ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB279

ON WIRE-CORRECTOR OPTIMIZATION IN THE HL-LHC AND THE APPEARANCE OF SPECIAL ASPECT RATIOS

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Abstract

For the two high-luminosity insertions of the Large Hadron Collider (HL-LHC), current bearing wire correctors are intended to mitigate the detrimental effect of long-range beam-beam interactions. Two special locations corresponding to the special values 2 and 1/2 of the beta-function aspect ratio have been previously shown to provide simultaneous cancellation of multiple two-dimensional Resonance Driving Terms. This paper attempts to explain the appearance of these special aspect ratios.

INTRODUCTION

Two-dimensional amplitude-independent Resonance Driving Terms (RDT), based on the coefficients c_{pq} in the multipole expansion of the beam-beam kick delivered to the weak-beam particle have been used in [1] to describe the effects of long-range beam-beam interactions (l.r.bb, or simply bb) in the HL-LHC, and also optimise the wire correctors. An analytic formula is presented to compute the optimum parameters of the wires. The formula follows from imposing the condition for simultaneous cancellation of a *target* pair RDT (indices p_1, q_1, p_2, q_2, where $p_i \geq 0$) and produces, for two wires located left-right symmetrically w.r.t. the IP, two equal optimum parameters: integrated current and distance to the axis. These guarantee that the target pair RDT, and also the symmetric one (q_1, p_1, q_2, p_2), vanish over a single turn. Further, by varying the longitudinal location of the wire, it is found that for some special such locations many, in fact nearly all, other (*residual*) driving terms are eliminated, besides the target pair. These locations correspond to two special values of the *beta*-function aspect ratio $\frac{\beta_{x,y}}{\beta_{y,x}} = 1/2$ and 2. It has been long inferred [2] that particular values of the aspect ratio at the bb collisions, positioned the closest to the opposite beam, specific to the optics, are responsible for the occurrence of special locations. This paper, see also [3], proposes a mathematical formalism (p-norm) to explain the existence of special ratios. The results confirm the above conjecture.

WIRE CORRECTION USING RDT

We consider IR5 of the HL-LHC with nominal round-beam optics, equal emittances ϵ in both planes and, formally, $\epsilon = 1$, leading to somewhat unusual notations: $\sigma_{x,y} \equiv \sqrt{\beta_{x,y}}$, $\beta_{x,y}$ being the weak-beam beta functions; normalized separations $\psi_x = \frac{|\mathrm{D}_x|}{\sigma_x}, \psi_y = \frac{|\mathrm{D}_x|}{\sigma_y}$, where D_x is the full separation. With these, the (p, q)-coefficient [1] for the n-th bb, or either left (L) or right (R) wire corrector can be rewritten as

$$
\begin{aligned}
c_{pq}^{(n)} &= \frac{\beta_x^{(n)\,\frac{p}{2}} \beta_y^{(n)\,\frac{q}{2}}}{|\mathrm{D}_x{}^{(n)}|^{p+q}} = (\psi_x^{(n)})^{-p}\,(\psi_y^{(n)})^{-q}, \qquad (1)\\
c_{pq}^{\mathrm{w,R}} &= N^{\mathrm{w,R}} \frac{\beta_x^{\mathrm{w},R\,\frac{p}{2}} \beta_y^{\mathrm{w},R\,\frac{q}{2}}}{|\mathrm{D}_x{}^{\mathrm{w},R}|^{p+q}} \text{ and similar for L.}
\end{aligned}
$$

Here $N^{\mathrm{w,R(L)}}$ are proportional to the integrated current of a wire.

