Two-Stream Effects in Coherent Beam-Beam Oscillations in VEPP-2000 Collider Near the Linear Coupling Resonance

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INTRODUCTION

VEPP-2000:

- Operating point is on the main coupling resonance
- High beam-beam parameter ξ

Nominal beam-beam parameter $\xi \gtrsim 0.3$;

Bunch length $l = 3.5\ cm$

Beta function in the interaction point $\beta^* = 10\ cm$

Available theories of the beam-beam coherent motion involve synchro-betatron motion and two-stream interaction of colliding bunches, however, without the betatron coupling. In round-beam colliders such as VEPP-2000 the betatron coupling resonances are inherent in the collider optics, that is why treatment of coupled coherent beam-beam oscillations is necessary.

In earlier studies* the two-stream interaction of colliding bunches was successfully included, as well as their synchro-betatron motion, the theory similar to the TMCI was constructed including the effect of the lattice chromaticity and the transverse impedance of the machine.

In the present study we introduce the betatron coupling in the theoretical formalism based on the circulant matrix approach. This technique exploits an essentially linear (macroparticle) construction in the time domain where the transport of synchro-betatron modes of the bunch motion is efficiently done with the circulant matrices. The coherent mode tunes and waveforms are found from the one-turn map spectrum.

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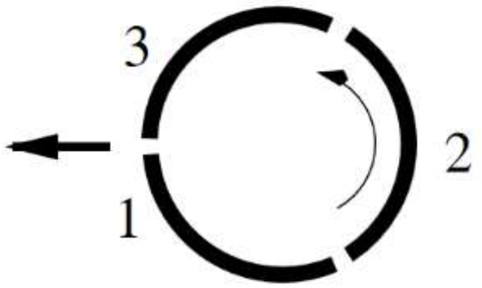
* [1] E.A. Perevedentsev, in Proc. of the 1999 PAC, New York, 1999, Vol. 3, p. 1521;
[2] E.A. Perevedentsev and A.A. Valishev, Phys. Rev. ST Accel. Beams 4, 024403 (2001);
[3] I.N. Nesterenko, E.A. Perevedentsev, and A.A. Valishev, Phys. Rev. E 65, 056502 (2002).

Theory: circulant matrix approach



Division of synchrotron phase into 3 boxes

It is convenient to use phazors: $\tilde{x} = x + \frac{ic}{w}x'$. I.e., for solution of the equation of motion

$$\ddot{d}_i + w_\beta^2 d_i = -2w_\beta D_{ik} d_k, \qquad (1)$$

we use the ansatz $d_i = a_i e^{-iw_\beta t} + c.c.$ and, performing standard averaging, deal with shortened equations:

$$i\,\dot{a}_i = D_{ik} a_k \qquad (2)$$

where $\dot{a}_i$ is a full derivative w.r.t. t. Then, replacing the derivative with respect to the synchrotron phase by N-point approximation – circulant matrix γ_{ik}, we get matrix equation:

$$i\,\dot{a}_i = (C_{ik} + D_{ik})\,a_k, \qquad (3)$$

where $C_{ik} = -w_s\gamma$. In order to restore omitted betatron frequency (write an equation for $a_i e^{-iw_\beta t}$), we need to add the diagonal matrix W composed of $-w_\beta$.

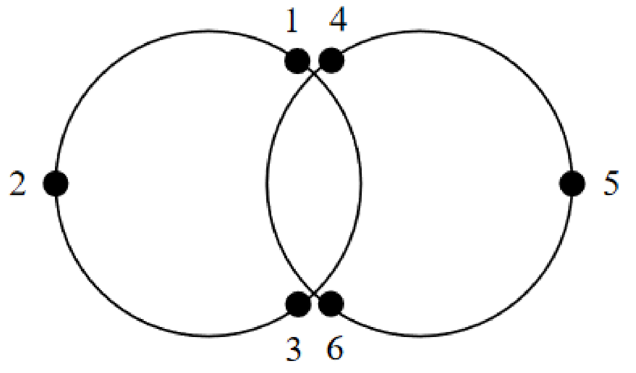
Substituting $a_i(t) = v_i e^{-i\Omega t}$ in eq. (3), we find mode frequencies – eigenvalues of the matrix $C_{ik} + D_{ik} + W$.

