MODELING SHORT RANGE WAKEFIELD EFFECTS IN A HIGH GRADIENT LINAC

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Summary

The framework

- High brightness **electron beams** from a C-band hybrid photoinjector driving an ICS (Inverse Compton Scattering) source
- Acceleration in a high gradient linac causes **wakefield interaction**: possible phase space quality dilution or instabilities (BBU)

L. Faillace *et al.*, *Beam dynamics for a high field C-band hybrid photoinjector*, this conference.

MILES tracking code

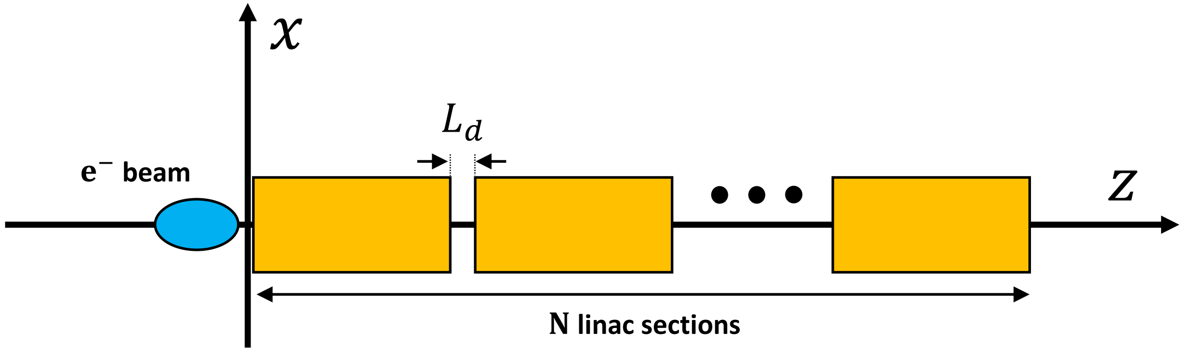
- Development of a tracking code based on **simple models** in order to investigate the effects of transverse wakefields in linacs
- The tool is meant to be **light** and **flexible** **(≲ 1 min. typical runtime)**: inspection of machine's properties finding possible operation limits
- In addition to the applied fields and **dipole wakefields**, the code accounts also for **space charge** forces through the analytical ellipsoidal model

Modeling Instabilities in Linacs with Ellipsoidal Space charge

Basic Beam Dynamics

Linac

- Sequence of **accelerating structures** separated by field-free regions
- Smoothed energy **gradient** $\gamma' = \langle eE_z \rangle / mc^2$
- The **focusing** forces come from non-synchronous RF space harmonics in the SW cavities



Single particle transverse optics

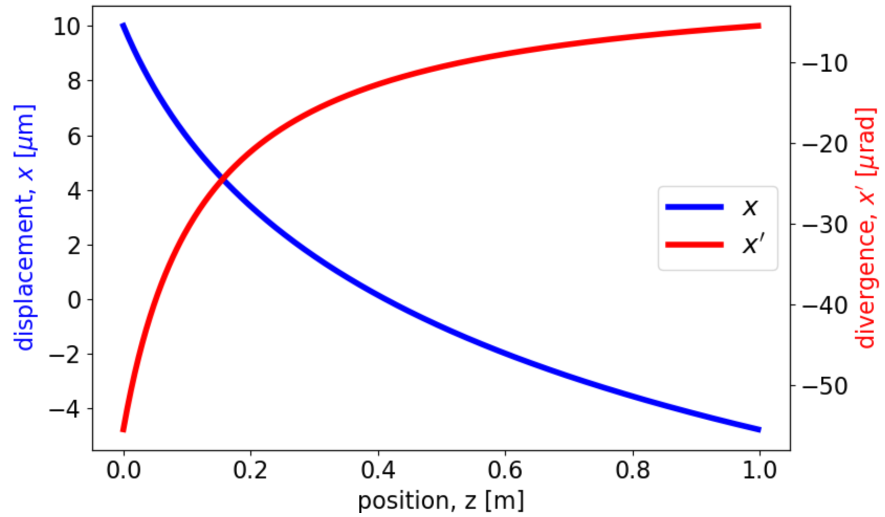
- Radial **kick** from field-free to non-zero field regions and vice versa **(a)**
- Transfer map for an **accelerating cell**, i.e. $\gamma_1 \mapsto \gamma_2$ **(b)**

$$\begin{pmatrix} x \\ x' \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 \\ \mp \frac{\gamma'}{2\gamma} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix} \quad \textbf{(a)}$$