Denote with N_{bb} the number of l.r.bb on the right of IP5 (nominal value $N_{\mathrm{bb}} = 18$). Further, using the bb spacing (= half bunch distance), define the bb domain ($n \in$ bb) shown on Fig. 1, and in the same way, but with double spacing, the bb +wire domain, extending over +201.96 m from IP5, shown on Fig. 2, top. Using only lattice parameters on the right of the IP is sufficient, [1], since with the assumed exact anti-symmetry of the insertion, for fixed n these appear as L-R pairs: $\sigma_{x,y}^{(n),\mathrm{L}} = \sigma_{y,x}^{(n),\mathrm{R}}$, $\mathrm{D}_x{}^{(n),\mathrm{L}} = -\mathrm{D}_x{}^{(n),\mathrm{R}}$ and hence $\psi_{x,y}^{(n),\mathrm{L}} = \psi_{y,x}^{(n),\mathrm{R}}$. The RDT depends on two vectors of length N_{bb}. These can be chosen $\boldsymbol{\psi}_L$ and $\boldsymbol{\psi}_R$, with components $\psi_{\mathrm{L}} = \frac{\mathrm{D}}{\sigma_x^{\mathrm{L}}}$, $\psi_{\mathrm{R}} = \frac{\mathrm{D}}{\sigma_x^{\mathrm{R}}}$ ($\mathrm{D} \equiv \mathrm{D}_x{}^{\mathrm{R}} > 0$). These are different since outside the drift region $\sigma_x^{\mathrm{R}} \neq \sigma_x^{\mathrm{L}}$. Preferably, one of the vectors can be replaced by $\boldsymbol{r}$ whose components are the sigma aspect ratios $r \equiv \sigma_x^{\mathrm{R}}/\sigma_x^{\mathrm{L}} = \psi_{\mathrm{L}}/\psi_{\mathrm{R}}$. The two vectors are shown on Fig. 1, where $\boldsymbol{\psi} \equiv \boldsymbol{\psi}_L$. With all the above, the RDT becomes

$$
\begin{aligned}
\Sigma_{pq} &= \sum_{n \in \mathrm{LR}} c_{pq}^{(n)} = \boldsymbol{\psi}_L^{-p} \cdot \boldsymbol{\psi}_R^{-q} + \boldsymbol{\psi}_R^{-p} \cdot \boldsymbol{\psi}_L^{-q} = \\
&= \sum_{n \in \mathrm{bb}} \psi_n^{-(p+q)} (r_n^p + r_n^q). \qquad (2)
\end{aligned}
$$

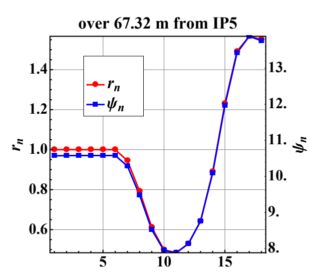

Figure 1: Values of the *sigma* aspect-ratio r_n and normalized separation ψ_n over the bb-domain in IR5 Right.

Similarly, the contribution to the RDT of two wires set at distance D^{w} from the axis, and with same effective charge $N^{\mathrm{w}} = N^{\mathrm{w,L}} = N^{\mathrm{w,R}}$ is

$$
\Sigma_{pq}^{\mathrm{w}} = N^{\mathrm{w}} (\psi_{\mathrm{w,L}}^{-p} \psi_{\mathrm{w,R}}^{-q} + \psi_{\mathrm{w,R}}^{-p} \psi_{\mathrm{w,L}}^{-q}). \qquad (3)
$$

The cancellation condition is then

$$
\Sigma_{p_1,q_1} + \Sigma_{p_1,q_1}^{\mathrm{w}} = 0, \quad \Sigma_{p_2,q_2} + \Sigma_{p_2,q_2}^{\mathrm{w}} = 0. \qquad (4)
$$

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Omitting their derivation, the solutions of (4) are

$$N^{\mathrm{w}}(r) = (A_1^{-P_2} A_2^{P_1})^{\frac{1}{P_1-P_2}}, \tag{5}$$

$$\mathrm{D}^{\mathrm{w}}(r) = \sigma_{\mathrm{w,L}}\ (A_1^{-1} A_2)^{\frac{1}{P_1-P_2}}. \tag{6}$$

One needs to know what remains of any other driving term [1]. The non-corrected (residual) RDT is given by

$$R_{pq}(r) = A_1^{-\frac{P_2-P}{P_1-P_2}}\ A_2^{\frac{P_1-P}{P_1-P_2}}(r^p + r^q) - \Sigma_{pq}. \tag{7}$$

Here $P = p+q$, $P_1 = p_1+q_1$, $P_2 = p_2+q_2$, where $P_1 \neq P_2$ and r is a continuous variable defined within the wire domain. The formulae (5) and (6) are the same as in [1]. Eq. (7) was first presented in [3]. Importantly, all three expressions Eqs. (5), (6) and (7) depend on a single function of r:

$$A_{pq}(r) \equiv \frac{\Sigma_{pq}}{(r^p + r^q)} = \sum_{n\in\mathrm{bb}} \psi_n^{-(p+q)} \frac{r_n^p + r_n^q}{r^p + r^q}\ \ (= \sum_{n\in\mathrm{bb}} V_n^P). \tag{8}$$

with the notations $A \equiv A_{pq},\ A_1 \equiv A_{p_1q_1},\ A_2 \equiv A_{p_2q_2}$. The meaning of vector $\boldsymbol{V} = (V_1, \ldots, V_n)$ is to be clarified later.

