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On Possibility of Alpha-Buckets Detecting at the KIT Storage Ring KARA (Karlsruhe Research Accelerator)

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Absrtact

Computer studies of longitudinal motion have been performed with the objective to estimate the possibility of detection of alpha-buckets at the KIT storage ring KARA (Karlsruhe Research Accelerator) [1]. The longitudinal equations of motion and the Hamiltonian were expanded to high order terms of the energy deviation of particles in a beam. Roots of third order equation for three leading terms of momentum compaction factor and free energy independent term were derived in a form suitable for analytical estimations. Averaged quadratic terms of closed orbit distortions caused by misalignment of magnetic elements in a ring lead to orbit lengthening independent of particle energy deviation. Particle transverse excursions were estimated and are taken into account. Simulations have been bench-marked on existing experiments at Metrology Light Source (MLS) in Berlin (Germany) and SOLEIL (France). A computer model of KARA was used to predict behavior and the dynamics of possible simultaneous beams in the ring.

MOMENTUM COMPACTION FACTOR THEORY

pass length variation of beam orbit in a ring split into two parts; one independent on momentum deviation (χ); the other dependent on high order terms of momentum offset δ

$$\Delta L/L_0 = \alpha(\delta)\cdot\delta + \chi \qquad (1)$$

momentum compaction factor itself depends on energy offset

$$\alpha(\delta) = \alpha_1 + \alpha_2\delta + \alpha_3\delta^2 \qquad (2)$$

Linear and high order components of the momentum compaction factor depend on dispersion function terms

$$\alpha_1 = \frac{1}{L_0}\oint\left(\frac{D_0}{\rho}ds\right) \qquad (3)$$

$$\alpha_2 = \frac{1}{L_0}\left[\oint\left(\frac{D_0^2}{2\rho^2} + \frac{D_1}{\rho} + \frac{D'^2_0}{2}\right)ds\right] \qquad (4)$$

$$\alpha_3 = \frac{1}{L_0}\left[\oint\left(\frac{D_0 D_1}{\rho^2} + \frac{D_2}{2\rho} + D'_0 D'_1\right)ds\right] \qquad (5)$$

The momentum independent term of relative orbit lengthening (χ) includes betatron oscillations and COD errors [10]

$$\chi = \frac{1}{2L_0}\oint\left(x'^2_\beta + z'^2_\beta + x'^2_{cod} + z'^2_{cod} + \frac{x^2_\beta}{\rho^2} + \frac{x^2_{cod}}{\rho^2}\right)ds \qquad (6)$$

Condition of <u>fixed</u> momentum offset $\frac{\partial H}{\partial\delta} = \frac{d(\Delta\varphi)}{dt} = 0$ of Hamiltonian of longitudinal motion [4,12] is realized for two phases $\varphi = \varphi_s$ and $\varphi = \pi - \varphi_s$.

Condition of <u>fixed</u> phase $\frac{\partial H}{\partial(\Delta\varphi)} = -\frac{d\delta}{dt} = 0$ is fulfilled when the relative orbit lengthening is zeroed

$$\alpha_3\delta^3 + \alpha_2\delta^2 + \alpha_1\delta + \chi = 0 \qquad (7)$$

For χ=0 equation (7) is split in two parts. <u>**one**</u> root on-momentum synchronous particles ($\delta_1 = 0$) and phase φ_s (**RF bucket**) Second part provides another <u>**two**</u> roots with momentum offset (**α–buckets**)

$$\delta_{2,3} = \frac{1}{2\alpha_3}\left[-\alpha_2 \pm \sqrt{\alpha_2^2 - 4\alpha_1\alpha_3}\right] \qquad (8)$$

Roots are real if $|a_2| > 2\sqrt{\alpha_1\alpha_3}$ or first α_1, and third α_3 order terms are of different sign and $(a_1 a_3) < 0$

At some conditions, for ($\chi \neq 0$) [13] three <u>**real**</u> roots of cubic (CARDANO) equation might be found [1,4]

$$\delta_1 = -\frac{a_2}{3a_3} + \frac{2}{\sqrt[3]{2}(3a_3)}\left\{\sqrt[3]{\sqrt{\Delta_1^2 + |-\Delta|}}\cos\left(\frac{\varphi}{3}\right)\right\} \qquad (9)$$

$$\delta_2 = -\frac{a_2}{3a_3} - \frac{1}{\sqrt[3]{2}(3a_3)}\left\{\sqrt[3]{\sqrt{\Delta_1^2 + |-\Delta|}}\left[\cos\left(\frac{\varphi}{3}\right) + \sqrt{3}\sin\left(\frac{\varphi}{3}\right)\right]\right\}$$

$$\delta_3 = -\frac{a_2}{3a_3} - \frac{1}{\sqrt[3]{2}(3a_3)}\left\{\sqrt[3]{\sqrt{\Delta_1^2 + |-\Delta|}}\left[\cos\left(\frac{\varphi}{3}\right) - \sqrt{3}\sin\left(\frac{\varphi}{3}\right)\right]\right\}$$