For example, matrix C_3 (with 3 boxes), is written as follows:

$$C_3 = \frac{i\,w_s}{\sqrt{3}}\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

Theory: application of circulant matrix approach

Mapping matrix: In order to include beam-beam and coupling to the circulant formalism, one needs to write the one-turn matrix as a matrix exponent of the resulting circulant matrix $C_{ik} + W$.



Two colliding bunches in the synchrotron phase space in terms of circulants

Betatron coupling: the extended map matrix is multiplied by coupling matrix. The study of uncompensated rotations in the final-focusing solenoids is of practical value for the BEPP-2000 case. The coupling matrix is written as follows:

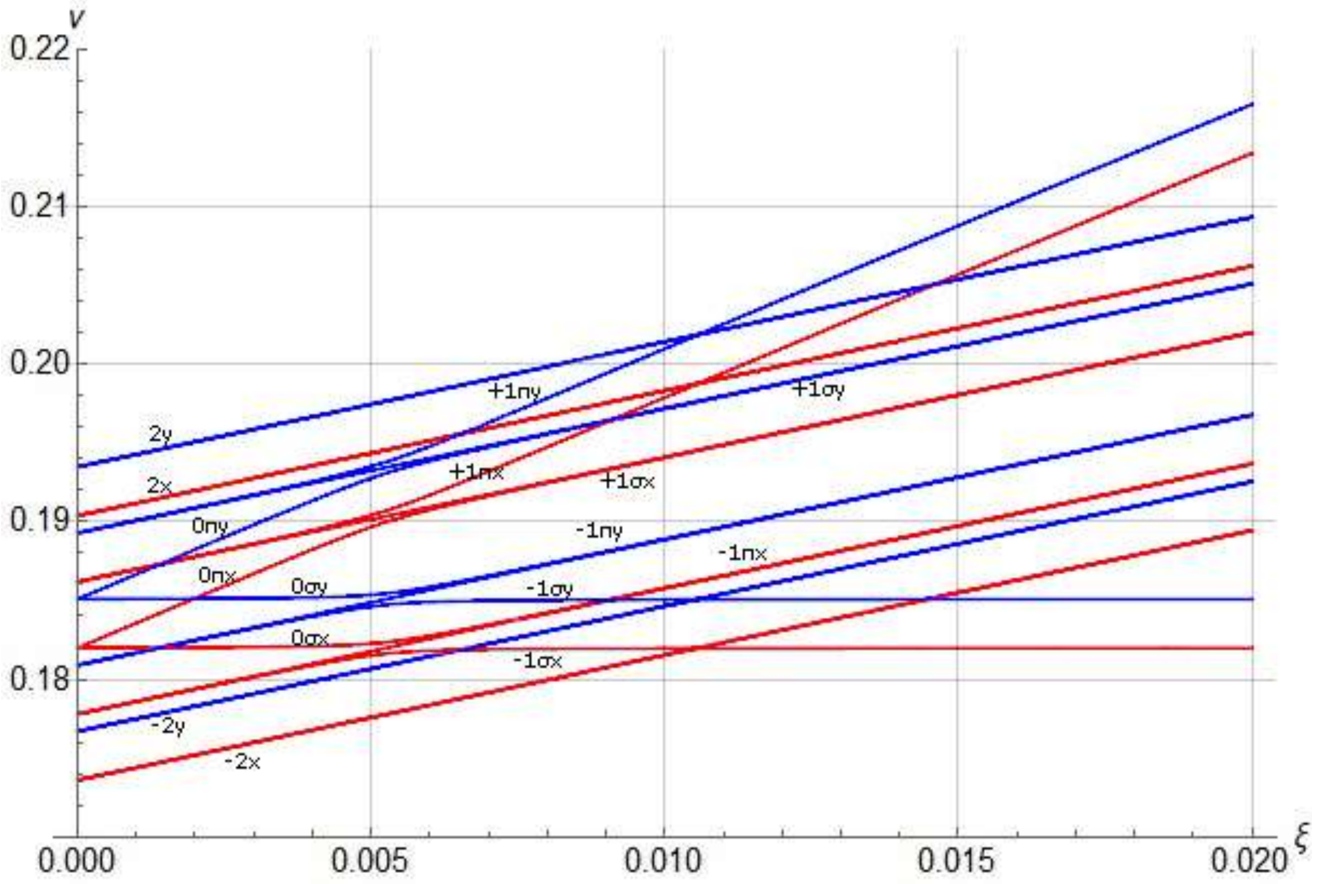
$$\begin{pmatrix} 1 & i\,L \\ -i\,L & 1 \end{pmatrix}$$