Notice that, by definition of a residual term, $R_{p_1q_1} = R_{p_2q_2} = 0$. E.g., take $p = p_1$ and $q = q_1$; then since $P = P_1$,

$$R_{p_1q_1} = A_1^{-\frac{P_2-P_1}{P_1-P_2}}(r^{p_1} + r^{q_1}) - \Sigma_{p_1q_1} = \Sigma_{p_1q_1} - \Sigma_{p_1q_1} = 0.$$

For illustration, Fig. 2, using three sample target pairs results in plots identical to the ones in [1].

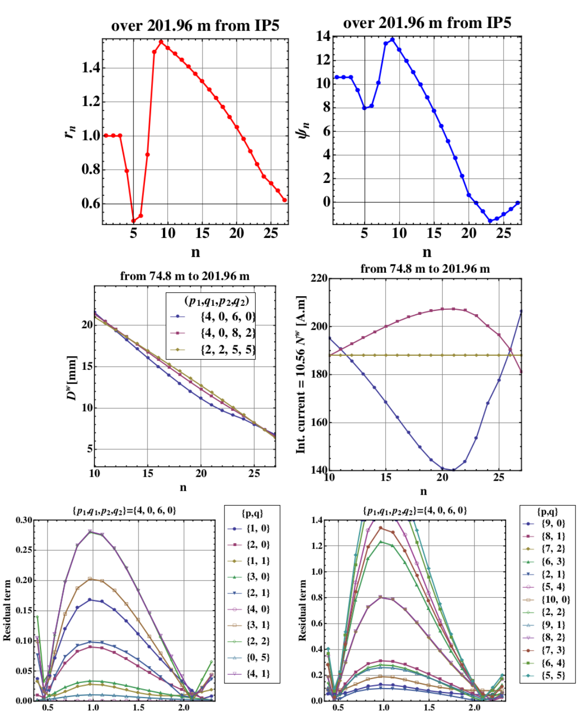

Figure 2: **Top**: Values of the aspect-ratio r_n and normalized separation ψ_n over IR5 Right. The distance from IP5 is here $S[m] = 7.48 \times n$ (a double bb-spacing is used). **Middle**: Solutions of Eqs. (5) and (6) shown over the wire domain only. **Bottom**: Relative residual driving term $R_{pq}(r)/\Sigma_{pq}$ Eq. (7) as a function of r^2 over the wire domain.

Let us recall the meaning of Eqs. (5), (6), and (7). Looking at Fig. 2, take the first target pair $(p_1, q_1, p_2, q_2) = (4, 0, 6, 0)$. Combined with any residual pair (p, q) from the bottom plot, it forms a group referred to below as a "triad' of RDT": Σ_{pq}, $\Sigma_{p_1q_1}$ and $\Sigma_{p_2q_2}$. If two wires with parameters $N^{\mathrm{w}}(r_{\mathrm{w}})$ and $\mathrm{D}^{\mathrm{w}}(r_{\mathrm{w}})$ are installed at symmetric locations at which the aspect ratio is $r = r_{\mathrm{w}}$ (and $1/r_{\mathrm{w}}$), then $\Sigma_{p_1q_1}$, $\Sigma_{p_2q_2}$ are cancelledat these locations, while the dependence of the residual Σ_{pq} on r is given by (7). It is further clear that some special locations provide solutions for both multiple target pairs and residual terms (bottom), say $n = 12$, where $r \sim \sqrt{2}$ (top, left), The questions are: **1)** why this may be true for every such triad; and **2)** is this related to the extreme, i.e. minimum and maximum values of r_n being $0.48 \sim 1/2$ and $1.56 \sim 3/2$ within the bb domain.

EQUATION FOR THE SPECIAL RATIOS

Special aspect ratios r, for which the residual term may also (besides the target pair) be canceled, satisfy $R_{pq}(r) = 0$, hence from (7) and (8), these are roots of the equation

$$A_1(r)^{-\frac{P_2-P}{P_1-P_2}}\ A_2(r)^{\frac{P_1-P}{P_1-P_2}} = A(r). \tag{9}$$

Notice the identity $-P_1\frac{P_2-P}{P_1-P_2} + P_2\frac{P_1-P}{P_1-P_2} = P$. By using it, it can be shown that 1) if r is a solution, then $1/r$ is also a solution; 2) for a single bb collision occurring at a flatness r_n, the solution is $r = r_n$ and inverted (same flatness at wire as at the bb).