J. Rosenzweig and L. Serafini, *Transverse particle motion in radio-frequency linear accelerators*, Physical Review E 49, 1599 (1994).

$$\begin{pmatrix} x \\ x' \end{pmatrix} \mapsto \begin{pmatrix} \cos\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) & \frac{\gamma_1}{\nu\gamma'} \sin\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) \\ -\frac{\nu\gamma'}{\gamma_2} \sin\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) & \frac{\gamma_1}{\gamma_2} \cos\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix} \quad \textbf{(b)}$$

Input kick + accelerating section: (x, x') vs z

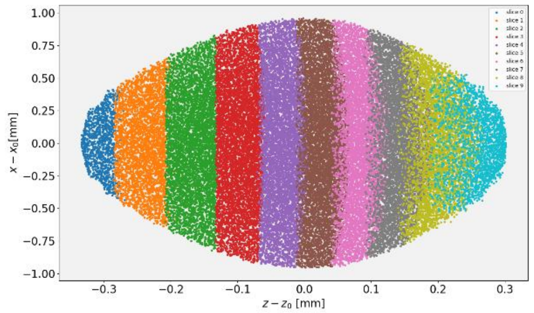


Space Charge Effects

- During its propagation, the beam exhibits a nearly-**ellipsoidal** shape

$$\mathcal{E}: \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$$

$$\begin{cases} a = \sigma_x \sqrt{5} \\ b = \sigma_y \sqrt{5} \\ c = \gamma \sigma_z \sqrt{5} \end{cases}$$



M. Carillo *et al.*, *Three-dimensional space charge oscillations in a hybrid photoinjector*, this conference.

- The **electrostatic field** (beam-frame K') produced by an uniform ellipsoid of charge

$$\mathbf{E}'_\perp(\mathbf{r}) = \hat{x}\ 2A(0)x + \hat{y}\ 2B(0)y$$

O. Kellogg, *Foundation of Potential Theory*. Dover, 1953.

- A Lorentz transformation provides the **transverse force** in the laboratory frame and, hence, the change in transverse momentum

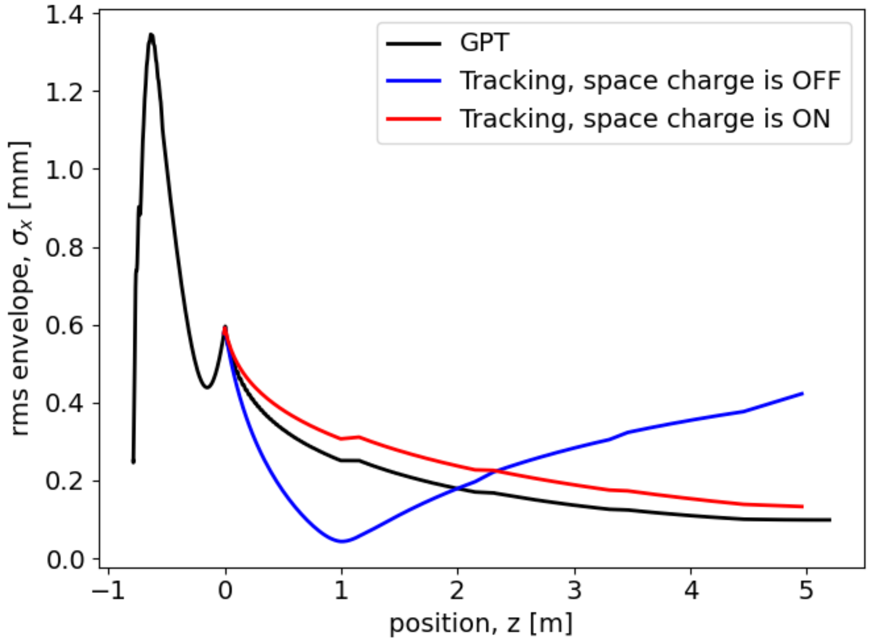
$$\Delta p_x = \int dt F_x \approx \frac{eE'_x}{\gamma'\beta c} \ln\left(\frac{\gamma + \gamma' L_c}{\gamma}\right)$$