Beam-beam: once the map matrix is written, only we need to do, is to extend it to two bunches and multiply by beam-beam linear effect (it’s matrix), which is written in terms of phasors as follows:

$$\begin{pmatrix} 3 & 0 & 0 & -1 & -1 & -1 \\ 0 & 3 & 0 & -1 & -1 & -1 \\ 0 & 0 & 3 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & 0 & 0 \\ -1 & -1 & -1 & 0 & 3 & 0 \\ -1 & -1 & -1 & 0 & 0 & 3 \end{pmatrix} \xi\, k,$$

where ξ is the beam-beam parameter, and k is the known constant.

Theory: predictions of the circulant method for VEPP-2000



Spectrum dependency on the beam-beam parameter for round colliding beams

Synchro-betatron spectrum dependency on the beam-beam parameter.

In this case, phase advances on the beam length are equal due to the equality of transverse beta-stars.

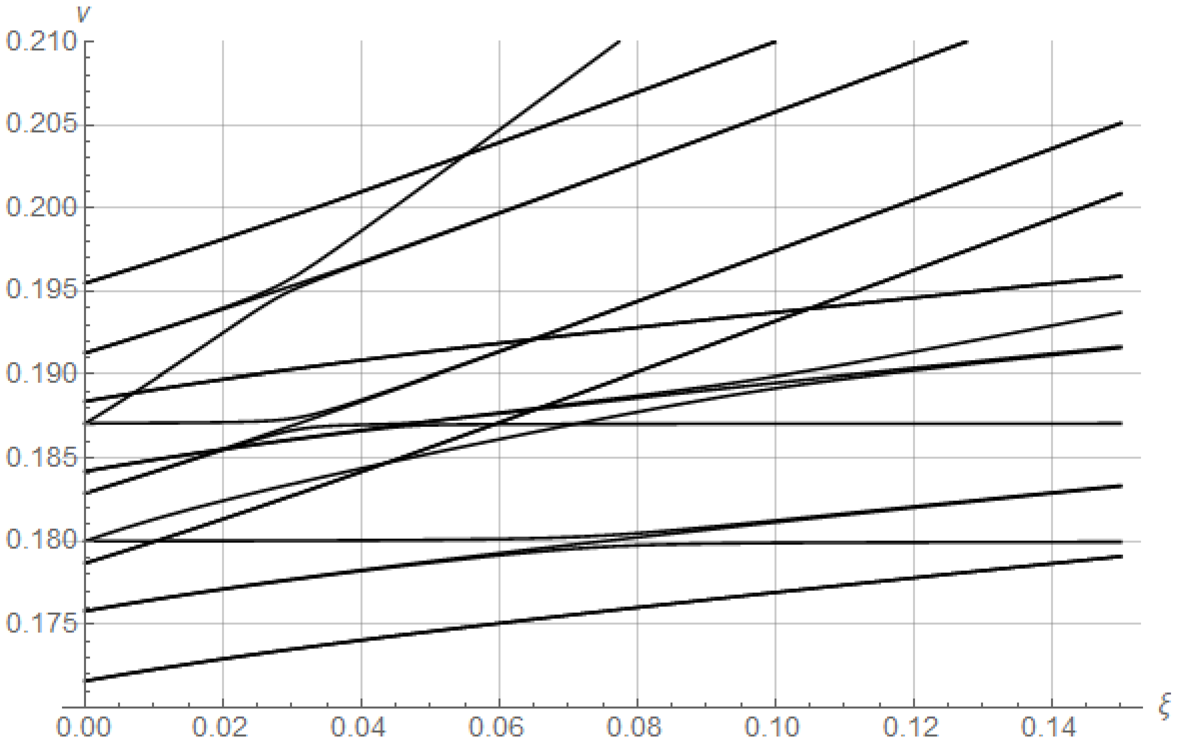
Transverse beam-beam forces are also equal due to the round beams.

x-transverse modes are plotted as red, and y modes are marked in blue.