Assume that there exist r such that Eq. (9) is fulfilled for *any* triad. Since this Eq. (9) depends only on $A(r)$, this function must have some special property. To find it, introduce, besides $P = p + q$, the difference $M = p - q$ (a measure of coupling). Here $P \geq M$. We are now looking for r such that Eq. (9) is fulfilled for arbitrary both P and M.

Notice that (9) will be true for any P, if A is of the form $A = S^P$ (and $A_1 = S^{P_1}$, $A_2 = S^{P_2}$). This again follows from the above identity. In such case, vector $\mathbf{V}$ satisfies:

$$\sum_{n\in\mathrm{bb}} V_n^P = S^P;\ \ V_n \equiv \psi_n^{-1}\left(\frac{r_n^{(P+M)/2} + r_n^{(P-M)/2}}{r^{(P+M)/2} + r^{(P-M)/2}}\right)^{1/P}, \tag{10}$$

since $p = \frac{1}{2}(P+M)$ and $q = \frac{1}{2}(P-M)$. Thus the above S turns out to be the p-norm [4] of the vector $\boldsymbol{V}$:

$$S = \left(\sum_{n\in\mathrm{bb}} V_n^P\right)^{1/P}. \tag{11}$$

Next, since M must also be arbitrary, S should not depend on M. Require that the derivative of S over M is zero $\frac{\partial}{\partial M}\left(\sum_{n\in\mathrm{bb}} V_n^P\right)^{1/P} = 0$. Using the chain rule:

$$\frac{\partial S}{\partial M} = \frac{1}{P\ S^{1-\frac{1}{P}}} \sum_{n\in\mathrm{bb}} \frac{d}{dM} V_n^P = \frac{1}{S^{1-\frac{1}{P}}} \sum_{n\in\mathrm{bb}} V_n^{P-1}\frac{dV_n}{dM} = 0.$$

When $M < P$ (p, q are positive integers), then V_n^{P-1} is almost independent of n and can be taken outside the sum:

$$\sum_{n\in\mathrm{bb}} \frac{dV_n}{dM} = 0. \tag{12}$$

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB279

The signs of $\frac{dV_n}{dM}$ are important as for some n they may cancel, Fig. 3. By substituting here the derivatives of V_n:

$$\sum_{n\in \mathrm{bb}} \frac{1}{2P\psi_n}\left(\frac{(r_n^{-\frac{1}{2}(M+P)}r^{\frac{1}{2}(M-P)}(r_n^M+1)}{r^M+1}\right)^{\frac{1}{P}}\times \left(-\frac{r_n^M-1}{r_n^M+1}\log r_n+\frac{r^M-1}{r^M+1}\log r\right)=0. \qquad (13)$$

In the limit $P\longrightarrow\infty$, Eq. (13) becomes:

$$\sum_{n\in \mathrm{bb}} \psi_n^{-1}\left(\frac{r_n^M-1}{r_n^M+1}\sqrt{r_n}\log r_n-\frac{r^M-1}{r^M+1}\sqrt{r}\log r\right)=0. \qquad (14)$$

The only two roots are easily found numerically. Assume for a moment that P and M are arbitrary and >0. Fig. 4 left, shows that the roots reach the large-P limit at $P\approx M$. For $P>M$, both roots are independent of P, but their value at saturation value exhibits, for large M, a small shift or spread, which is larger for the upper root (near $\sqrt{2}$). This agrees with what is seen on the bottom plot of Fig. 2.

Weight Function

At least for the optics considered, the special roots are nearly independent of ψ_n and by further ignoring the square root (≈ 1), the large-P limit Eq. (14) can be replaced by

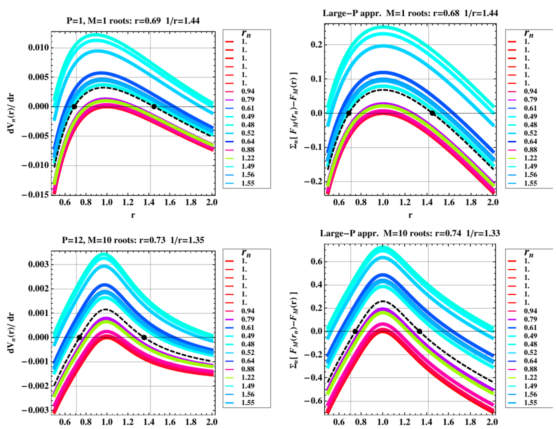

Figure 3: Plotted are the 18 terms in Eq. (12) versus r and their average: sum/18 (black dashed curve). This curve is seen to be at zero near $r\approx 1/\sqrt{2}$ and $\sqrt{2}$, which fact is marked with black dots. Left: Eq. (13), Right: Eq. (15).