where L_c is the cell length and $E'_x \propto x$

An example of tracking with space charge

- Linac starts at $z = 0$
- Comparison with GPT(General Particle Tracer)

Rms beam envelope σ_x vs position z



Without SC: overfocusing
With SC: laminar motion

Wakefield Matrix Formalism

- Fit of the wake-function with a set of superimposed effective **resonant modes** $(\omega_0, R_\perp, Q)$

$$w_\perp(s) = \frac{\omega_0 R_\perp}{Q} e^{-\alpha s} \sin(\omega_n s/c)$$

- Particles' collective interaction described by means of the so-called **wakefield matrix (*)**
- The same formalism allows to account for **long-range** interaction through HOMs in multi-bunch operation

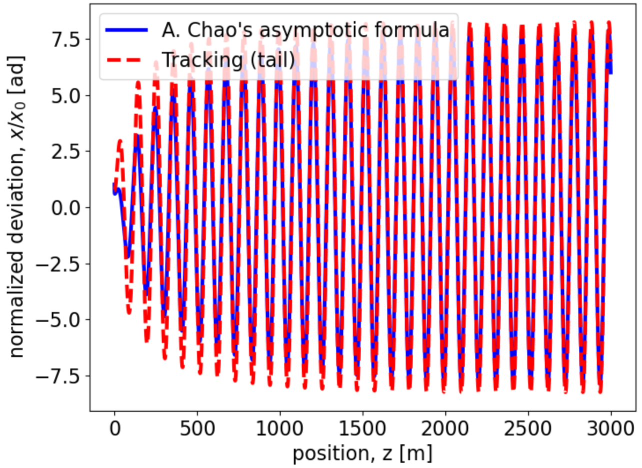
(*) M. Migliorati and L. Palumbo, *Multibunch and multiparticle simulation code with an alternative approach to wakefield effects*, Phys. Rev. ST Accel. Beams 18, 031001 (2015).

M. Migliorati *et al.*, *Instability studies at the CERN Proton Synchrotron during transition crossing*, Phys. Rev. ST Accel. Beams 21, 120101 (2018).

A reference case for BBU:

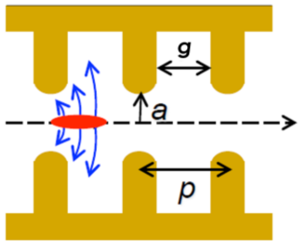
- A. Chao's asymptotic formula(**)
- Linear increase of dipole wake strength from head to tail
- Head is injected off-axis with x_0 offset

Example of tracking with wakefields



(**) A. W. Chao, *Physics of collective beam instabilities in high energy accelerators*. Wiley, New York, 1993.