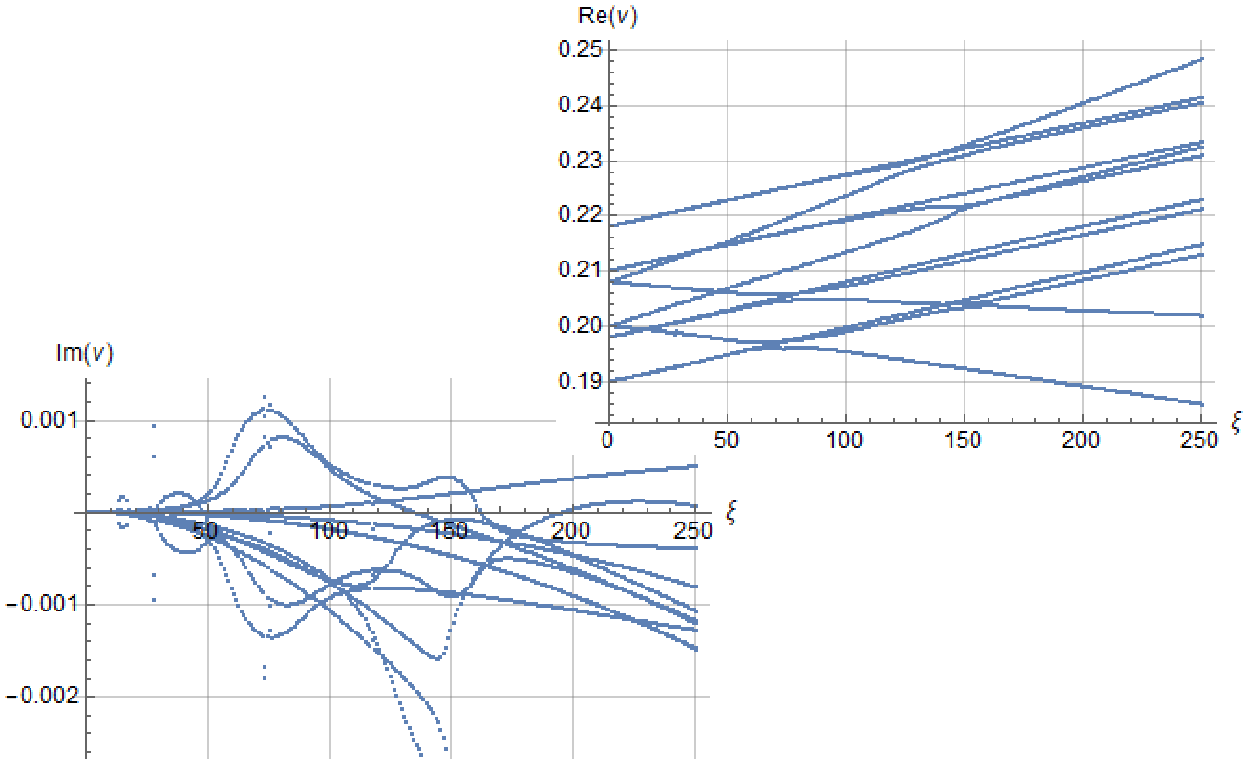
Wake functions are turned off.

One can see from the picture, that there is no instability in the case when only two-stream effect is present. It is clearly seen from the repulsion of 0π and +-1 modes. Moreover, there is no coupling between x and y modes in the case of round beams – lines are just crossing each other without excitation of imaginary parts.

Theory: predictions of the circulant method for non-round beams and with wakes turned on



Spectrum dependency on the beam-beam parameter for non-round colliding beams



Real an imaginary parts of the spectrum dependency on the beam-beam parameter for round colliding beams with wakes turned on

With the different transverse forces, i.e., for non-round beams (picture on the left), the coupling between x and y modes are observed.

With the wakes turned on (pictures on the right), of course, an instability can occur

Tracking

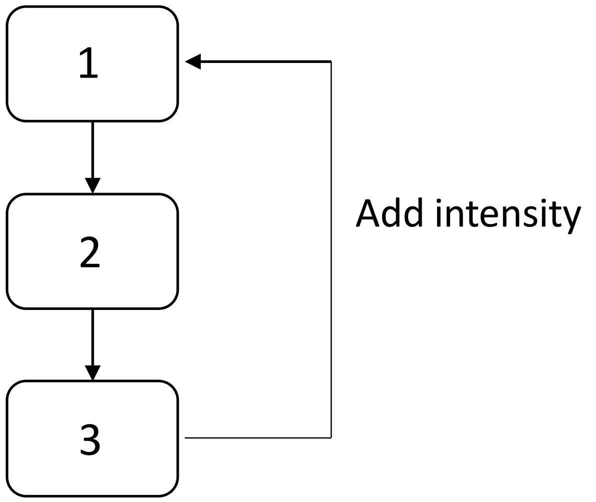
Simple tracking program:

- Period transformation single-particle equations or one-turn matrix
- Beam-beam effects – with zero and finite bunch length
- Wakes support

In this paper the one-turn matrix and full linear beam-beam model (with finite bunch length) were used

Simulation is divided into several steps:

1. setting the initial parameters – distributions of two bunches, their intensities;
2. mapping the bunches through a given number (N) of machine turns;
3. calculating the frequencies and writing them to the output file;
4. repeating steps 1-3 for the set of beam's intensities.



