Examination of Semi-Analytic Model for Mode Coupling Instabilities

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Square Well Model / Airbag Square Well

- Semi-Analytic model for studying beam dynamics
- Longitudinal dynamics assume airbag beam distribution confined in square potential well
  - Simplified longitudinal systems gives mode coupling
- Quickly solvable for large ΔQ_{sc}
- Possible to generalize this model to a more complex system

SWM Equations of Motion

$$\frac{dx_+}{d\tau} = -\frac{i}{\nu_0}[x_+(\Delta Q_x - \xi\nu_0) + F + (x_+ - x_-)\Delta Q_{sc}/2]$$

$$\frac{dx_-}{d\tau} = \frac{i}{\nu_0}[x_-(\Delta Q_x - \xi\nu_0) + F + (x_- - x_+)\Delta Q_{sc}/2]$$

$$\frac{dF}{d\tau} = -\alpha F + (x_+ + x_-)\kappa W_0/2$$

Burov, Alexey. "Convective instabilities of bunched beams with space charge," *Phys. Rev. Accel. Beams*, vol. 22, p. 034202, 2019.

Blaskiewicz, Michael. "Fast head-tail instability with space charge," *Phys. Rev. ST Accel. Beams*, vol. 1, p. 044201, 1998.

Multi Loop Square Well

MLSW Equations of Motion

$$g_1(z,\dot{z}) = -Q_x^2 + \xi(\dot{z}) + C_{sc}\Sigma_{j=1}^{2m}\rho_j^n$$

$$g_2(t,z) = -C_{sc}\Sigma_{j=1}^{2m}\rho_j^n\overline{\hat{x}}_j^n + \Sigma_{k=1}^{\kappa}\hat{F}_k^n$$

$$\frac{d\bar{\hat{x}}_j^n}{dz} = \frac{i\omega_0}{2Q_x\dot{z}_j^n}[(2Q_x\Delta Q_x\bar{\hat{x}}_j^n + Q_x^2\bar{\hat{x}}_j^n + g_1(z,\dot{z}))\bar{\hat{x}}_j^n + g_2(t,z)]$$

$$\frac{d\hat{F}_k^n}{dz} = \gamma_k\Sigma_{j=1}^{2m}\rho_j^n\overline{\hat{x}}_j^n - \alpha_k\hat{F}_k^n$$

- Possible to expand on SWM/ABS
- Adding Multiple wells and loops of current yield more realistic longitudinal phase space

Multiloop Square Well Phase Space

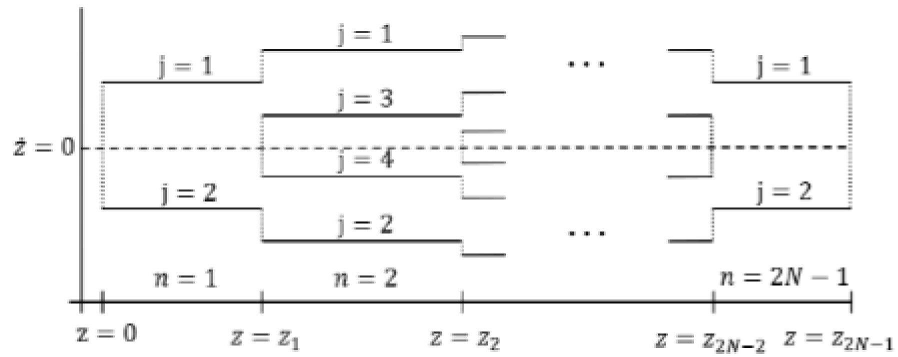

Figure 1: Diagram of arbitrary longitudinal phase space. n corresponds to subdomain and j to half loop of current.

