Two-dimensional Beam-Beam invariant with applications to HL-LHC

2D Nonlinear invariant for beam-beam

We present a formula for the geometric distortion of the two-dimensional nonlinear (Courant Snyder) invariant valid to lowest order of the beam-beam parameter $\lambda$. The $n$-th collision ($n = 1, .., N_{1\pi}$), is described by the coefficients $C_{mk}(a_x, a_y; \theta^{(n)}_{\text{str}})$ in the Fourier-expansion of the long-range beam-beam Hamiltonian, written in terms of action-angle coordinates $\vec{J}, \vec{\phi}$ of the unperturbed motion: $H(\vec{a}, \vec{\phi}; \theta^{(n)}_{\text{str}})$. Here $m, k$ are integers, $N_{1\pi}$ is the total number of $1\pi$ around the ring, $\vec{a} = (a_x, a_y)$, where $a_{x,y} \equiv \sqrt{2J_{x,y}/\epsilon}$, are the test particle normalized amplitudes, $\epsilon$ is the emittance and $\theta^{(n)}_{\text{str}}$ are the strong-beam lattice parameters at this (longitudinal) location.

Effective Hamiltonian and invariants $W_x, W_y$

\[
h(\vec{J}, \vec{\phi}, \vec{\mu}) = -\mu_x J_x - \mu_y J_y + S(\vec{J}, \vec{\phi}, \vec{\mu}); \quad (4)
\]
\[
S(\vec{J}, \vec{\phi}, \vec{\mu}) \equiv \lambda \sum_{n=1}^{N_{1\pi}} \sum_{m=-N_c}^{N_c} C_{mk}(a_x, a_y; \theta^{(n)}_{\text{str}}) \times \frac{(m\mu_x + k\mu_y) e^{i(m\phi_x + \mu_x^{(n)}) + i(k\phi_y + \mu_y^{(n)})}}{2 \sin \frac{1}{2}(m\mu_x + k\mu_y)}.
\]

\[
W_x(J_x, \phi_x) \equiv J_x - S_x(J_x, J_y^0, \phi_x, \frac{\pi}{2}),
\]
\[
W_y(J_y, \phi_y) \equiv J_y - S_y(J_x^0, J_y, \frac{\pi}{2}, \phi_y).
\]
Hamiltonian Fourier Coefficients

In previous papers [7], [8], expressions for $C_{mk}$ were presented valid at large amplitudes and large $\sim 12$ normalized separations, as required by the nominal beam-beam layout and a round collision optics in the HL-LHC.

Results

Figure 2: In-plane tracking around the ring ($N_{1x} = 4 \times 18$) of the weak-beam particle using MadX (red dots) and the projected invariants computed with 6 coefficients $N_c = 6$, Eqn. (5) (blue). The particle is launched in either $X$ (left) or $Y$ (right) planes with $a_x (y) = 4, 6, 8$ and 10. It penetrates the strong beam core at $\sim 8 \sigma$. 
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Hamiltonian driving-terms (HDT). Verify wire correctors with HDT
Simultaneous (for all m) cancellation of Hamiltonian driving terms in IR5 in case of the in-plane left-right independent wire correction:

![Graphs showing IR5 Right, IR5 Left, and IR5 with HDT](image)