Electron Dynamics

Classical Cyclotron Motion
\[ \frac{d\vec{v}}{dt} = -\frac{e}{m_e} \vec{v} \times \vec{B} \Rightarrow \Omega_{c0} = \frac{e B_0}{m_e} \]

Resonant Interaction
\[ \frac{d}{dt} (\gamma \vec{v}) = -\frac{e}{m_e} \left[ \vec{E} + \vec{v} \times \vec{B} \right] \Rightarrow \Omega_c = \frac{e B_0}{m_e \gamma} \]

ECR Condition: \( \Omega_c = \omega \Rightarrow \) Acceleration Band: \( \frac{\pi}{2} \leq \phi \leq \frac{3\pi}{2} \)

Temporal Autoresonance
\[ \omega = \Omega_c = \frac{e B(t)}{\gamma m_e} \]

\( \vec{B} = B_z \hat{k} \) and \( \vec{E} = E_0 \cos(\omega t) \hat{j} \)
Gyroresonant Acceleration (Gyrac)

Gyrac Model

Considering:
\[
\vec{E} = E_0 \left[ \sin(\varphi) \hat{r} + \cos(\varphi) \hat{\theta} \right] \quad \text{and} \quad \vec{B} = B_0 \left[ 1 + b(t) \right] \hat{k}
\]

Energy and phase-shift evolution:
\[
\dot{\gamma} = -g_0 \left( 1 - \frac{1}{\gamma^2} \right)^{1/2} \cos(\varphi)
\]
\[
\dot{\varphi} = \left[ b(\tau) - (\gamma - 1) \right] \frac{1}{\gamma} + g_0 \left( \gamma^2 - 1 \right)^{-1/2} \sin(\varphi)
\]

Gyrac Regime: \( b(\tau) = \alpha \tau \Rightarrow \alpha \leq 1.19 g_0^{4/3} \) where \( g_0 = -\frac{E_0}{B_0 c} \) and \( B_0 = \frac{\omega m_e}{e} \)

Cylindrical Mode TE\(_{111}\)

\[
\vec{E}^{hf}(\vec{r}, t) = \frac{E_0}{J_1(p_{01})} \frac{J_1(q_{01}/R)}{R} \sin \left( \frac{\pi}{L} z \right) \cos(\omega t) \hat{\theta}
\]
\[
\vec{B}^{hf}(\vec{r}, t) = \frac{E_0}{J_1(p_{01})} \left[ \frac{\pi}{L \omega} J_1 \left( \frac{q_{01}/R}{r} \right) \cos \left( \frac{\pi}{L} z \right) \sin(\omega t) \hat{r} - \frac{q_{01}/R}{\omega} J_0 \left( \frac{q_{01}/R}{r} \right) \sin \left( \frac{\pi}{L} z \right) \sin(\omega t) \hat{k} \right]
\]

where \( q_{01} = 3,83171, p_{01} = 1,84118, R = 7.84 \text{ cm}, L = 20 \text{ cm}, E_0 = 1 \text{ kV/cm and } f = 2.45 \text{ GHz.} \)
Physical Scheme and Simulation Model

Physical scheme: (1) Electron injection point, (2) Cylindrical Cavity and (3) Cross section $z = L/2$.

Electromagnetic Field

Cylindrical Mode $\ TE_{011}$ \Rightarrow \ \vec{E} = \vec{E}^{hf} \ y \ \vec{B} = \vec{B}^{hf} + \vec{B}^{ext}$

Simulation Model

- Gyrac Model: Runge-Kutta Fourth Order Method.
- 2D Relativistic Newton-Lorentz equation: Boris integrator.

Numerical experiments

1. An electron is released from rest at point 1 using a set of $\alpha$ parameters.
   \[\alpha = \{1.0 \times 10^{-4}, 1.5 \times 10^{-4}, 2.0 \times 10^{-4}, 2.5 \times 10^{-4}, 2.75 \times 10^{-4}, 3.0 \times 10^{-4}\}\]
2. Particle System: Ring-like electron injection from rest using said set of $\alpha$ parameters.
Results

**Fig 1:** Evolution of $\gamma$ and $\varphi$.

**Fig 2:** Evolution of $\gamma$ and $\frac{B_z}{B_0}$.

**Fig 3:** Evolution of $\gamma$ for different $\alpha$ parameters.

**Fig 4:** Evolution of $R_L$ for different $\alpha$ parameters.

**Fig 5:** Guide center trajectory.
Results

Evolution of the particle systems after 4.65 µs for different α parameters.
Conclusions

• It was showed by numerical experiments that it is possible to accelerate electrons under electron cyclotron resonance conditions in time-varying magnetic fields using the TE_{011} cylindrical mode.

• A set of values for $\alpha$ parameter that allow to maintain the resonance condition over time was determined.

• It was found that there is a region ring-like ($3R/8 < r < 9R/16$) where the electrons are captured in the autoresonance regime.

Future Works

We will study the 3D dynamic of an electrons cloud in magnetic fields varying in time using the cylindrical mode TE_{011}.
References

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