Block-scheme of the tracking

Tracking realization

The period map (step 2)

period matrix + beam-beam interaction + wakes effect (optional)

The beam-beam interaction (step 2):

First, we need to move particle to their initial collision points:

$$x = x_0 - z\, p_x, y = y_0 - z\, p_y$$

Then, the beam-beam kick is applied:

$$p_1 = {p_1}_0 + k\, \xi\, (x_1 - x_2), p_2 = {p_2}_0 - k\, \xi\, (x_1 - x_2)$$

In the end, of course, particles should be moved the same distance z back:

$$x = x_0 + z\, p_x, y = y_0 + z\, p_y$$

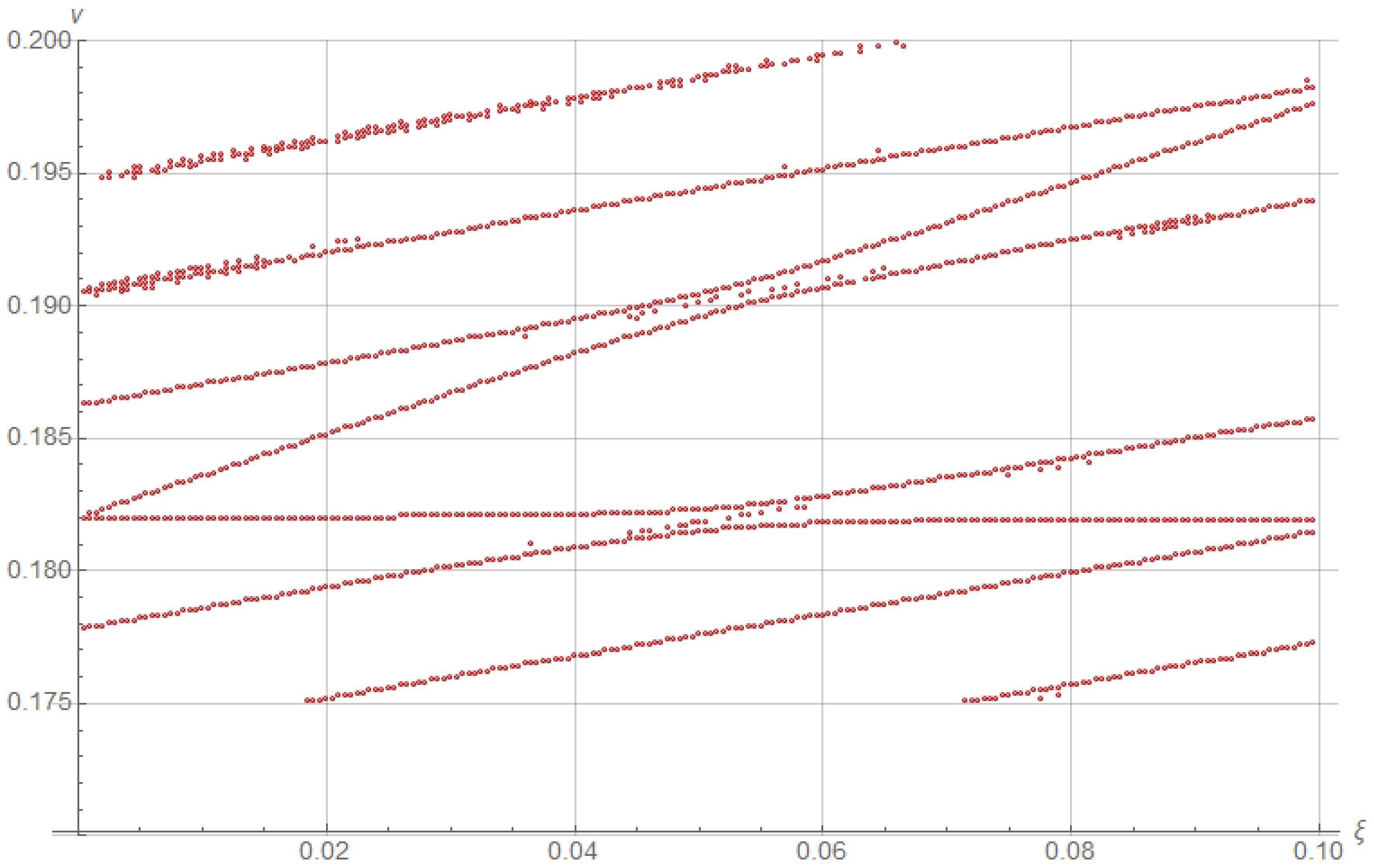
Spectrum (step 3):