Second order determinant $(-\Delta_0) = 3a_1a_3 - a_2^2$

Third order determinant $\Delta_1 = 2a_2^3 - 9a_1a_2a_3 + 27a_3^2\,\chi$

General determinant $(-\Delta) = \Delta_1^2 + 4(-\Delta_0)^3$

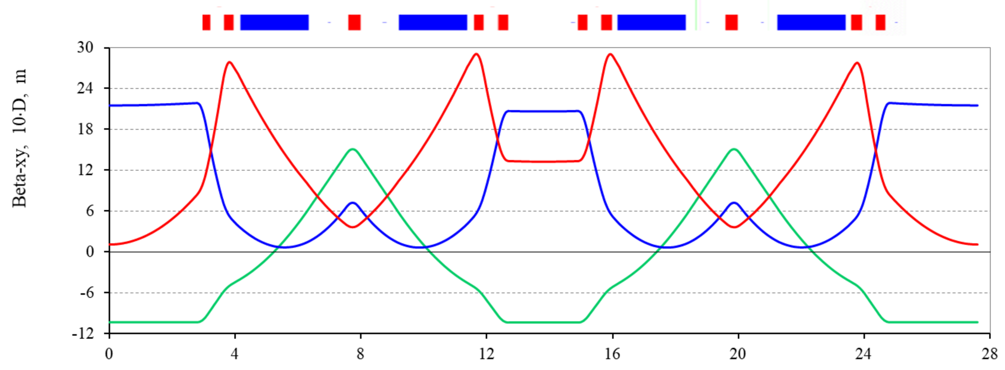
The synchrotron tune, F_s , depends on high order terms of the momentum compaction factor where $\alpha = \partial(\Delta L/L_0)/\partial\delta$ [1,4,14,15]

$$F_s(\delta) = F_0\sqrt{\frac{h_{rf}\,eU_{rf}(-\cos\varphi_s)}{2\pi\beta_0^2E_0}}\cdot\sqrt{(\alpha_1 + 2\alpha_2\,\delta + 3\alpha_3\delta^2)} \qquad (10)$$

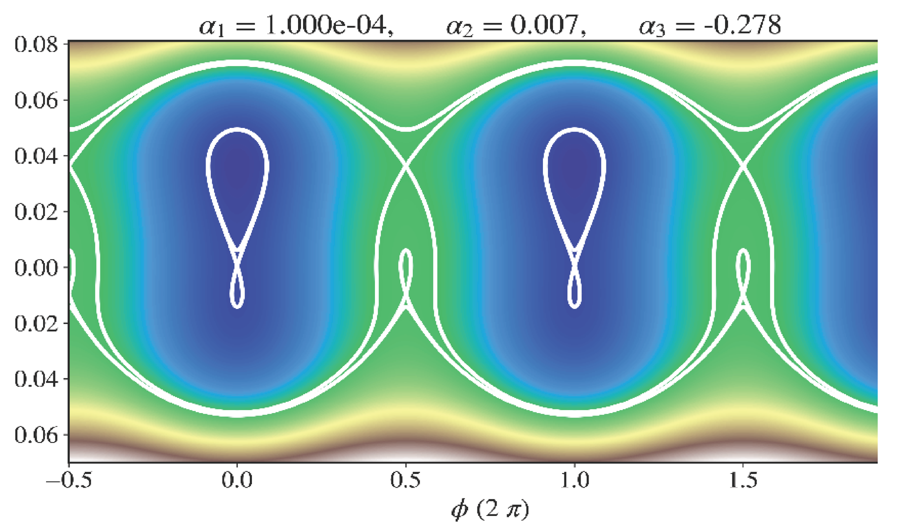
MA limited by zero synchrotron tune (F_s= 0) – low–α ($\alpha_1\cdot\alpha_3$<0)

$$MA_{(Fs=0)} = \delta_{(Fs=0)} = \frac{1}{3\alpha_3}\left[-\alpha_2 \pm \sqrt{\alpha_2^2 - 3\alpha_1\alpha_3}\right] \qquad (12)$$

