudinal and transverse actions of macro-particles. We found that macro-particles with smaller transverse amplitudes have a much larger transverse beam size growth rate than macro-particles outside the bunch core, which is not seen in the weak-strong simulation. We conclude that macro-particles in the bunch core contribute most of the artificial emittance growth observed in the strong-strong simulation for the EIC.

REFERENCES

[1] C. Montag *et al.*, "Electron-Ion Collider Design Status", presented at the IPAC'22, Bangkok, Thailand, Jun. 2022, paper WEPOPT044, this conference.

[2] J. Qiang *et al.*, " A parallel particle-in-cell model for beam–beam interaction in high energy ring colliders", *J. Comput. Phys.*, vol. 198, p. 278, 2004.

[3] Y. Luo, "SimTrack: A compact c++ code for particle orbit and spin tracking in accelerators", *Nucl. Instrum. Meth. Sec. A.*, vol. 801, pp. 95-103, 2015.

[4] Y. Luo *et al.*, "Numerical Noise Error of Particle-In-Cell Poisson Solver for a Flat Gaussian Bunch", presented at the IPAC'22, Bangkok, Thailand, Jun. 2022, paper WEPOPT040, this conference.