- Logarithmic scale
- Window function $\sin^2(\frac{\pi\, n}{N-1})$

Getting frequencies from the spectrum (step 3):

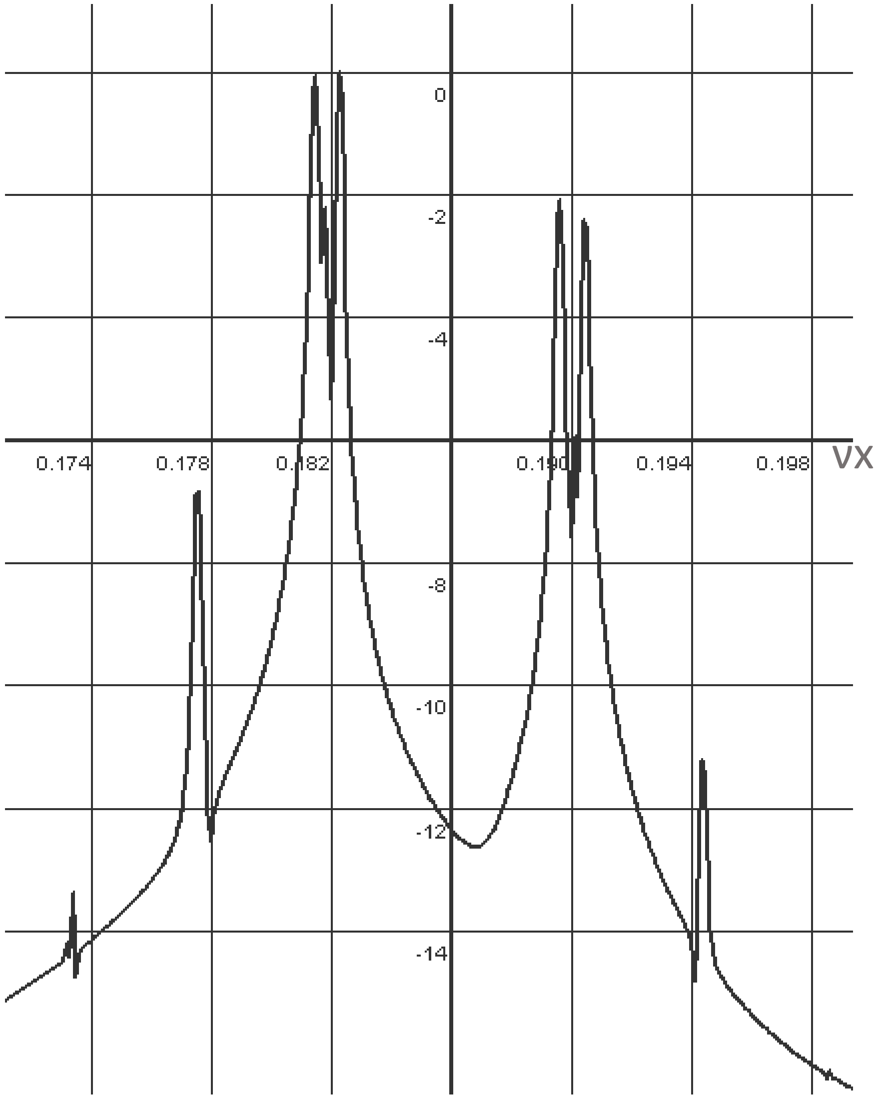
The point is checked for a local spectral maximum, if the spectrum wavering in it’s close proximity is greater, than the averaged on some section wavering, which width depends on the value of the spectrum in the point.