CONDITIONS FOR α–BUCKETS AT KARA RING

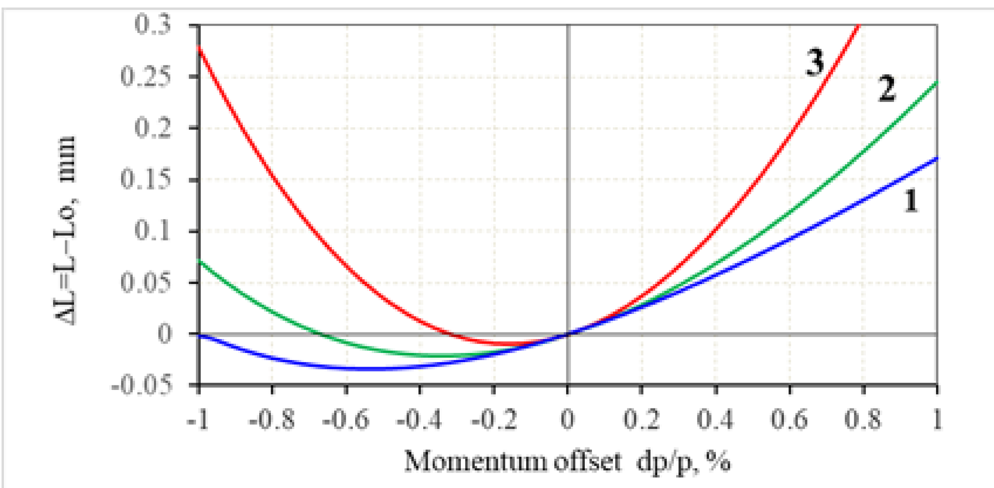


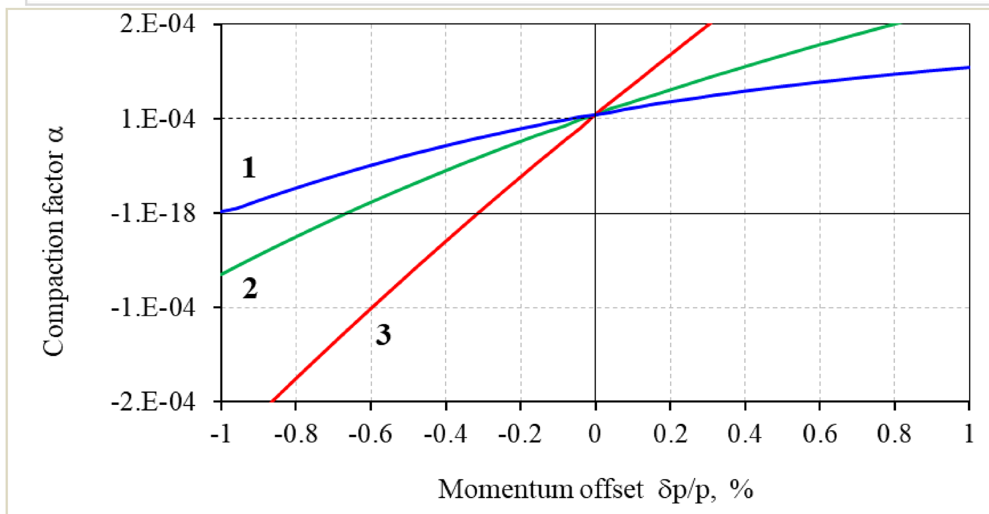
One cell of KARA lattice at low–α (α=+1.0E-4). Dispersion is stretched from +1.4 m down to –1 m in order for negative contribution to compensate positive one inside bending magnets. **Blue** curve is horizontal beta-function **βx**, **red** – vertical **βy**, green zoomed in 10 times dispersion **Do**



Courtesy of A.-S. Müller

Images of RF and α–buckets in longitudinal phase space (ϕ, δ) for chosen values of low compaction factor terms α_1=1.0·10^{-4}, α_2= 7·10^{-3} and α_3= –0.278. RF bucket corresponds to reference energy (δ_1=0) while α–buckets are shifted in energy at (δ_2=+3.6%) and (δ_3=–1.0%). α–buckets are displaced in RF phase with respect to RF bucket at π.