Qualitative Effects of Multiple Loops

- Multiple Wells change dynamics of system
- Exact Changes depend on exact shape of approximated distribution and potential
- Several obvious changes to model
  - Each loop of current increases the number of possible eigenvalue solutions, but can be degenerate
  - Standing waves favored by SWM/ABS harder to drive

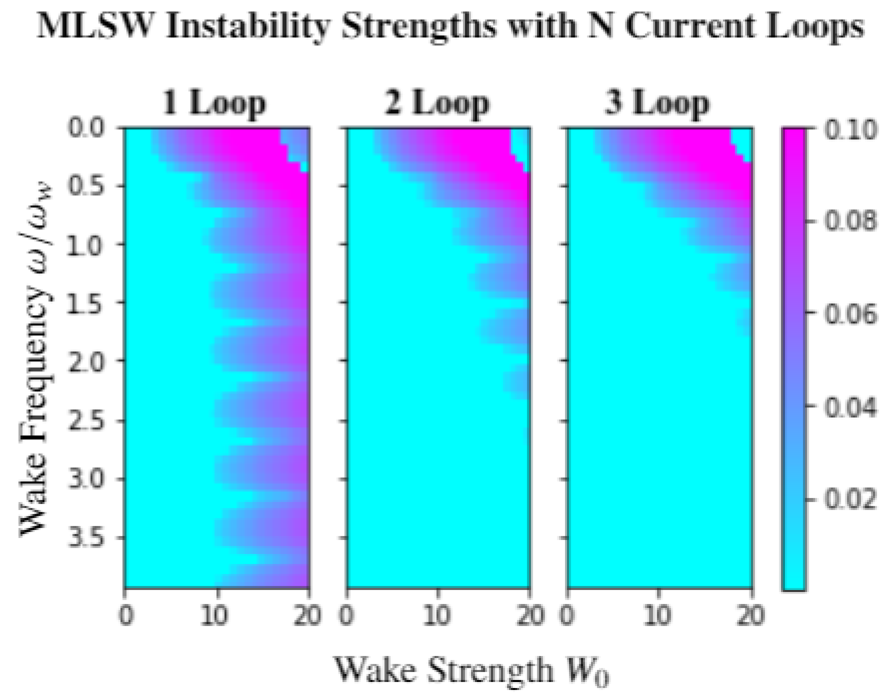

Figure 2: Instability strengths as a function of dimensionless wake strength and frequency. System is modelling an even distribution of longitudinal COM energies centered around $E_{z,COM} = 0$ and a z^2 potential. Higher order modes are more difficult to drive with multiple current loops, satisfying foundational assumptions.

Possible MLSW Convergence

- Complexity of the system increases with N^2 where N is the total number of wells/loops of current
- Important to know how large N must be to get valid solution
- Looks as though 5 wells may be sufficient to model a Gaussian Distribution

M. Blaskiewicz, “A Multipurpose Coherent Instabiliity Simulation Code,”in *Proc. PAC07*, Albuquerque, NM, USA, June 2007, pp. 3690–3692.