- Dipole short range **wake-function** for a periodic accelerating structure (Diffraction theory, K. Bane *et al.*)



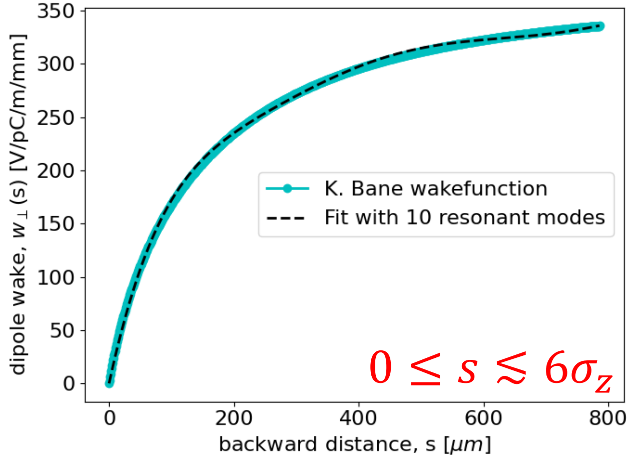
$$w_\perp(s) = \frac{4Z_0 c s_0}{\pi a^4}\left[1 - \left(1 + \sqrt{\frac{s}{s_0}}\right)\exp\left(-\sqrt{\frac{s}{s_0}}\right)\right], s \geq 0$$

$$s_0 = 0.169\frac{a^{1.79} g^{0.38}}{p^{1.17}}$$

a [mm]	g [cm]	p [cm]
2.0	$2.0(<\lambda/2)$	$2.624(\lambda/2)$

K. Bane, "Short-range dipole wakefields in accelerating structures for the NLC," 2003.

K. Bane's wake-function

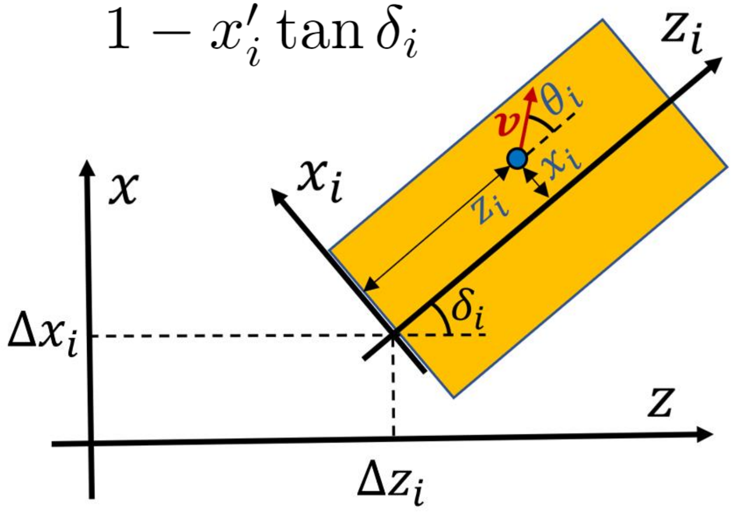


Misaligned Sections

- Misaligned sections can be described by a **local** reference system (x_i, z_i)
- Coordinate **transformations** to the nominal machine axis

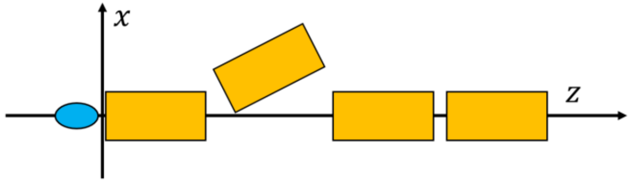
$$\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} \cos\delta_i & \sin\delta_i \\ -\sin\delta_i & \cos\delta_i \end{pmatrix} \begin{pmatrix} x_i \\ z_i \end{pmatrix} + \begin{pmatrix} \Delta x_i \\ \Delta z_i \end{pmatrix}$$