Simulation results



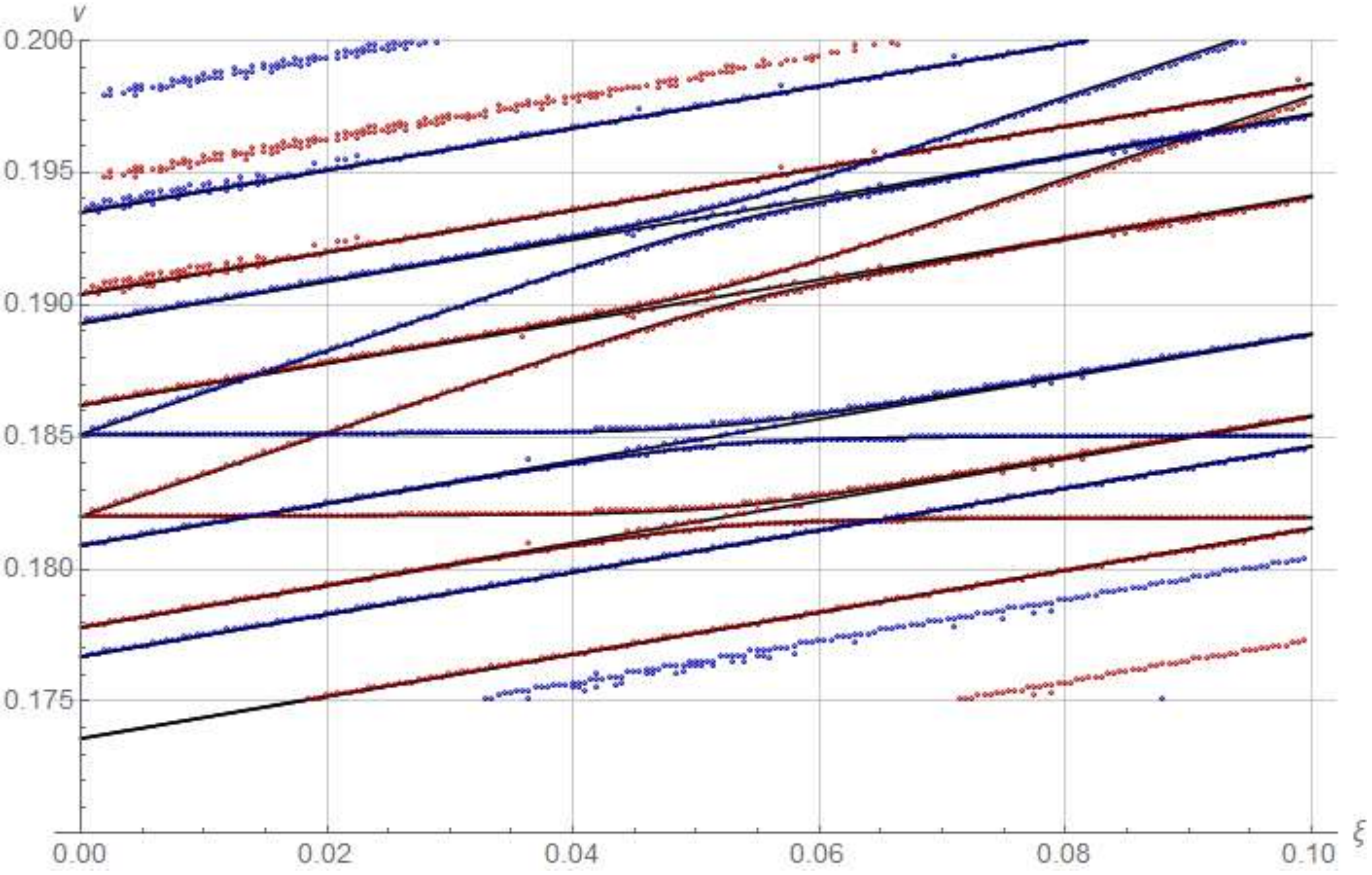
Part "x" of the tracking spectrum for round colliding beam parameters:

