END-TO-END RMS ENVELOPE MODEL OF THE ISAC-I LINAC

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ABSTRACT
A full end-to-end simulation of the ISAC-I linear accelerator has been built in the first order envelope code TRANSOPTR. This enables the fast tracking of rms sizes and correlations for a 6-dimensional hyperellipsoidal beam distribution defined around a Frenet-Serret reference particle frame, for which the equations guiding envelope evolution are numerically solved through a model of the machine’s electromagnetic potentials. Further, the adopted formalism enables the direct integration of energy gain via time-dependent accelerating potentials, without resorting to transit-time factors.

TRANSOPTR and RMS envelope tracking
The Courant-Snyder Hamiltonian for a relativistic, charged particle is used [1]:

\[ H = -\frac{q_A}{\gamma} \sqrt{ (\frac{E - \Phi}{c})^2 - m^2 c^2 - (P_z - q_A)^2 - (P^\perp - q_A)^2 } \]

Beams of charged particles are treated using the first and second moments of the distribution [2]:

\[ \langle X \rangle = \frac{1}{N} \sum_{n=1}^{N} x_n \] (centroids)

\[ \sigma = \frac{1}{N} \sum_{n=1}^{N} (x - \langle X \rangle)^T (x - \langle X \rangle) \] (beam matrix)

The evolution of the ensemble forming the beam is:

\[ \frac{dX}{ds} = F(s)X \]

2. Radiofrequency Quadrupole Linac

\[ F(s) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \]

\[ a_Y = \frac{qV_0 \sin (\omega t + \phi)}{4\beta c} \left( A_0 \cos (\omega t + \phi) \right) \]

\[ C = \frac{qV_0 (\omega / (\beta c))^2 A_0 \cos (\omega t + \phi)}{4\beta c} \left( \cos \psi \cos^2 (\omega t + \phi) - 2 \sin (\omega t + \phi) \right) \]

Input: RFQ parameters \((a, m, k)\) [4]

3. IH-DTL

\[ F(s) = \begin{pmatrix} 0 & \frac{1}{\beta c} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\beta c} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \]

\[ A_Y = -\frac{q}{2\beta c} \left( E' s - E s \sin \phi \right) \]

\[ B_Y = \frac{qE_0 s \sin \phi}{\beta^2 c^2} \]

Figure 1: Overview of the ISAC-I linac

TRANSOPTR is fast: subsecond execution time. All components of the ISAC-I linac are now in the code.

4. No TTF

To first order in TRANSOPTR, the potential directly modifies the canonical energy [3,4,5]:

\[ c^2 P \frac{dP}{dx} = (E - \Phi) \Delta E \]

Energy gain is directly integrated from the field:

\[ E(s) = E_0 + qV_s \int_0^s E(s) \cos (\omega t + \phi_0) ds \]

This allows for a straightforward integration of the reference particle energy, without resorting to transit time factor approximations.

5. Conclusion

The envelope code TRANSOPTR has now been extended to represent the entire ISAC-I linear accelerator. This notably includes a novel RFQ simulation capability, only requiring the vane modulation parameters as input. In addition, the code possesses an axially symmetric linac feature.

The model is now being used at TRIUMF-ISAC for investigations and studies of the linac’s tune and performance.

Bibliography