$$x' = \frac{x'_i + \tan\delta_i}{1 - x'_i \tan\delta_i}$$

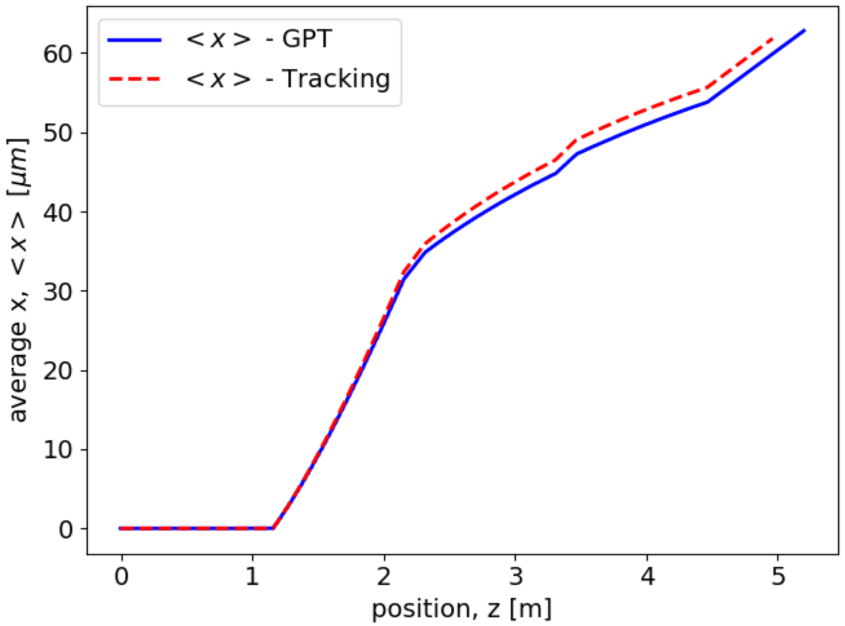


An example of tracking with a misaligned section

- $\Delta x_2 = 50\ \mu\text{m}$
- $\delta_2 = 50\ \mu\text{rad}$
- $\Delta x_i, \delta_i = 0,\ i \neq 2$
- The test focuses on the optics: wakefields are excluded



Average displacement $\langle x \rangle$ vs position z

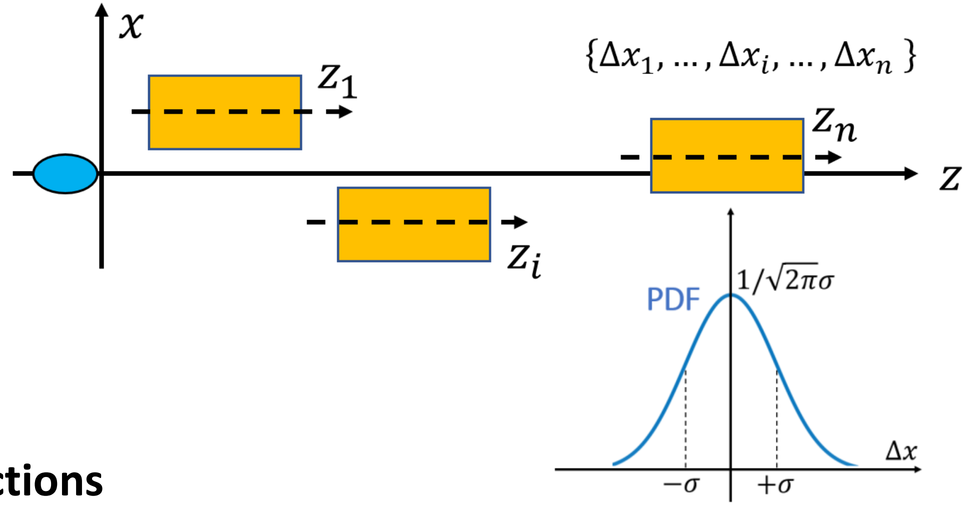


Linac sections with **arbitrary** offset/tilt sequences can be easily investigated