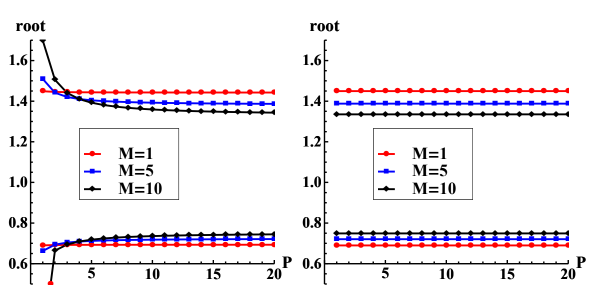

Figure 4: Solutions of the exact Eq. (13) (left) and the approximate Eq. (15) (right). Only the range $P\geq M$ is meaningful.

$$\sum_{r_n\neq 1} F_M(r_n)=N_{bb}F_M(r),\ \text{with}\ F_M(r)\equiv\frac{r^M-1}{r^M+1}\log r. \qquad (15)$$

Here still $N_{bb}=18$, but since $F_M(1)=0$, the sum is is in fact only over the “flat” collisions. F_M has the meaning of a *weight function*: r is such that the value of F_M at r equals the average contributions of all collisions taken with weights $F_M(r_n)$. It shows how much the flatness at the beam-beam collision contributes to the root. It depends on the coupling parameter M.

Subsets of Long-Range Collisions

A fundamental property of the p-norm is that for large P it can be replaced by the maximum (modulo) of its elements:

$$\lim_{P\to\infty} S=\max_{1\leq n\leq 18}|V_n|, \qquad (16)$$

This suggests that the sum Eq. (12) may be well represented by appropriate subsets of n. These would produce the same roots as all the 18. An indication is that on Fig. 3 there are three well separated groups of curves: top, mostly corresponding to $r_n\sim 1/2$, middle – $r_n\sim 3/2$, and bottom – the overlapping red ones for $r_n\approx 1$. Such subsets indeed work and they turn out to be a mix of several round-beam l.r. bb with l.r. positioned near its extremes over the bb domain: $r_n\approx 1/2$ or $r_n\approx 3/2$ (see Fig. 1). The condition found is "two round l.r. per flat one" – Fig. 5.

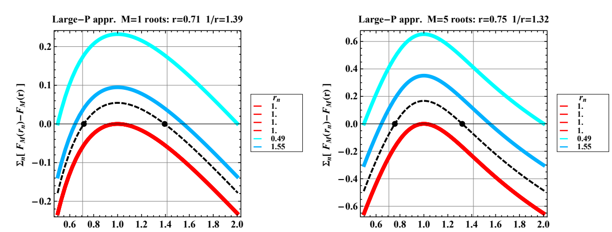

Figure 5: Sample subset containing four round ($r_n=1$) and two flat ($r_n\neq 1$) l.r.bb collisions with $r_n\approx 1/2$ and $3/2$.

CONCLUSIONS

The findings in [1]: the two locations at which multiple driving terms are cancelled correspond to the magic values$\sim\sqrt{2}$ (and inverted) of the sigma aspect parameter r are confirmed and explained with properties of the p-norm. It is further found that: **1)** depending on how far r is from the magic value, low- and high-order terms are canceled to a slightly different degree because of the spread occurring for large $p-q$ (weaker coupling), Fig. 4. Namely, higher order terms are better cancelled a little below $\sqrt{2}$ and a little above $1/\sqrt{2}$. The spread is larger for the location with $r\sim\sqrt{2}$ (beta aspect ratio ~ 2), as also observed in [1]. The other location, near Q5, is therefore preferable. **3)** For the optics considered, the magic values can be explained with the min/max values of r in the beam-beam region being $\approx\sqrt{2}$ and $\approx 3\sqrt{2}$.

This work would not have been possible without the discussions with S. Fartoukh and his critical comments. The author is thankful to Y. Papaphilippou and R. Baartman for the discussions and support.

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