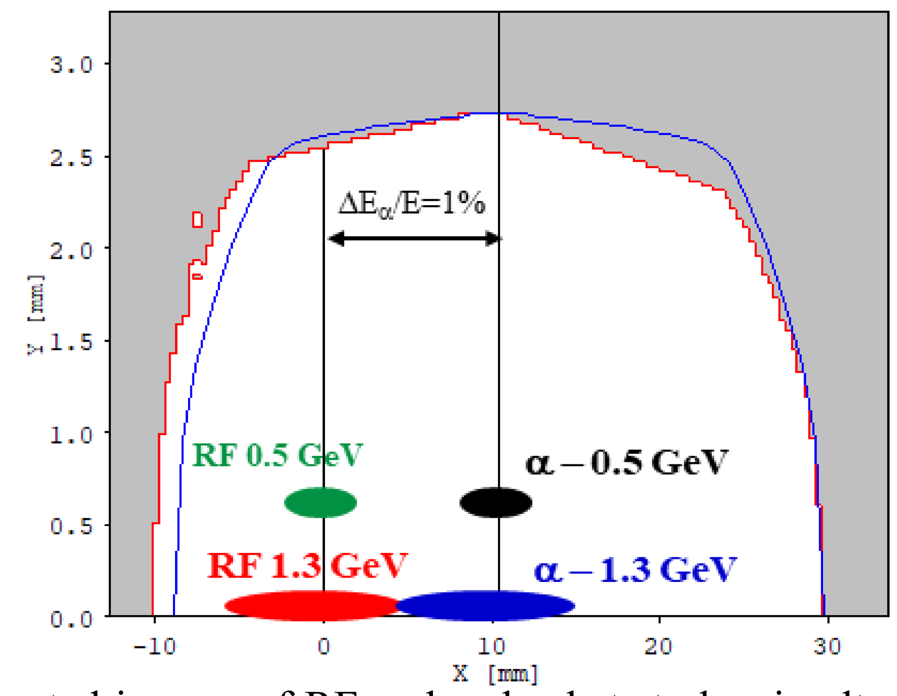




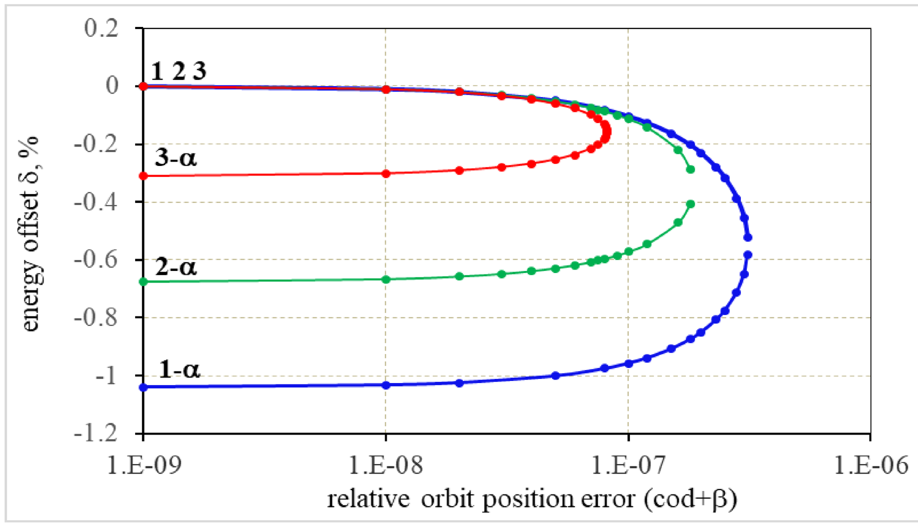
Reduction of the momentum offset of α–buckets with respect to the energy of the reference orbit at low–α mode with α_1=+1·10^{-4} by variation of second term of momentum compaction factor: (a) orbit lengthening as a function of momentum offset; (b) compaction factor vs energy deviation. Curve 1 (**blue**) corresponds to second term α_2=+7.13·10^{-3}, curve 2 (**green**) – α_2=+1.37·10^{-2} and curve 3 (**red**) – α_2=+3.26·10^{-2}.

CONCLUSION

Results of analytical studies based on second and third order equations were compared with high order computer tracking, and benchmarked against existing experiments at KARA, SOLEIL and MLS rings. We showed that for certain conditions, strong dependence of the synchrotron tune on the energy offset might limit the momentum acceptance and lifetime at low and negative–α operation. The authors are grateful to the operation, diagnostics, and power supply groups of KARA storage ring for their support during low and negative–α experiments.



Expected images of RF and α–buckets to be simultaneously stored at KARA during low–α operation where the first term of momentum compaction factor is α_1=+1·10^{-4}. Bunches at 0.5 and 1.3 GeV are split in the vertical plane for clarity. Dispersion D≈–1m at observation point. It was assumed that momentum offset between RF and α–buckets is ~1%.



Energy offsets of α–buckets in the presence of momentum independent coherent orbit errors (χ). Positive low–α optics of the KARA storage ring with first term α_1=+1·10^{-4} was chosen as an example. Curves marked in **blue** represent RF (1) and α–buckets (1-α) with second term α_2=+7.13·10^{-3}, curves marked in **green** – RF (2) and α–buckets (2-α) with α_2=+1.37·10^{-2} and curves marked in **red** – RF (3) and α–buckets (3-α) with α_2=+3.26·10^{-2}.

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