$\nu_x = 0.18,\ \nu_y = 0.1851,\ \nu_s = 0.0042,\ \frac{l}{\beta^*} = 0.35$

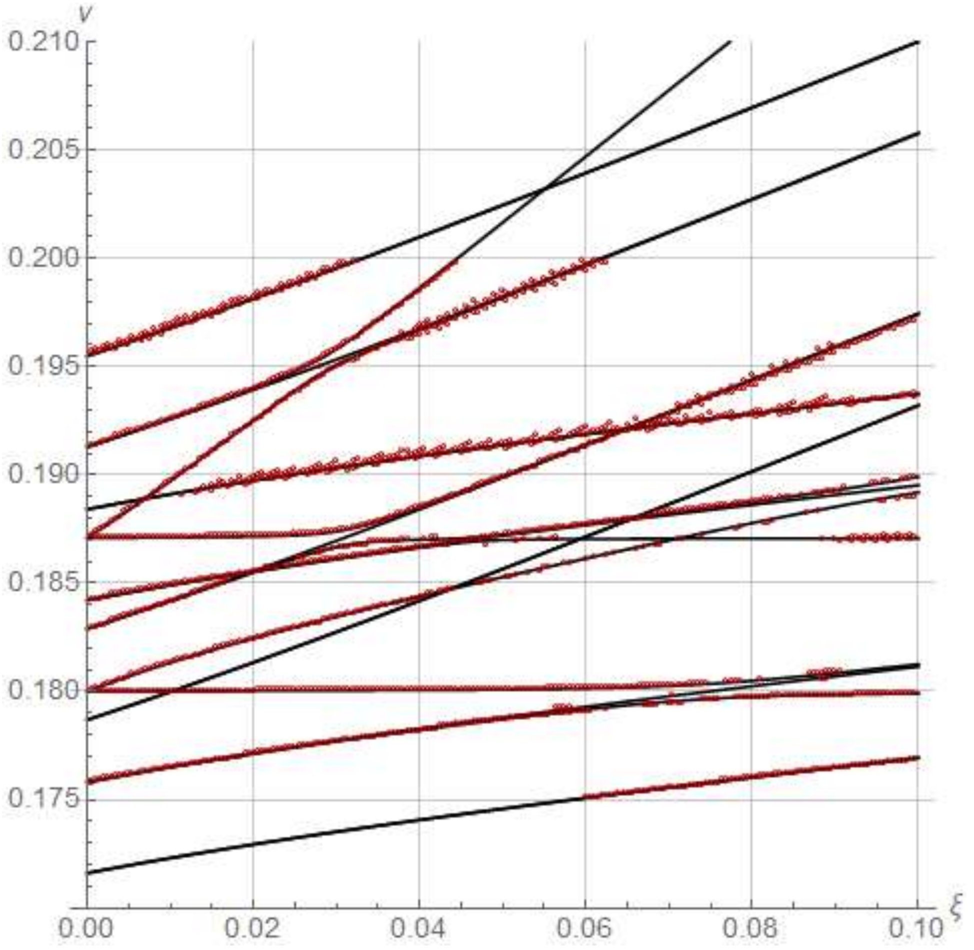


Spectrum for beam-beam parameter $\xi = 0.05$

Comparison of spectra from circulants and tracking



Circulant's and tracking spectra for round beams. $\nu_x = 0.18, \nu_y = 0.1851,$ $\nu_s = 0.0042, \frac{l}{\beta^*} = 0.35$



Circulant's and tracking spectra for non-round beams ($\xi_y = 4\xi_x$). $\nu_x = 0.18,\ \nu_y = 0.1851, \nu_s = 0.0042, \frac{l}{\beta^*} = 0.35$

Conclusions

- In the round-beam collider VEPP-2000, the x-y betatron tunes lie in the band of the main coupling resonance, and their separation is small. This is a difficulty for conventional perturbative approach.

- On the other hand, a time-domain approach using circulanat matrices can be applied for constructing a full set of the synchrobetatron modes of colliding bunches which interact in a two-stream manner due to finite lengths of the colliding bunches. The beam-beam kick is linearized, and the mode-set spectrum is obtained from the one-turn map, to conclude on the stability of the beam-beam coherent oscillations. Neither x-y coupling nor closeness of the tunes cause any problem in this formalism.

- In this paper we studied the coherent beam-beam oscillations using the circulant-matrix approach. We have made theoretical predictions on synchrobetatron mode spectrum vs. the beam-beam parameter for round-beam colliders. We assumed the operating point located near the main coupling resonance (e.g. for e^+e^- collider VEPP-2000).

- The results have been compared with a simple macro-particle simulation and have shown a good agreement.

- The wakefields if taken into account have been shown to cause instability. However, the wake effects are small compared with beam-beam effects and do not have to be worried about.

- The obtained results can help us to better understand the role of coherent oscillations in the beam-beam limit problem in colliders.