Theoretical Analysis of the Conditions for an Isochronous and CSR-Immune Triple-Bend Achromat with Stable Optics*

Chengyi Zhang¹,², Yi Jiao³, Cheng-Ying Tsai³
1. Key Laboratory of Particle Acceleration Physics and Technology,
2. Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, 100049, China and University of Chinese Academy of Sciences, Beijing, 100049, China
3. Department of Electrotechnical Theory and Advanced Electromagnetic Technology, Huazhong University of Science and Technology, Wuhan, 430074, China

Abstract: Transport of high-brightness beams with minimum degradation of the phase space quality is pursued in modern accelerators. For the beam transfer line which commonly consists of bending magnets, it would be desirable if the transfer line can be isochronous and coherent synchrotron radiation (CSR)-immune. For multi-pass transfer line, the achromatic cell designs with stable optics would bring great convenience. In this paper, based on the matrix transfer formalism and the CSR point-kick model, we report the detailed theoretical analysis and derivation process of the condition for a triple-bend achromat (TBA) with stable optics in which the first-order longitudinal dispersion (i.e., \( R_{\text{e}} \)) and the CSR-induced emittance growth can be eliminated (i.e., \( R_{\text{e}}=0 \) and \( \Delta \varepsilon=0 \)). The derived condition suggests a new way of designing the bending magnet beamline that can be applied to the FEL spreader and ERL recirculation loop.

Derivation of the \( R_{\text{e}}=0 \) and \( \Delta \varepsilon=0 \) conditions for a TBA

A midpoint symmetric TBA structure that consists of three identical dipole magnets (see Fig.1) is studied. Here only the transverse motion in the horizontal plane and the longitudinal motion of the particle are considered for simplicity. The transfer matrix for the phase-space coordinate vector \( (x, x', y, y', z, \delta) \) is used, which is part of the typical 6 × 6 transfer matrix that describes the motion of \( (x, x', y, y', z, \delta) \) though we still adopt the subscripts 5 and 6 for the longitudinal dispersion functions \( R_{\text{e}} \).

\[
\begin{align*}
\beta_0 &= \beta_0, \\
R_{\text{d}} &= R_{\text{d}} \\
M &= \begin{pmatrix}
m_{11} & m_{12} & 0 & 0 \\
m_{21} & m_{22} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
\end{pmatrix}, \\
M_{\text{t}} &= R_{\text{d}}/2MR_{\text{d}}R_{\text{d}}, \\
\end{align*}
\]

Cross points of 1st order isochronous & CSR-induced emittance growth cancellation condition

The two cross points locate at

\[
m_{11} = 1 - \cos \theta, \\
m_{11} = \frac{1}{2} \theta. \\
\]

When \( \theta \ll 1 \), Taylor expand with respect to \( \theta \) and to the lowest order

\[
m_{11} = 4, \\
m_{12} = \frac{15}{2} \theta. \\
\]

Discussion in the stable area

The cross point B satisfies the stability criterion

\[
|M_{\text{e}}(1) + M_{\text{e}}(2)| \leq 2.
\]

However, the periodic solution of \( \beta \) will diverge at cross point B

\[
\Delta \varepsilon = 0.
\]

But both the minimized \( R_{\text{e}} \) and \( \Delta \varepsilon \) can be found in the vicinity of Cross point B