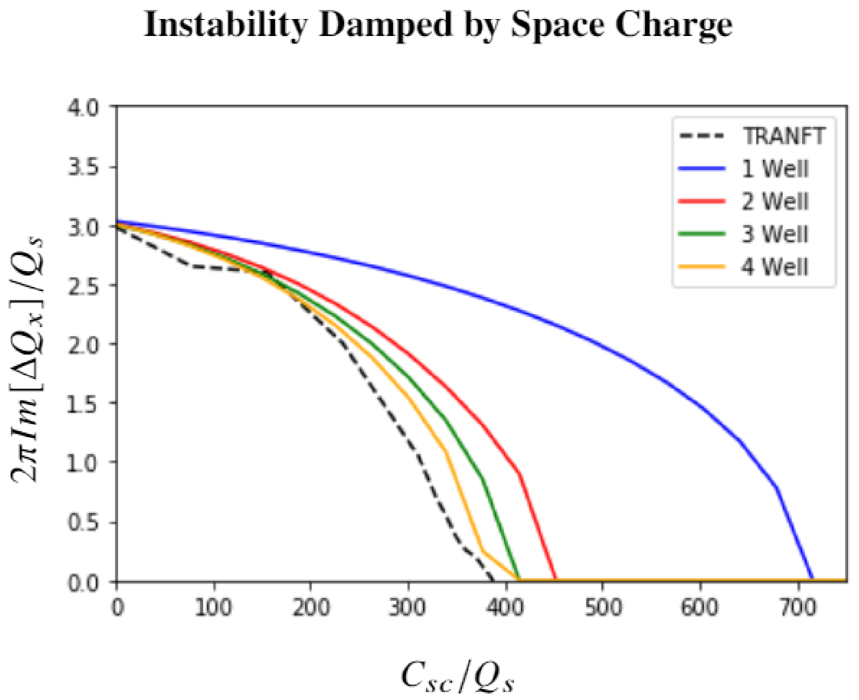

Figure 3: Instability strength of most unstable eigenmode of Gaussian distribution for TRANFT and MLSW under a step wake two times instability threshold. Space charge is strengthened to eliminate instability. Adding square wells causes the solution to approach TRANFT value indicating possible convergence. Note for the TRANFT case, C_{sc} is defined in terms of form factor $f = 3\sqrt{\pi/2}$

Comparison With Rigid Beam Equations

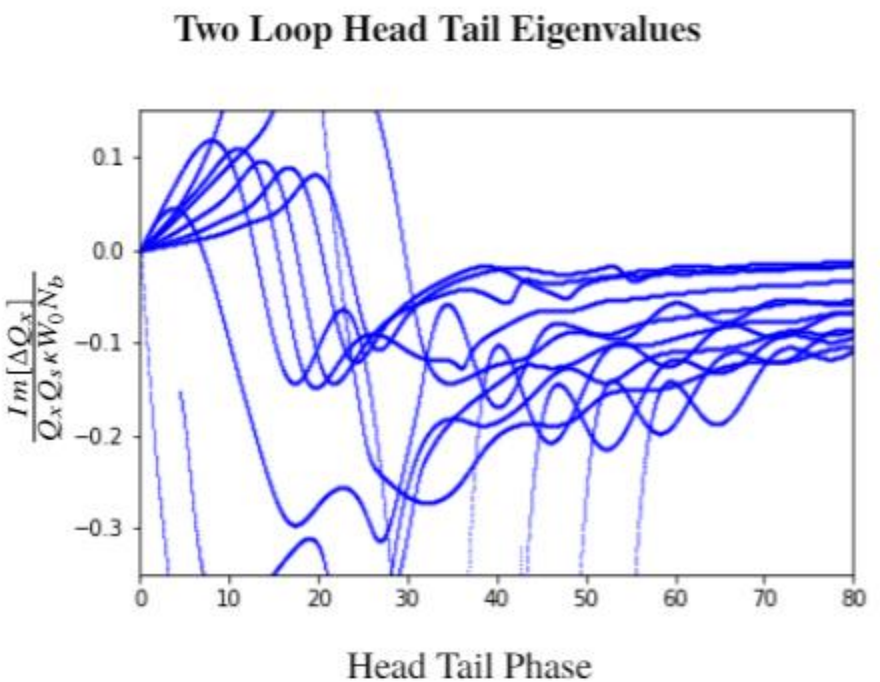

Figure 4: Rescaled imaginary components of two loop MLSW eigenvalues. These have a similar form [5] Fig. 3. $N > 1$ loop MLSW has more eigenvalues than Rigid-Beam making this sort of analysis more difficult in a many loop case.

- Comparisons between ABS and Rigid-Beam Equations by Burov show good agreement
- MLSW has more eigenmodes
  - We do not expect these to have the same solution
  - Similar Eigenvalues implies similar physics
- Can compare Gaussian Rigid Mode Equations to 2 loop MLSW
  - Can get good agreement between specific modes.

A. Burov, "Head-tail modes for strong space charge," *Phys. Rev. Accel. Beams*, vol. 12, p. 044202, 2009.

Thank You

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