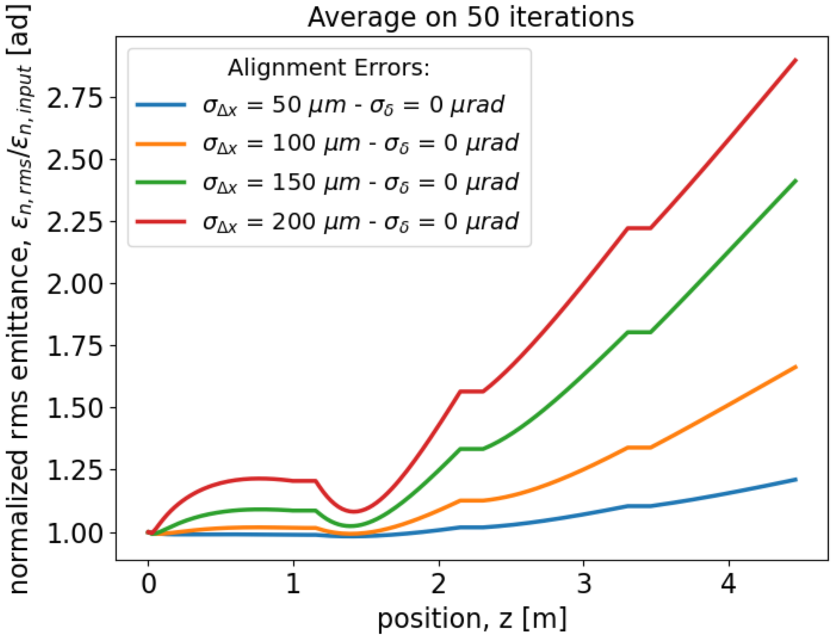
Random Misalignments

Procedure

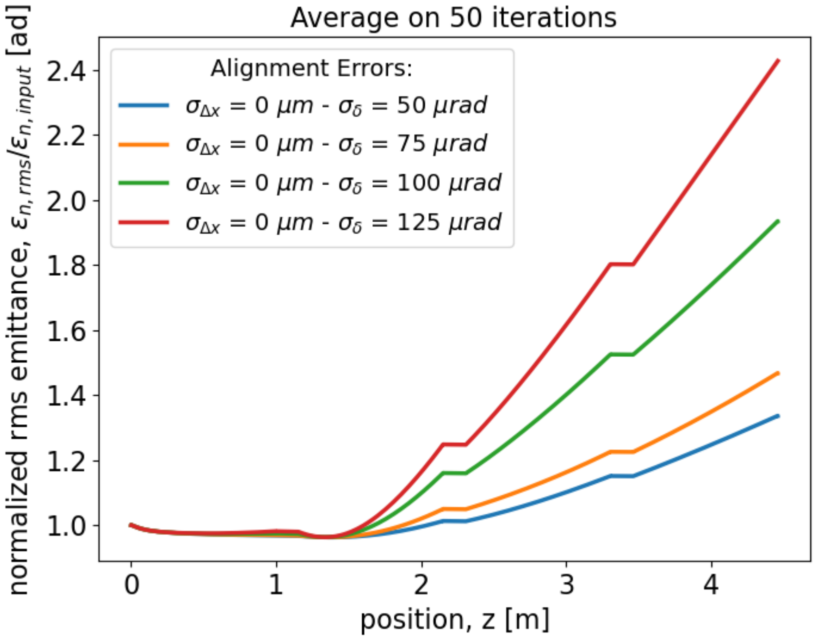
- Assume a gaussian distribution of errors and fix a **standard deviation** $\sigma_{\Delta x}$ (i.e. a characteristic error)
- Perform a large number (e.g. 50) of **simulations** each time using a new set of random misalignments
- Calculate the **average** normalized emittance to mitigate statistical fluctuations



Randomly shifted sections



Randomly tilted sections



- Average on several simulations: **flexibility** is crucial
- Suitable for predictions on **corrector schemes** (beam based alignment)

Conclusions

- MILES describes the **transverse dynamics** for electron beams in linacs accounting for *dipole wakefields* and *space charge* forces
- Based on **simple** models, it requires very short run times
- **Benchmarking** of the code has shown good results when compared with pre-existent tools or models

Further steps

- Code **upgrade** including longitudinal dynamics aspects causing energy spread and chromatic effects
- Investigate **correction (BBA) schemes** to relax tolerance requirements
- Investigate **multi-bunch** operation subjected to long-range wakefield interaction

Thank you for joining!

(Questions are welcome)